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DESIGNING FOR INSTRUMENTALIZATION: CONSTRUCTIONIST PERSPECTIVES ON INSTRUMENTAL THEORY

Chronis Kynigos* & Giorgos Psycharis**

*Educational Technology Lab, School of Philosophy, University of Athens, Greece
**Department of Mathematics, University of Athens, Greece

Abstract. In this paper we aim to address the problem of fragmentation of theoretical frameworks within the field of technology enhanced mathematics while exploring the dialectic between constructionist tool design and students' instrumental genesis. We report findings from design-based research aiming at shedding light on the meanings about angle in 3d space generated by 13 year olds while controlling and measuring the behaviours of angular constructions with the use of a specially Turtle Geometry with dynamic manipulation microworld. The analysis indicates that the combined use of constructionism and instrumental theory enhances our efficiency to explore the links between tool design and students' learning in technology-rich environments designed to favour instrumentalisation.

1. INTRODUCTION

The goal of the ATATEMLO symposium is "to share and integrate competences, methodologies and ideas of researchers in the field of Activity Theory (AT) and mathematical teaching and learning, in order to better address issues related to orchestrations of mathematics learning with technology." (Symposium ATATEMLO, Call for proposals, p.1). The operationalization of theoretical constructs within the AT framework is central in this process. Recent reflections amongst researchers, however, have expressed concern around the fragmented and unconnected emergence of constructs and frameworks, each inevitable bound to the context within which it was produced and used. Researchers have recently highlighted the need to find ways to generate connections amongst frameworks and constructs, to enhance their explanatory power and the efficiency of communication of research results emerging from studies taking place in different research and didactic contexts (Artigue et al., 2009).

Along this line of thought, our contribution to the symposium concerns an attempt to discuss possible connections between constructionism and instrumental theory. These two frameworks which emerged through the study of ways technology can be designed and used to enhance mathematical learning have different historical roots and were developed in different research cultures: constructionism (Harel and Papert, 1991) was developed as a vision of using digital media to generate leaning by doing cultures and was based on the notion of digital microworlds originally associated with expression of mathematical ideas through programming with Logo-based Turtle Geometry while the French ergonomic approach (i.e. instrumental theory, Verillon & Rabardel, 1995, Artigue, 2002) was developed in the direction of AT over a series of research studies concerning the didactic exploitation of CAS. Recent reflections upon the constructionist vision (Healy & Kynigos, 2010) indicate that constructionist approaches for interpreting students' interactions within the evolving affordances of computational settings have been moved over the years from a primary interest in the individual learner to a growing focus on the situated and embodied nature of mathematics learning in different communities of practice. This direction has taken the notion of constructionism into to realm of AT. Thus, a fruitful line for further integrative theoretical work has been established for situating constructionism in the light of contemporary AT frameworks in the field. However, a prime characteristic of constructionism is a specific epistemology for mathematics and mathematical learning emphasizing mathematics as a fallible discipline and the generation of mathematical meaning as the main stance for the nature of learning mathematics.

In the present study we reflect on the extent to which connections can be found between the essence of instrumental genesis within the AT framework and the possible uses of constructionist approaches to learning with digital media. We report findings from design-based research aiming at shedding light on the meanings about angle in 3d space generated by 13 year olds while engaged in controlling, measuring and changing virtual perspectives of their angular constructions with a specially designed Turtle Geometry with dynamic manipulation microworld. We describe our study where we adopted some aspects of instrumental theory and used them at a practical level for analyzing how students' intuitions and ideas concerning angle as a spatial visualisation concept were challenged as they worked with a set of activities we designed under a constructionist theoretical perspective (Kafai & Resnick, 1996, Kynigos, 2007). The analysis revolves around the mutual transformation of learners and tools with an emphasis on the links between students' conceptualizations of mathematical knowledge and tool use in particular learning situations. We pay particular attention on the dialectic between design objectives envisioned by the designer of an artifact for student learning and learners' situated activity in a particular computer-based educational setting.

2. THEORETICAL FRAMEWORK

Instrumental genesis, the process through which an artifact is transformed to an instrument, provides a powerful framework for analysing student learning in computer-based settings. It allows to describe student-tool reciprocal change as a process leading to more successful integration of the tool in students' practices for the accomplishment of particular tasks. In a similar way, a key consideration of constructionism is the two sided relationship between tool and learner. Constructionist research approaches concerning the educational use of artefacts in mathematics focus primarily on student's constructions of mathematical meanings as abstractions (i.e. *situated abstractions*, Noss & Hoyles, 1996) emerging in activity and the ways in which these structure and are structured by the use of available tools.

We consider this common attention to the reciprocal relationships between tools and thinking as providing a basis for networking the two theories so as to better address issues linked to design, implementation and analysis of research studies in technology enhanced mathematics. If we consider the instrumental approach in relation to constructionism, we might say that the term "shaped by tools" concerns "instrumentation" and "shaping tools" "instrumentalisation". In our view, an issue that needs further investigation is related to the fact that instrumental approach was developed through the educational exploitation of CAS whose design was given to researchers by CAS designers (Monaghan, 2007). Thus, the issue of design was not a point of attention per se in the first steps of the development of the ergonomic approach. This might explain in part the fact that most of the analyses of mathematics learning with digital tools is primarily centered on instrumentation rather than instrumentalisation (Healy & Kynigos, 2010). But, tool design does matter both at the didactic and the research level since it represents a point of intersection between an artifact which embodies an objective envisaged by the designer(s) and the instrument emerging from students' activity in carrying out particular tasks (White, 2008). It has recently been indicated that there is also a dialectic relationship between epistemology and tool design according to which the theoretical foundations of the designers' choices and the evolution of meaning-making based on these choices can co-emerge (Pratt & Noss, 2010).

As far as constructionism is concerned, the constructionist emphasis on learning as design suggests a rather particular stance on instrumentalisation. According to this, constructionist computational environments incorporate the idea of instrumentalisation by design, i.e. not seen as finished or ready-made tools but developed to invite changes by the student (see for instance the idea of *half-baked microworlds*, Kynigos, 2007). Thus, investigating the process of shaping the tool in ways that also influence their potential – built-in – uses is at least as important as focussing on instrumentation as far as understanding the relationships between tool design and mathematics learning is concerned.

This contribution reports classroom research aiming to explore 13 year-olds' construction of meanings for the notion of angle as a spatial visualisation concept during the implementation of a set of activities activity involving the construction and manipulation of programmable geometrical figures. The students worked in collaborative groups of two using *MaLT (MachineLab Turtleworlds)*, a microworld environment for geometrical constructions which combines a 3d Logo Based Turtle Geometry with (a) dynamic manipulation of procedure parameter values and the resulting constructed figures and (b) dynamic manipulation of the students' viewpoint inside the simulated 3d space containing the constructed objects (Kynigos et al. 2009, Latsi & Kynigos, 2011).

Angle can be defined from at least three different perspectives: (a) angle as a geometric shape, i.e. formed between two geometrical objects embodying directionality which can be either segments or 2d geometrical figures (b) angle as a dynamic notion, indicating a change of one direction both as a turn and as the result of a turn; and (c) angle as a measure represented by a number (Henderson & Taimina (2005). It is clear from previous research that students have great difficulty in coordinating the different facets of the angle embedded in various physical angle contexts involving slopes, turns, intersections, corners, bends, directions and openings (Mitchelmore & White, 1998). Traditionally, teaching of angles at school revolves around static approaches of angle while the notion of angle as turn is usually underrepresented although it is considered the most natural, the most instinctive aspect of angle (Freudenthal, 1983). Even in cases where angle is approached as turn this is done only through static 2d representations, which, no matter how cleverly designed, may delay the development of dynamic aspects of the concept and their integration with the static ones (Clements et al, 1996). In the reported experiment our purpose was (a) to relate students' construction of meanings for the notion of angle in 3d space explicitly to their physical angle experiences and (b) to throw light on the paths by which students might come to integrate their various angle concepts in 3d space when they have specially designed tools at the disposal allowing the construction and dynamic manipulation of 2d geometrical objects and observation of objects from various perspectives. We also used the idea of half-baked microworlds as means to foster creative instrumentation and instrumentalisation processes from the part of the students. A central part of our analysis concerns if and how such an approach with a strong design element might contribute to productive instrumental genesis.

3. THE COMPUTER ENVIRONMENT

MaLT is a programmable environment designed to extend Turtle Geometry to 3d by adding particular kinds of turn commands and providing tools for the creation and exploration of interactive 3d geometrical constructions. It consists of the following interconnected components (see Figure 1).

• Turtle Scene (TS). This is a 3d grid-like projection of a simulated 3d space in which a 3d turtle is visualised when a command (or a procedure) is executed. Whenever the turtle moves it leaves behind it a trace which is a selectable 3d cylindrical line.

• Logo Editor (LE). This is the Logo-like programming interface which is linked to the TS. The user is able to write, run and edit Logo procedures to 'drive' the construction of geometrical objects by the turtle using a specially designed version of Logo. A distinctive feature of this specific version is that it provides an extension of Logo commands in 3d space including two new types of turtle turns (Reggini, 1985): 'UPPITCH/DOWNPITCH n degrees' ('up/dp n') which pitches the turtle's nose up and down on a plane perpendicular to the one defined by right-left turns and 'LEFTROLL/RIGHTROLL n degrees' ('lr/rr n') which moves the turtle around its own axis.

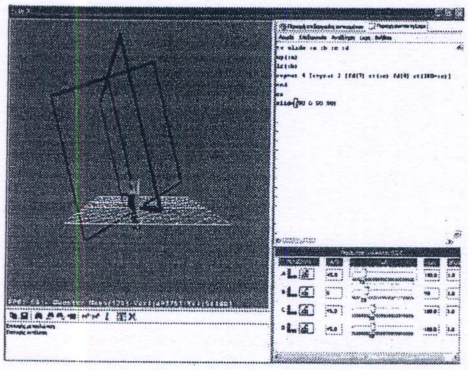

Figure 1: A revolving door simulation in MaLT.

• Variation tools which recognise the procedure responsible for any figural construction and afford dynamic manipulation of variable values resulting in DGS-style change in the figures. Figure 1 shows the Uni-dimensional Variation Tool (1dVT) whose main part consists of 'number-line'-like sliders (see at the bottom of Figure 1), each corresponding to one of the variables used in a Logo procedure. After a variable procedure is defined and executed with a specific value, clicking the mouse on the turtle trace activates the tool, which provides a slider for each variable. Dragging a slider has the effect of the figure dynamically changing as the value of the variable changes sequentially. Another feature also affords dynamic manipulation but this time what is changed is the users' viewpoint of the Turtle Geometry space (a) by a toggle fashion by using buttons to pick among 3 default views (front, side, top-down) (Fig. 3) and (b) by dragging a specially designed vector tool, which we called 'the active vector', where the user can define the camera's direction or position.

4. RESEARCH SETTING AND TASKS

The work reported in this paper is part of a design-based experiment (Cobb et al., 2003) concerning the exploration of the mathematical nature of angles in 3d space by groups of 13-year-old students who work collaboratively in the classroom setting. The experiment took place in a secondary school with one class of twenty 7 grade students (13 year-old) and one experimenting teacher who acted also as a researcher in the classroom. The class had totally 18 teaching sessions with the experimenting teacher over two months. The activity sequence was divided in two phases with a strand of two tasks each. In designing the tasks for the first phase of the activity we took into account the constructionist theoretical implication that angle as turn (which inevitably involves directionality) in the simulated 3d space of MaLT is closely related to real world experiences as well as to body-syntonic activities such as walking or piloting a flying object. In task 1 (phase 1) students were asked to move the turtle in the right and left corner of the 3d space and then to bring it back at its initial position. It was not specified on purpose what was meant by 'the left and right corner', so as to leave students to explore 3d space and develop their own 'navigation' strategies. In task 2 (phase 1) students were asked to navigate the turtle in such a way so as to simulate the take-off and the landing of an aircraft. In the second phase of the activity angle was approached as a dynamic quantity through the simulation of real 3d objects in the TS. In particular, in task 3 students were asked to construct rectangles using parametric procedures in at least two different planes of the TS simulating the windows of a virtual room. In task 4 students were asked to develop a parametric procedure so as to simulate the opening and closing of a door. It was expected that students would reflect on the previous experience of creating rectangles in different planes and connect it to the turn commands ('LEFTROLL/RIGHTROLL n degrees). We were interested to see if and how students might see 'the need for turning' the rectangles and in which ways they might integrate the turn commands in their new procedures.

```
to slide :a :b :c :d
up(:a)
lr(:b)
repeat 4 [repeat 2 [fd(7) rt(:c) fd(4) rt(180-:c)] lr(:d)]
end
```

Figure 2: The Logo code of the 'slide' half-baked microworld.

In task 5 students experimented with one half-baked microword (Kynigos, 2007), i.e. they were given a 'buggy' procedure and were asked to experiment, figure out what was wrong or superfluous in the code and correct it. In particular, in task 5 students were asked to use the 1dVT to control the four variables in the procedure 'slide' which corresponded to turtle turns so as to create the simulation of a revolving door with four rectangular flaps. The procedure was designed to have more than the variables needed. Students had firstly to identify the role of each variable and which values could be given to them. Thus, they had to make changes to 'slide' so as to develop a procedure that creates the simulation of a revolving door with the least possible variables.

5. METHOD

During the implementation of the activity a second researcher participated in the classroom acting as a co-researcher. The adopted methodological approach was based on participant observation of human activities taking place in real time. We used a screen capture software (i.e. Hypercam) which allowed us to record students' voices and at the same time to capture all their actions on the screen. For the analysis we transcribed verbatim the audio recordings of three groups of students (focus groups) throughout the teaching sequence and we also selected significant learning incidents from the work of all groups in the classroom. Background data were also collected (i.e. observational notes, students' electronic and written works). In the analysis a phenomenological approach (Nemirovsky & Noble, 1997) was adopted according to which researchers' interpretations were focused on the student's interactions with the available representations and the meanings they attached to these from their own points of view while struggling to relate the turtle's move/turn and/or specific 2d representations in forming angular relationships in 3d space. The episodes were selected (a) to have a particular and characteristic bearing on the students' interaction with the available tools and (b) to represent clearly aspects of the students' construction of meanings for particular aspects of angle in 3d space (i.e. geometric shape, dynamic amount and measure) emerging from this use. The results presented here are based on the work of two groups of students.

6. ANALYSIS

6.1 Instrumenting 'perspective taking' in 3d space

At this paragraph we highlight meaning generation for angle at different phases of students' engagement in instrumenting their perspective of graphical objects in TS. More in particular, the analysis signals the students' shift from driving the turtle having a particular fixed perspective of the TS and the graphical outcome to controlling their views of the 3d space and the turtles' moves through their engegement in a process of instrumenting the available artifacts. The following episodes have been reported in different papers up to know (Kynigos et al., 2009, Latsi & Kynigos, 2011). In the present analysis the instrumental aspects of students' activity serve as an opportunity to examine students' construction of meanings based on the use of artifacts amenable to instrumentalisation.

During task 1 students have constructed simple crooked lines limiting turtle's motion either on the 'ground plane' or on the 'screen plane' (vertical to the ground plane) while their experimentation concerning the notion of angle was developed around how to find out the measure of turtle turns. Their preference in moving in a 2d plane was tightly linked to their difficulty in conceptualising the role of the rotation commands - especially rightroll/leftroll – while moving the turtle around in the TS. However task 2 seemed to have provided a more fruitful context for the experimentation of students with both new types of turtle turns. It seemed that the simulation of the take-off and landing of an aircraft brought in the foreground the concept of angle as a turn in 3d space with particular measure involving also angle as a slope, an aspect of angle difficultly recognised by students as the one supporting edge is missing (Mitchelmore & White, 2000). As the students were engaged in navigating the turtle so as to simulate the take off of an aeroplane, in the initial stages of exploration students appeared not to focus on the change of planes or the angles (internal or external) of the crooked line that the turtle had drawn but rather at the angle that was drawn by the turtle in relation to the horizontal ground plane. This is evident in the following excerpt in Group A students' confusion over the way in which the commands up(45) and lt(50) affected/determined the generated graphical outcome visualized in Figure 3. These students were reflecting upon the commands given to the turtle so as to explain why the aircraft collided to the ground (Figure 3).

Researcher: Hey, nice take off!! I see you hit the ground!
Student: Look there is a slope up(45) and then a slope of lt(50).

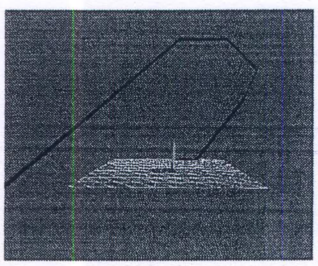

Figure 3: Simulating the taking-off and flying of an aircraft.

```
fd(5)
rt(90)
fd(2)
up(45)
fd(5)
lt(50)
fd(6)
rr(30)
up(90)
fd(6)
up(45)
fd(8)
```

A closer look at the respective Logo code reveals that students, drawing upon their everyday experiences, considered the horizontal ground plane and the directions of up and down as fixed. This may be a possible explanation of their insistence to use the up command in order to get height regardless of the opposite graphical outcome. Apart from the lack of previous familiarisation with the new kinds of turtle turns, at the instrumentation level the preference of students to move the turtle only in one plane (during task 1) or their difficulty in coordinating turtle's turns and trace with the notion of angle as a slope could possibly be interpreted in the light of the fact that students, accustomed to work with 2d representations of geometrical figures, might have had difficulties in understanding either the conventions used to represent a 3d object in the computer screen (Lowrie, 2002) or the conventions that underlie the move of an entity – here the turtle – in a simulated 3d space. Thus, at that time the students seemed to instrumentalise the turtle's turn by developing schemes in accordance with their earthly experiences and the established 'up' and 'down' directions linked to the gravitational effect. The students' difficulty in applying the above mentioned set of commands in an intrinsic way indicates also that students' instrumentation of perspective taking at that time appears strongly interrelated to body-syntonicity. Progressively however the information they gleaned through their experimentation and the provided feedback helped the students to explore other artifacts for taking various views of the simulated 3d space so as to explain/justify the graphical results of specific turtle's turns.

The next episode finds this group of students engaging in investigating the role of the :d variable, which determines the measure of turtle's turning and respective position in the 3d space before drawing each successive door of the revolving door model. But this time the students begin to make use of the artifact provided to facilitate taking different views of the TS and thus of the current geometrical construction. The students conjectured about the number of the visible rectangles (doors) if the value given to :d is 720. However they did not find the front default view convenient and after testing all the available default views they chose to continue working with the top-down view active, where the doors/rectangles created by the turtle were more clearly visible.

S1: Lets see how many doors there are if the value is Only one? This perspective is not convenient, I will change it (*He activates successively all the 3 default views and opts for the top –down one*).

S2 Yes, exactly like in the case of 360. It turns two rounds.

S1: Yes, it collects all of them in one. When we move it, the doors are changing position. They are sticking together or they are unsticking.

S2: We can't say that. With d we determine their place. Look, If it is 90° they are turning and they are forming a cross, they form right angles, yes right angles, with 360° or 720° they are placed together in the same line.

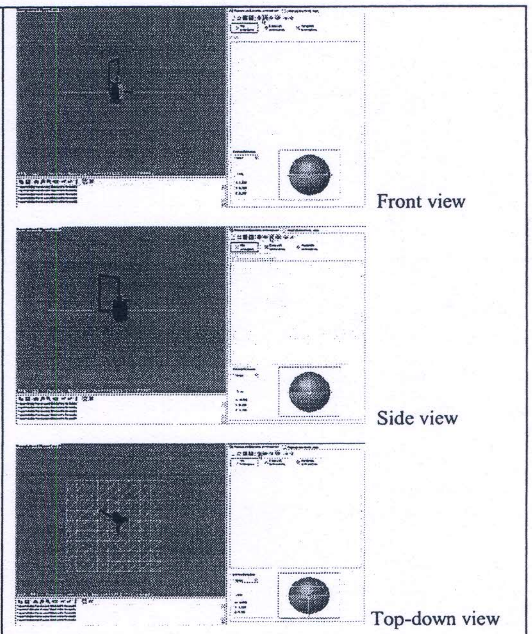

Figure 4: Episode 2 and the 3 default views of the 3d simulated space in MaLT.

These excerpts suggest that the group's instrumentation of the default views of the simulated 3d space hinged on the visualization of the various turn commands descriptors of evolving geometrical objects' place and orientation in 3d space. To be able to support their emergent explanations for the graphical outcome in relation to the measure of a leftroll turn command students chose the top – down view since it allows them to observe the turtle's rolling and to specify the number of the drawn rectangles. In this view the dihedral angle between the parallelograms is rather more easily discernable as it is closer to students' representation of 2d geometrical figures in the traditional setting. At an instrumentation level, students enacted certain assumptions about the 'input-output' correspondence between specific measures of turtle's turning and the position of the four constructed rectangles around the x-axis. More in particular, students were able to coordinate turtle's turning – one round for 360° and two rounds for 720°- with the static geometrical figure of 360° and 720° angle where the position of the two rays that form the angle coincide. These assumptions such as "with 360° or 720° they are placed together in the same line" are situated abstractions guiding this emergent scheme through which the students worked to more effectively orient their instrumented actions toward the purposeful use of the default views provided by the software. We highlight this episode in order to show the evolution of students' perspective taking towards the graphical outcome and the turtles' moves as situated in a larger process of instrumenting the available tools for specific default views.

6.2 Instrumenting dynamic manipulation through the simulation of 3d objects

The episodes in this paragraph are taken form the work of another group of the same classroom (Group B). These episodes will be used to detail the use of 1dVT as an integral part of students' attempts to achieve the simulation of 3d objects encountered in everyday situations (e.g. doors, revolving doors). From an instrumental perspective, dynamic manipulation through 1dVT constitutes one way that allows students take control of the construction, animation and perspective taking of 3d geometrical objects in MaLT.

In task 3 the need to design figures in different planes of the 3d space challenged students to move the focus of their attention from directed turns between lines and planes to directed turns between two similar geometrical figures which is related to the conceptualisation of a dihedral angle in 3d space. For example, most groups of students recognised that in order to construct windows in two consecutive walls (planes in mathematical terms) the use of the commands 'rr/lr' or 'up/dp' was needed. During this construction process students easily identified the dihedral angle defined by the two consecutive windows (i.e. rectangles) and used the terminology familiar to them from 2d geometry in order to describe it. However, some of the groups of students had difficulties in identifying its measure. For instance, students of group B characterized the dihedral angle drawn by the turtle as an acute and not as a right one as it was the case, although they had commanded the turtle to leftroll 90 before drawing the second window. It seemed that these students had focused more on the visual characteristics of the figural representation and were confused by the 'distortion' of the dihedral angle as a result of the use of a vanishing point in the line of horizon of the TS designed to strengthen the sense of depth in the representation.

As we saw in the previous paragraph group A students exploited the available affordances for the different views of TS in order to 'solve' a rather similar problem of visualisation in 3d space. However, in this case the use of 1dVT allowed these students to view their constructions from different perspectives which might have minimized the 'distorting' effects of static 3d representation and prompted them to focus on the measure of the turtle's turn in the Logo code. The more the students appeared accustomed to the conventions used in the 3d simulated space the more they were able to coordinate the visual characteristics of the drawn angles with their measure related to turtle's turns. For instance during the dynamic handling of the revolving door simulation (task 5) the students of group B were able to overcome the difficulties faced earlier during task 3 and to recognize the four consecutive right dihedral angles created between the four rectangles around the common vertical side of the four rectangles (see Figure 1). Experimenting with the variables of the procedure 'slide' (see Figure 2), which was given ready-made to them, so as to create a revolving door moving around group B students progressively got able to handle different aspects of angle simultaneously. In the next episode students had constructed four parallelograms around the x–axis by placing slider :d on the value 90. In the next excerpt S1 recognized the need to construct four rectangles by dragging on the slider :c on the 1dVT.

S1: Wait, we should move it here first. It's the angle of the rectangle [*moves the slider :c to the value 90 so as to construct 4 rectangles*], so as to become like this (*i.e. the door*) and then probably turns with this [*moves the slider :b*]. Let's see…
S2: Yes, it definitely turns around with this [*i.e. slider :b*] as it has lr.
S1: Yes, but we don't only want it to turn, we also want it to move even further down.
S2: I should change here [*He puts the slider :a to the value 90 so as to have the simulation in a vertical position*].
S1: Yes, 90 is fine.
S2: Now, with this [*points to the slider :b*] it turns around normally.

These successive articulations of the students' attempts to normalize (see Psycharis & Kynigos, 2009) the buggy model of a sliding door by dragging on the variable sliders, reveal steady refinements in the group's instrumentalization of the variation tool. These students manipulation of the variation tool in relation to the Logo code reflects the purposeful way in which the computational setting provided a web of structures (Noss & Hoyles 1996), which students could exploit in shaping the available resources to specify angular correlations based on particular values of each variable. The meaning of the available tools emerged only through the course of their application to a specific task, namely the group's effort to determine the angular relations underlying a 'correct' model of a sliding door so as to prevent the distortion of the shape and confirm that the dynamic rolling around of the door around the x axis actually works. To achieve this students' instrumentation of the available tools hinged in particular aspects of angle as a spatial 3d concept. More in particular, the above excerpt suggests that students created meanings in relation to angle (a) as a constitutive element of a figure which is defined and stay fixed (variable :c), (b) as a means to move from the horizontal plane to the vertical one in relation to the viewing axis of the user which is again defined and stay fixed (variable :a) and (c) as a means of constantly changing planes in 3d space (variable :b) around the common vertical side of the four rectangles (Figure 1).

7. CONCLUSIONS

Our concern was the fragmentation of theoretical frameworks within the field of mathematics education and its negative consequences in terms of appreciating research results and validating the constructed knowledge. Being interested in situating constructionism in the light of instrumental theory we focused on the dialectic between tool design and students' situated activity in the classroom. We particularly addressed some of the meanings generated by the 13 year olds as they interacted with multiple interlinked representations of angular concepts in 3d space. Under the constructionist theoretical perspective the above incidents illustrate the dialectic relationship between the evolution of instrumental genesis and student's progressive focusing on angular relationships underlying the current geometrical constructions and representations. In the first cluster of episodes, students' intrumentalisation of the available default views of the simulated space indicated their evolving success in using these resources as meaningful problem solving tools with greater utility for coordinating the figural results of the turtle's turning in 3d space. In the second cluster of episodes, students' struggling with the representational convention used and the identification of the angles corresponding to turtle's turning lead them to progressively approach angle as a dynamic entity for simulating 3d objects through the use of variables. This kind of experience was critical for the coordination of the various facets of the angular relations simultaneously present in the subsequent revolving door simulation where angle was approach either as a geometric shape with a fixed measure (e.g. between 2d geometrical figures) or as a dynamic amount with a varying measure. Thus, constructionist focus on design seemed to add a productive set of tensions to the instrumental dialectic—namely between the design choices at the level of educator-designers of classroom tools and the students goal-oriented use of those tools. It is indicated that technology-rich environments designed to favour instrumentalisation require evaluation of their design features and constrains in the light of empirical investigation based on the combined use of different frameworks. The approach adopted in the present study is clearly a first step towards intertwining constructionism and instrumental theory for addressing how tool and activity design in technology enhanced mathematics could enrich co-development of students and tools in particular learning environments.

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