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Teachers' attempts to address both mathematical challenge and differentiation in whole class discussion

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In this paper we investigate how a group of six mathematics teachers in Greece deals with the need to balance work on mathematically demanding tasks and differentiation in lesson planning and enactment. Videotaped lessons and pre and post reflection interviews were analysed with a specific focus on whole class discussion. The findings show certain teaching practices that appear to promote both mathematical challenge and differentiation and emerging patterns of actions that make the challenge accessible to students.

Keywords: Mathematically challenging tasks, whole class discussion, differentiation.

Introduction

Research on challenging tasks and differentiation has contributed to deepen our understanding of teaching. The former explored ways to engage students in rich mathematical activity through tasks advancing their thinking and reasoning (Stein, Smith, Henningsen, & Silver, 2000). The latter pertains to research in differentiating teaching so as to engage all the students in mathematically productive learning experiences (Sullivan, Mousley, & Zevenbergen, 2006). Bridging challenging tasks and differentiation constitutes an area that is not explored at the research level. However, it seems to be at the core of everyday teaching since working with rich mathematical tasks requires taking into account explicitly how all students could be engaged with them both at the level of lesson planning and enactment. Whole class discussion can be seen as a terrain providing fertile opportunities to study in what ways teachers might work at the intersection of mathematical challenge and differentiation. During whole class sessions a teacher faces the need to preserve a mathematically productive discussion and at the same time to respond to diverse students' abilities, learning styles and paces (Sullivan, Mousley & Zevenbergen, 2004). The present study aims to address this issue by focusing on mathematics teachers' attempts to work at the intersection of mathematical challenge and differentiation while designing tasks and orchestrating whole class discussions in their classrooms. The research question is: "How do teachers attempt to balance mathematical challenge and differentiation in whole class settings?"

Theoretical framework

Challenging tasks are those that require students to: process multiple pieces of information with an expectation that they make connections between those pieces and see concepts in new ways; explain their strategies and justify their thinking to the teacher and other students; engage with important mathematical ideas; and extend their knowledge and thinking in new ways (Stein et al., 2000). Working with challenging tasks seems to be rather demanding for the teachers at the level of design and enactment. Lesson planning involves design of tasks that support a learning trajectory and extend students' thinking (Sullivan et al., 2006). Handling the enactment of challenging tasks during whole class sessions requires specific actions by the teacher to support students' mathematical discourse. These involve talk moves such as revoicing, repeating, reasoning and adding on (Chapin, O'Connor, & Anderson, 2009) as well as key practices such as anticipating, monitoring, selecting, sequencing, and making connections between student responses (Stein et al., 2008). Stein et al. (2000) proposed three paths to describe how teachers handle the demands of challenging tasks in their lessons: lowering, maintaining, or increasing the challenge.

Differentiation is a process of aligning learning targets, tasks, activities, resources to individual learners' needs, styles and paces (Beltramo, 2017). In supporting teachers in dealing with differentiation issues at the level of lesson planning existing literature suggests focusing on content, process, and product (Tomlinson, 2014) or providing enabling prompts to support students experiencing difficulty and posing extension tasks for students who finish the tasks quickly (Sullivan et al., 2006). Researchers suggest that the goal of addressing differences between students in lesson enactment can be effectively carried out by building of "a sense of communal experience" (ibid, p. 118) where all students benefit from participation on common discussions.

Whole class discussion constitutes a phase of the lesson enactment where mathematical challenge and differentiation can be at interplay providing opportunities for further study. According to Dooley (2009), whole class discussion is both emergent because the outcome cannot be predicted in advance and collaborative because it is the outcome of the collective activity between the teacher and the students. Focusing on the tensions involved in teachers' attempts to build on students' ideas for mathematically productive class discussion, Sherin (2002) highlights the need to maintain the student-centered process of mathematical discourse and to direct the content of the mathematical outcomes. In the present study our focus is on teachers' actions to balance mathematical challenge and differentiation in this rather complex part of the lesson.

Methodology

EDUCATE is a professional development (PD) European project aiming to support teachers to engage all their students in challenging tasks. To achieve this goal, four partners/university teams from different countries collaborate to develop teacher education activities and materials. The current study took place in the introductory phase of the project where the partners' aim was to explore teachers' needs and challenges in relation to working with challenging mathematical tasks and differentiation without being engaged in any teacher education/PD activity. In this paper, we use the data collected in Greece and it involves the teaching of six practicing secondary school mathematics teachers (Adonis, Eugenia, Markos, Gianna, Kosmas, Takis, all pseudonyms) who participated in the study in a volunteering base. All of them were experienced, qualified teachers but without any PD professional development experience related to address together mathematical challenge and differentiation. Their years of experience ranged from 10–25 years at the time of the study. Three of them worked in experimental schools and the rest of them in typical public schools. Also, three of them were teaching in lower secondary and the rest in upper secondary school classes. One of them had a doctoral degree in mathematics education (Kosmas), three held a master's degree in mathematics education (Adonis, Markos, Gianna) and one held a master's degree in pure mathematics (Takis). All teachers had collaboration with the university team either during their studies or through their participation in other research projects. Participants were informed about the project rationale/aims and they were asked to design challenging tasks and enact them as part of their everyday teaching with the goal to engage all their students. They were also informed that we were interested in the challenges and difficulties encountered by them when implementing these tasks without further details on how to proceed with this goal.

Data collection included: video-recording of two lessons for each teacher (12 video-recordings in total); pre- and post-lesson reflections/interviews (12 in total); teachers' designs for their lessons (e.g., worksheets, digital resources) and students' work. The recorded lessons lasted one teaching hour (45-50 minutes) and were carried out between December 2017 and January 2018. Under a grounded theory approach (Charmaz, 2006), we analysed lesson planning and enactment with a particular focus on whole class sessions considering the setting up of the task and the discussion of students' solutions. Balance of mathematical challenge and differentiation in teachers' designs was addressed by triangulating the analysis of teachers' lesson plans, teaching materials and pre-lesson reflections focusing on (a) the designed tasks, (b) the used resources and (c) the decisions related to classroom implementation (i.e. group work, role of teacher, classroom norms). In the videotaped lessons, we identified episodes indicating a balance between mathematical challenge and differentiation. Mathematical challenge was originated either by the task itself, a student's query or the teacher's extending prompt or question. Differentiation was expressed by the teacher's attempts to make the challenge accessible to all students. Shifts in the students' engagement with the challenge in the episodes were identified (by us) mainly by interpreting students' progressive participation in the mathematical discourse.

Results

In the first part of this section we analyse teachers' designs. In the second part we present selected episodes from classroom observations indicating a balance between mathematical challenge and differentiation. Our focus is on the teaching actions and their interplay that indicate a process of making the challenge accessible to all students and not on cases where this was not evident.

How mathematical challenge and differentiation were balanced in lesson planning

Each lesson was based on a sequence of tasks and activities to be undertaken by students so as to support each teacher's learning goals. Most tasks (21 out of 25) can be characterized as challenging offering opportunities to students to: model an everyday situation through arithmetic, algebraic and geometrical relations (Markos – 7^{th} grade, Adonis – 7^{th} grade, Gianna – 10^{th} grade, Eugenia – 8^{th} grade); link algebraic and geometrical representations (Kosmas, 10^{th} grade); or conjecture and prove a geometrical property (Kosmas – 10^{th} grade, Takis – 10^{th} grade).

Analysis of teachers' pre-lesson interviews allowed us to identify their goals and actions for proactively planning the level of mathematical challenge and support provided to meet different students' needs. As regards the mathematical challenge, the teachers attempted to integrate it in their didactical designs and make it accessible to all students through the following planning actions: (a) *Designing tasks with multiple solutions and different entry points*. For example, Kosmas asked students to use both algebraic and geometrical ways to solve equations with absolute values. The teacher knew that there were students (e.g., Maria) who could easily handle the geometrical way while they had difficulties in the algebraic. The teacher considered the use of the geometrical way as challenging as it required a deeper understanding of the meaning of absolute value. Offering different entries for the students was related to the process of exploration often in the context of open and/or modelling problems. For example, Eugenia designed a modelling task on estimating the height of the classroom as an application of the tangent trigonometric notion. (b) *Using different kinds of resources (e.g., manipulatives, digital applets, diagrams, typical and non-typical measuring instruments)* to facilitate the making of connections between different representations. Markos, for instance, used digital tools (*Algebra Arrows* applet) to facilitate students' focus on the structure of arithmetic and algebraic expressions. Eugenia offered a hand-made protractor (a measuring instrument originated in the ancient Greek mathematics) and she commented about the critical role of this tool in mediating the conceptualization of the notion of tangent ratio. (c) *Creating an inclusive and mathematically challenging learning environment* by encouraging students to share their work in groups and in whole class discussions and avoiding evaluative comments. These actions were included in different ways in all teachers' didactical agendas. For example, Eugenia attributed emphasis to the roles and responsibilities she assigned to the students in each mixed ability group according to their mathematical backgrounds and interests.

How mathematical challenge and differentiation were balanced in lesson enactment

The episodes that follow are chosen to indicate illustrative ways by which the teachers stimulated the mathematical challenge and attempted to balance it with differentiation in different phases of the lesson. Episode 1 is selected from the setting up of the task and the challenge is based on the teacher's strategy. Episodes 2 and 3 are taken from the discussion of students' solutions and the starting point was students' difficulties or unexpected responses. In parentheses (italics) we characterize teachers' actions and at the end of each episode we summarize the teachers' actions.

Episode 1: Stimulating the key mathematical idea by exploring the validity of students' responses

This episode took place in one of Kosma's lessons in Geometry (10th grade). The task given to the students was the following: "How many degrees is the sum of the three angles of a triangle? How can we be sure about the answer?" (In the worksheet, there is an oblique triangle drawn and a triangle with an obtuse angle). In this task the challenge concerned the students' engagement in appreciating the need for proof. In the episode, we see how Kosmas promotes the challenge and at the same time the way that he formulates the task to allow the engagement of all students.

The teacher asks the students: "Are we sure that the sum of the angles of a triangle is 180 degrees?" (*stimulating the challenge*). Although most of the students reply that they know it from previous grades, Alexis, one of the students having usually limited participation in the lesson, suggests to prove it by measuring. The teacher asks him "If you measure the angles do you think that you will find 180 degrees?" and he asks all students to draw triangles and measure their angles by the use of a protractor (*valuing students' ideas by addressing them to the whole class; encouraging empirical solutions*). Alexis finds 179 degrees and other students 178, 179, 180, 181. The teacher writes these responses on the board (*recording and discussing all students' answers*) and asks students: "How can we be sure? It seems that we cannot be sure by measuring" (*refuting the empirical solutions*). Through these actions the teacher seems to point out the key mathematical idea to the students by building on Alexis' suggestion for an empirical measurement. He intentionally accepts Alexis' suggestion and invites all students to perform measurements in different triangles. Next, he records all students' answers on the board as a way to question the validity of the approach. In his pre- and post-lesson reflection the teacher mentioned that he targeted the empirical justification to be discussed: "The discussion was what I wanted as the responses varied. It also went well since most students were involved, also by working in groups felt less exposed to evaluation" (Kosmas' post-lesson reflection). In this episode, the teacher stimulates the mathematical challenge of the task, encourages students to explore an inappropriate idea coming from a student, summarizes their responses, provokes the refutation and reinforces the challenge.

Episode 2: Using digital resources to address mathematical challenge and students' difficulties

This episode took place in one of Markos' lessons (7th grade) about the structure and equivalence of arithmetic and algebraic expressions through their connection to a realistic situation. The task and the corresponding questions revolved around the idea of describing an everyday situation (Maria's account balance after shopping) with different ways through simple arithmetic expressions (initially) and algebraic ones with the use of one and more than one variables (subsequently). The problem situation is described as follows: "Maria has 500€ in her bank account. She bought meat that cost 10€. She also bought fish that cost 20€. She used her debit card and received a message from her bank on her mobile, informing her that her account balance is 470€". The challenging dimensions of the task were related to the use of a realistic situation as a context of reference in conjunction with a multi-representational applet (*Algebra Arrows*) to identify the structure of arithmetic and algebraic expressions. We note that the construction of expressions in the environment is concretized by connecting Input/Output fields (including numbers or variables) to operation fields through the use of arrows. The environment provides the final result of the calculation as well as its structure (that is represented through arrows and symbolically). The students – who were familiar with the use of the applet - were given a worksheet involving the questions and a printed representation of the work area of *Algebra Arrows*. They worked in groups for about 5 minutes on a specific question with paper and pencil on the worksheet. After each group work session, the teacher used the *Algebra Arrows* representations (provided through a projector) to discuss students' solutions/reflections in a whole class discussion.

The episode took place after students describing the account balance and their relation [the expressions were: 500–10–20 and 500-(10+20)]. The class discussion concerned the students' solution to the questions: "If it is not known the initial amount of Maria's money in her account, construct through the use of *Algebra Arrows* two different expressions to describe the amount of money left in the account. What is the relation between the two expressions?" During the preceding group work session the teacher knew that the group 1 students (4 girls) had written correctly two expressions using variable for describing the situation – i.e. x-(10+20) and x-10-20 – but they faced difficulty to consider them as equal. The teacher invites the students to discuss about the solutions in a whole class session. Before constructing the two expressions with the applet, he asks the students what are these expressions (*stimulating all students to provide the solution*). One student answers: "x-(10+20) and x-10-20". Then Markos constructs the two expressions with the applet and asks students to provide the answer before this is projected (*using multiple and interconnected digital* *representations to justify an answer*). Next, he comes back to the group 1's concern posing the question they discussed during autonomous work to the whole class showing the two fields in the applet: "Girls, I remember that earlier you were concerned if it is the same 'x' that appears here [in the expression x-(10+20)] and the 'x' that appears there [in the expression x-10-20)]. This is a question for the whole class. I ask: Does this 'x' express the same thing?" (*posing an individual student difficulty to the whole class; stimulating all students to reflect on the provided representations*). One student (Nick) replies: "They are the same as they are connected with the same field – entitled Maria's Money: x - in the applet". The teacher indicates that both expressions are built by using the same field symbolized as 'x' (*revoicing the correct answer*): "Look the arrows starting from the cell containing Maria's initial amount of money. We speak about Maria's money in both expressions (*linking the digital representations to the realistic context*). In this episode, the teacher brings an individual student's difficulty in the whole class through the use of digital representations, stimulates all students to reflect on the provided representations, revoices a student's correct answer, strengthens the challenge by providing links to the realistic context.

Episode 3: Building on students' unexpected responses to extend the challenge for all students

This episode took place in one of Adonis' lessons (7th grade) about the exploration of the binary number system in the context of a real problem with three questions. The task context is a flour mill in the 19th century. The owner (the miller) has only one weight of 1kg, one of 2kg, one of 4kg, one of 8kg and one of 16kg. He claims that he can weigh sacks of flour until 31kg using these weights. In the first two questions, the students are asked how the miller weighs (1) a sack of 18kg and (2) sacks of 1, 2, 3, … 31 kg (table is given to be filled in). The third question is how many kilos were contained in a sack where the miller has written on it the number 1011. Before the episode, the teacher had explained how weigh scales were used in the 19th century. The students completed the first and the second question rather easily. The mathematical demand increased in the third question when the students had to identify how the binary system 'works'. The teacher draws a sack on the blackboard and besides he writes the number 1011 and asks: "What does the number 1011 mean?" (*stimulating the challenge*). Fenia provides the following answer: "The number 1011 means that the miller used 1 weight of 16kg, none of 8kg, 1 of 4 kg and 1 of 2kg". The teacher asks students if they agree with Fenia (*valuing students' ideas by addressing them to the whole class*). Students provide different wrong responses (e.g., "It means 1kg and 11gr", "It means the price, 10Euros and 11cents"). In the realm of the rich discussion where different opinions were expressed, Ian suggests: "Perhaps we have to start from the right. One weight of 1kg, 1 weigh of 2kg, none weight of 4kg and 1 weight of 8kg". This idea created uncertainty among the students, and it was also unexpected for the teacher ("I couldn't decide immediately how to handle it. I let the discussion to continue and I think that finally we reached a consensus", Adonis' post-lesson reflection). The teacher writes Fenia's and Ian's responses on the board to make them accessible to all students (*recording students' responses on the board*). Then he invites all students to compare these two solutions focusing on the place value of the digits (*stimulating the key mathematical idea*). He proposes to look at the tables they had filled on the distribution of weights from 1Kg to 31Kg in the previous question and asks students to find the place value of each digit (*linking the key mathematical idea to a previously answered question*). The teacher writes on the board upon each digit the values that the students provide (1, 2, 4, 8 from left to right).

The disagreement is still evident in the discussion and it is made explicit by one student who says: “We do not know if the miller represented the weights by writing the digits from left to right or vice versa”. Most of the students seem to consider it as a dilemma. The teacher invites – for first time - a high achiever (Tom) who raised his hand in all questions. Tom says: “The value of each digit depends on its place and the number system that the miller uses”. The teacher revoices Tom’s opinion emphasizing that the value of each digit depends on its place. There are students still providing wrong answers about the place value of the digits. The teacher writes on the board an integer (i.e. 1357) in the decimal number system and asks the students to find the place value of each digit (*simplifying the initial challenge through a familiar case*). A lot of students provide correct responses ,but the teacher brings back the challenge: “Why do I have to accept it? How do we know what is the value of each digit?” (*bringing back the challenge*). The teacher addresses students who are reluctant to respond (*giving voice to silent students*). One of these students says: “We know it as a rule” and another one adds: “We have defined it. We have agreed to use it this way”. The discussion continues this way and more students participate. The teacher continues to bring as examples integers from the decimal number system (*simplifying the initial challenge*) and invites students to compare the two systems (*extending the challenge*). In this episode, the teacher brings a challenging issue by an individual student in the whole class, stimulates all students’ reflection, gives voice to silent students, makes links to familiar representations and extends the challenge by inviting students to make comparisons.

Conclusions

In this study we focused on teachers’ attempts to balance mathematical challenge and differentiation. Mathematical challenge was targeted by all teachers through the design of mathematically demanding tasks and the use of different resources (e.g., realistic contexts, diagrams, concrete materials, digital representations) to engage students in exploring, connecting and reflecting. During lesson enactment the teachers’ approaches to stimulate the challenge involved building on students’ ideas (revoicing, rephrasing, reformulating) as well as scaffolding by simplifying (e.g., bringing a familiar case/situation) and extending in a dynamic way (e.g., comparing different approaches/solutions/representations). Valuing students’ contributions and addressing them to all students appears at the core of teachers’ orchestration of the whole class discussion. The teachers appear to use students’ ideas (e.g., difficulties, indications of high-level reasoning) as a basis for communal reflection through the following actions: making the challenge accessible to students; recording all students’ answers; inviting students to connect different solutions; questioning proposed ideas; and favoring the development of an inclusive learning environment (e.g., encouraging silent students to participate). As regards the existence of some patterns in teachers’ approaches to balance mathematical challenge and differentiation, our analysis reveals a dynamic interplay of actions moving back and forth between providing challenging questions and prompts. Far from being characterized as linear, this process indicates underlying ‘zig-zag’ patterns related to the complexity of teaching practice when the teacher aims to keep the challenge and at the same time to maximize learning opportunities for all students. This is an interesting finding that needs further exploration taking into account that in existing literature teachers’ attempts to enact highly demanding tasks have been characterized with an emphasis on the part of mathematical challenge at a global level (upgrading/downgrading, lowering/maintaining/increasing, Stein et al., 2000). Our analysis indicates the need to take a more detailed look on teachers' actions and on the spot decisions while working at the intersection of mathematically demanding tasks and differentiation.

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