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Critical incidents as a structure promoting prospective secondary mathematics teachers' noticing

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This study builds on the idea of using "critical incidents" as a tool for inquiry and reflection in the context of mathematics teacher education. The analysis was based on 22 prospective teachers' portfolios reporting and interpreting selected critical incidents on the basis of their observations of mathematics teaching conducted by other teachers and by themselves in the context of their field experiences. The critical incidents addressed a multiplicity of issues related to mathematics teaching and learning. Prospective teachers' noticing developed in terms of what and how they notice indicating a more relational way of conceptualizing mathematics teaching and learning.

Keywords: Noticing, critical incidents, prospective mathematics teachers, teacher education.

Introduction

In this paper, we study prospective teachers' (PTs') noticing of mathematics teaching in their initial field experiences through their engagement in identifying and interpreting critical incidents taken from everyday classroom situations in the context of a teacher education course. Critical incidents are everyday classroom events which have significance for the teachers, make them question their practice and seem to provide an entry for their better understanding of teaching-learning situations (Hole & McEntee, 1999). To observe and question mathematics teaching is a rather demanding task for both practicing and prospective teachers. A number of research studies have indicated that PTs face difficulties in identifying salient aspects of classroom instruction. For instance, they tend to describe the lesson as a chronological order of disconnected events (Sherin & van Es, 2005), they keep their attention primarily on the teachers rather than on the learning students (e.g., Van Es & Sherin, 2002) and they have difficulties in developing interpretative analysis of classroom instruction (Jacobs, Lamb, & Philipp, 2010).

Research suggests the need for the development of structures that foster teachers' systematic reflection on teaching practice and help to make the act of noticing critical aspects of classroom interactions more concrete (Mason, 2002). Examples of such supportive structures are: the use of theoretical tools to code teaching (Mitchell & Marin, 2015); the decomposition of video lessons in small parts (McDuffie et al., 2014); the identification of critical incidents from classroom teaching (Goodell, 2006). These structures have been exploited in situations where prospective mathematics teachers analyze teaching of others mainly through video noticing (e.g., Sherin & van Es, 2005) while few studies refer to PTs' reflection on their own teaching (e.g., Goodell, 2006). However, there is an open discussion on if and how reflecting on other teachers' practice transfers for reflecting on PTs' own practice (Stockero, 2008). Many research studies prioritize helping PTs to focus on students' mathematical thinking (e.g., Jacobs et al., 2010) while few studies aim to facilitate PTs' attention to other important features of mathematics teaching and their interrelation to students' learning (McDuffie et al., 2014). Linking students' learning opportunities to teacher's

discourse moves is a rather demanding task and it poses a research challenge in the area of prospective mathematics teachers' noticing.

In our study, we attempt to explore how critical incidents can be used as a structure to support PTs in reflecting on mathematics teaching recognizing interrelationships between teaching and learning. Our research questions are: (a) What is the nature of critical incidents that PTs identify while reflecting on mathematics teaching conducted by other teachers and by themselves? (b) How does PTs' noticing develop when identifying and interpreting critical incidents related to students' mathematical activity?

Theoretical framework

Under a community of inquiry perspective, Jaworski (2006) introduced the concept of critical alignment, in which participants align with the practice of mathematics teaching while critically questioning aspects of it. Critical alignment is promoted through the tool of inquiry. Inquiry is a process of encouraging critical reflection and promoting critical alignment (Jaworski, 2006). In this perspective, reflection is considered as a tool that allows participants to be engaged in a continual reconstitution of the practice of teaching. The reflective process involves "firstly, a recognition of questions to address, identifying some perplexity, making some aspects of teaching problematic; and, secondly, through some processes of enquiry, to seek solutions, or resolutions to, or new ways of understanding, the problems identified" (Jaworski, 1998, p. 7). This perspective is close to our view of a critical incident as a continuum involving identification, interpretation and potential action where critical questioning is a constituent element of it.

Researchers have been concerned about the introduction of sufficient structures for making the act of inquiry into teaching practice more concrete. An example of a structured framework for reflection on classroom episodes, are critical incidents. In mathematics education, the idea of critical events/moments in mathematics teaching has been used as an analytical tool in studying mathematics teaching and learning. Skott (2001) used the term "critical incidents of practice" to describe moments of a teacher's decision-making in which multiple and possibly conflicting motives of his activity evolved that challenged the teacher's own school mathematics images and provided learning opportunities for students. As a developmental tool, critical incidents have been used by Goodell (2006) in pre-service mathematics teacher education. She analyzed PTs' reports of critical incidents and she found that the issues raised concerned: teaching and classroom management; student factors; issues concerning relationships with colleagues, parents and students; and school organizational issues. She also identified that PTs fruitfully addressed important aspects of teaching for understanding such as the necessary conditions, factors facilitating teaching for understanding and barriers to teaching for understanding.

Noticing has been introduced to mathematics teacher education to study shifts in the structure of teachers' attention and, through this, to address different levels of awareness both in mathematics and in mathematics teaching (Mason, 2002). According to van Es and Sherin (2002), noticing is a more complicated action than just observing teaching. Rather, it requires teachers to notice what is significant in a classroom interaction, to interpret this noteworthy incident on the basis of their knowledge and experiences, and to link these with broader principles of teaching and learning. Van es (2011) proposed a framework for learning to notice students' thinking constituted of four levels

of noticing according to "what teachers notice" and "how teachers notice." As regards to what the teachers attend to, the four levels include: making general observations about the whole class environment (Level 1 – Baseline Noticing); focusing on teacher pedagogy and begin to attend to students' thinking (Level 2 – Mixed Noticing); attending to particular students' mathematical thinking (Level 3 – Focused Noticing); and interrelating particular students' mathematical thinking and teachers' teaching strategies (Level 4 – Extended Noticing). When it comes to how the teachers notice and provide interpretations, the four levels include: providing general impressions and descriptive comments (Level 1 – Baseline Noticing); providing primarily evaluative with some interpretative comments and beginning to refer to specific events and interactions as evidence (Level 2– Mixed Noticing); providing interpretative comments, referring to specific events and interactions as evidence and elaborating on events and interactions (Level 3 – Focused Noticing); and making connections between events and principles of teaching and learning and suggesting alternative pedagogical actions (Level 4 – Extended Noticing). This framework provides a base for teacher reflection as well as a tool to describe the development of teachers' noticing. The above studies indicate that noticing critical aspects of mathematics teaching of others and prospective mathematics teachers' own teaching seems to constitute a basis for professional learning.

Methodology

The research took place in the context of a 14-week mathematics education undergraduate course (taught in one semester by the second author) included in a university program of a mathematics department leading to a first degree in mathematics. Enrolling in the course in which the study took place, PTs had already successfully passed at least four courses on pedagogy and mathematics education. The aim of the course was to engage PTs in critical consideration of aspects of mathematics teaching as they emerge from the complexity of teaching practice in schools. Every second week for the entire semester PTs were asked to participate in a number of field activities (over six field activities-weeks) while each week following the activities in schools included a three-hour meeting at the university. PTs' field activities consisted of observing other teachers' mathematics teaching for six hours in total (first three field activities-weeks), designing and teaching a lesson in one group of students outside the classroom for one teaching hour (fourth field activities-week), and designing and teaching lessons in the whole classroom for two teaching hours (fifth and sixth field activities-weeks). The 22 PTs (9 males, 13 females), who served as participants in this study, were divided into pairs and carried out collaboratively the field activities under the supervision of eight postgraduate students of mathematics education who acted as mentors.

Inquiry into mathematics teaching was a rather new practice for PTs and was supported through the discussions in the university meetings and the field activities. Critical alignment with the practice of the mathematics teaching in which they were engaged through observing and teaching, was expected to be developed through the process of inquiry and questioning aspects of practice. Critical incidents were expected to facilitate this process. PTs' field activities were based on the cycle observing-reflecting-designing-implementing-reflecting. For instance, PTs were asked to: identify the specific content of a lesson in the curriculum and to trace it throughout the different grades; look for possible research evidence related to potential students' difficulties; keep systematic notes from and/or record the lessons; reflecting on their classroom experiences; and analyzing lessons. In this context, PTs were asked to select critical incidents and provide a reflective account on the basis of

justifying their selection, interpreting them and proposing potential teaching actions. Instructional practice in the university sessions aimed to support PTs' reflection on their recent field experiences and to link emergent issues with existing mathematics education research in order to develop deeper levels of awareness. PTs were introduced to the idea of critical incidents through (a) a brief presentation of Goodell's (2006) study (including the meaning of critical incidents, the classification of them and examples from PTs' written reports), and (b) analysis of transcripts of lessons to identify critical incidents and discuss/justify in the class their criticality. The teacher educator facilitated the discussion, but also challenged the PTs to justify their selection of the critical events, to provide evidence of their claims, to make interpretations, and describe their potential teaching decisions. The PTs themselves presented the analysis of the critical incidents and their reflections in the university meetings. Overall, PTs' field activities and the discussions in the university meetings revolved around the idea of critical incidents and thus they were compatible with our research focus.

The data for this study consisted of: (a) PTs' personal portfolios including their written accounts of critical incidents, and material related to the design, implementation, and presentation of the field activities in the classroom (e.g., worksheets, lesson plans, presentation files); (b) video recordings of all meetings at the university (8 in total) and (c) researchers' field notes. In this paper we analyse the data from the PTs' portfolios. The analysis was carried out in three levels. In the first level, we adopted a grounded theory perspective (Charmaz, 2006) and indentified thematic areas indicating what the PTs noticed (first research question). In the second level, we analysed the critical incidents, their interpretation and the potential actions that PTs reported in their portfolios for each week's assignment in terms of the levels of van Es' (2011) framework. Finally, we traced PTs noticing over time looking for shifts in what they noticed in students' activity and how they interpreted it.

Results

The nature of critical incidents from PTs' portfolios

In Table 1, we present a categorization of the critical incidents that the PTs identified in their reports in two cases; one is while reflecting on the observations of other teachers' teaching and the second while reflecting on their own teaching. The total number of critical incidents in the first case was 72, while in the second 54. In both cases, the incidents reported most often were related to students' activity (35 out of 72 - 49% in the first case, and 21 out of 54 - 39% in the second) and in particular, to their conceptual difficulties. Another category of incidents focused on teaching - especially on the interaction between teacher and students (e.g., how the teacher responded to students' questions and answers). Thirty-three out of seventy-two (46%) incidents in the first case fell in this category and eighteen out of 54 (33%) in the second case. A third category appeared mainly when PTs reflected on their own teaching, concerned students' learning in relation to teaching (5% in the observations and 22% in the personal teaching). A fourth category that emerged only in the second case included three incidents focusing on epistemological issues.

Below, we present some illustrative examples of the above categories and we elaborate on the issues emerging from the analysis of the critical incidents in relation to our research goals. Focusing on students' activity, the PTs recognized misconceptions and difficulties in using mathematical language, performing procedures, connecting representations, and developing problem solving strategies. For example, the confusion between perimeter and area was noticed by one prospective

teacher, Marina, while observing a lesson in an eighth grade class: "The teacher asked the students to draw a triangle and then to name the sum of the sides. One student answered 'area' and another one 'perimeter.' The first one seemed to confuse area and perimeter". As regards to the unexpected students' responses, one prospective teacher, Leonidas, reported students' innovative approaches in finding triangular numbers in the Pascal triangle: "One student discovered a personal algorithm to calculate triangular numbers only by observing the arrangement of numbers in the Pascal triangle".

Incidents from classroom observation (72)		**Incidents from personal teaching (54)**	
Students' activity	**35 (49%)**	**Students' activity**	**21 (39%)**
Difficulties	29	Difficulties	18
Unexpected responses	5	Unexpected responses	3
Motivation	1	Motivation	0
Lesson planning and teaching	**33 (46%)**	**Lesson planning and teaching**	**18 (33%)**
Teacher-students interaction	19	Teacher-students interaction	10
Classroom norms	5	Classroom norms	3
Quality of tasks and mathematical content	8	Quality of tasks and mathematical content	2
Teaching versus planning	0	Teaching versus planning	2
Dynamic character of teaching	1	Dynamic character of teaching	1
Linking teaching and students' learning	**4 (5%)**	**Linking teaching and students' learning**	**12 (22%)**
Relating interaction and learning	0	Relating interaction and learning	8
Relating task and learning	3	Relating task and learning	3
Relating norms and learning	1	Relating norms and learning	1
Epistemological issues	**0 (0%)**	**Epistemological issues**	**3 (6%)**

Table 1: Categorization of the PTs' critical incidents

Concerning teaching and in particular teacher-student interaction, the PTs commented on positive and negative ways that the teacher or PT reacted to students' contributions. A positive example was when Vassilis noticed that the classroom teacher acknowledged different solution strategies and discussed those in the classroom. Stella referred to a negative example from her own teaching: "One student proposed to find the requested area through transformations, which is a good approach. However, I directed her to follow the approach described in the textbook". Stella also noticed the classroom norms and their effect on the mathematical communication: "Although the students provided repeatedly wrong answers, the teacher did not evaluate them and encourage further discussion". The quality of the tasks in relation to the mathematical content was related to the teacher's choices of the content, its integration in the designed tasks, and its transformation in the classroom teaching. Anthi reported: "I was impressed by the way that the teacher introduced students to the idea of limit in the context of geometry. … This experience can help students to get an intuitive sense of the idea of limit". By being involved in designing and teaching, PTs started to consider the complexity of teaching. In particular, they started to recognize the gap between planning and teaching and the dynamic character of teaching as it is indicated in the following example from Sofia's reflection: "Although I had designed a realistic problem with the aim of engaging students in making sense by themselves of the notion of circle, during the implementation, I ignored the design. Actually, I took a directive stance to secure that the task would lead the students to the expected conclusions".

Moreover, the PTs started to relate different aspects of teaching such as classroom norms, classroom interaction, and nature of tasks to students' learning. For example, Alexandros, recognized the mediation of digital tools in supporting students' understanding while reporting on his classroom observations: "I noticed a student who had difficulty realizing that the ratios in Thales' theorem remain constant independently of the position of the non parallel lines. She understood this property through dragging these lines in Sketchpad". Another example is about the relation between the presentation of a task and students' engagement. In an application of the Thales theorem, Leonidas noticed that the complexity of a geometrical figure in the task he designed posed barriers to students' participation: "Students' participation dropped vertically when they were asked to discern ratios of segments in the shape. So, the weak students could not consider at all even simple questions such as 'Show me a line that intersects the parallels'".

Finally, in the category "epistemological issues" we include critical incidents that refer to the nature of mathematical content from an epistemological point of view. For example, Anna noticed in her teaching that some students did not verify the validity of their findings, a process that she considers important in mathematics: "I chose to discuss this incident because verification constitutes an important process in mathematics. However, students often are not engaged in this".

The growth of prospective teachers' noticing

Here, we use the van Es' (2011) framework to trace PTs' development of what and how they notice when observing teaching and reflecting on their own teaching. The analysis of the portfolios indicated that most PTs progressed to higher levels of the van Es' developmental trajectory where relations between teaching and learning were noticed and connections between events and principles of teaching and learning were made. Below, we illustrate this shift through a representative case of a PT (Katia).

Katia provided a written account of the critical incidents she selected as part of the course assignments involving observations and designing and teaching. During the observations, Katia offered general descriptions of the whole class environment and incidents related to students' difficulties. She shifted from a baseline noticing in her first two observations (level 1) to mixed noticing (level 2) in the third one both in what and how she notices. For example, in her written account based on the second observation she gave as a critical incident the students' lack of motivation to participate in the lesson due to the fact that some of them would not have been examined in mathematics in the university entry examinations. As regards how she notices the above critical incident, she provided descriptive and evaluative comments considering teaching independent of students' behavior. In reflecting on her potential teaching actions, she mentioned that she would insist on inviting students to pay attention. In her account based on the third observation, Katia focused on students' difficulty to transform the formula of the area of a trapezium $E = (B+b)\times h/2$ to an equivalent expression in terms of another variable (e.g., the height h). This time she provided evidence of this difficulty by specifying students' errors in algebraic manipulations. She also noticed that the teacher used numerical examples with the same structure to address these difficulties. Commenting on this critical incident, she wrote: "Although students do well with numbers and equations with one variable, they get confused when more variables are involved and they panic". It appears that Katia begins to notice students' thinking and refer to teacher-students interactions in the teacher's attempt to address students' difficulty. While she was

challenged by the teacher educator to look for further evidence to support and interpret her observation (by discussing with the classroom teacher and one student who demonstrated this difficulty after the lesson, and by reading a relevant research paper), she still confirms students' difficulty without offering an explanation.

Katia's noticing was further developed while reflecting on incidents selected from her own teaching. She started to attend to subtle aspects of tasks and the way they influence students' activity, to develop interpretations based on her classroom experiences and research readings and to deviate from her planning at contingency moments. Our analysis provides evidence that while reflecting on her own teaching she was able to consider teaching and learning in a relational way and to provide justified arguments and alternative pedagogical solutions reaching focused noticing (level 3) and extended noticing (level 4). The following example illustrates this finding. Katia designed a lesson for the teaching of area measurement in grade 7 by taking into account research findings on students' strategies on area measurement. Her main goal was to engage students in calculating the area of irregular figures by developing as a main strategy the dissection of the shape in other shapes whose area could be calculated by the known formulas. The students were really engaged in the process and developed different strategies. Katia reported as a critical event the fact the use of the word "irregular" in the given worksheet raised a lot of questions in the classroom: "I did not expect that the word "irregular" would create questions and negotiations. However, I exploited to see how students think about these figures". In her analysis of the phenomenon, Katia refers to specific student's ideas about the meaning of the word "irregular" and how this influenced students' work.

Discussion

The critical incidents that PTs identified in their portfolios addressed a multiplicity of issues related to mathematics teaching and learning focusing mainly on students' activity and on student-teacher interaction. A similar picture was also formed in the study of Goodell (2006) where students' conceptual understanding and classroom interaction were the most dominant categories of the selected critical incidents. As regards the context in which the selected incidents emerged, there were not distinct differences in the nature of critical incidents that the PTs selected through their observations of other teachers' teaching and of their own. At the level of classroom management, the PTs found it more difficult to focus on the teacher-student interaction in their own teaching than in other teachers' teaching. Nevertheless, when PTs reflected on their own teaching, they started to see more clearly the impact of teaching on students' learning. One possible explanation could be that PTs' engagement in analyzing other teachers' teaching provided them a reflective stance towards their own teaching. A similar finding has been reported by Stockero (2008) who identified that PTs' experiences in analyzing video lessons of other teachers can enhance deeper levels of reflection on their own teaching. Tracing PTs' critical incidents, their interpretations and suggested teaching actions indicated shifts in their ways of noticing. Most PTs reached levels 3 and/or 4 of the Van Es' framework (2011) in terms of what and how they notice realizing interrelationships between teaching and learning. This finding adds to existing research on developing structures in teacher education facilitating PTs' noticing and enriches discussions that have taken place in previous CERME conferences (e.g., Potari et al., 2011). Integrating selection and reflection on critical incidents in teacher education provides a structured way that helps PTs to become aware of significant classroom interactions and to develop a critical way of addressing them.

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