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# Community documentation targeting the integration of inquiry-based learning and workplace into mathematics teaching

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*In this paper we describe the process of designing and implementing inquiry-based tasks relating mathematics and workplace context, through the collaboration of four teachers from different schools. Our focus is on the factors that influence the forms of inquiry integrated in community documentation. A common open task took different forms of inquiry during the implementation depending on factors such as the students' mathematical abilities, the specificities of the different school contexts and the prerequisite mathematical knowledge. The process of teachers observing each other's classroom implementations (hetero-observation) became the incentive for subsequent transformations of the shared resources, favoring the shifts from one form of inquiry to another.*

*Keywords: Inquiry-Based Learning, Workplace context, Community documentation.*

## Introduction

The reported study took place in the context of the European-funded project Mascil (www.mascil-project.eu) that targeted mathematics and science teachers' professional development (PD) through the integration of inquiry-based learning (IBL) and workplace contexts into their teaching. In this paper, we study the process of design and implementation of resources by a group of mathematics teachers from different schools who collaborated to conceive IBL tasks stemming from the 'Kite making industry' that was selected by them as the workplace context to develop their design. The importance of introducing inquiry into the teaching and learning process has been recognized by the existing research (Boaler, 1998) and constitutes a main aim of many mathematics curricula and teacher education programmes. Also, there is a growing interest concerning the nature of resources and contexts facilitating the integration of inquiry into classroom teaching (Doorman, 2011). This integration increases the complexity of teaching as it brings to the fore issues such as the nature of the designed tasks, the teaching management and the students' learning (Artigue & Blomjoi, 2013), thus teachers are reluctant to use IBL in their everyday practices (Bruder & Prescott, 2013). At the same time, it has been indicated that workplace contexts may offer rich opportunities for teachers to introduce authentic situations into their teaching (Wake, 2014) through open problems. The design and implementation of resources for mathematics linking IBL and workplace contexts constitutes an innovation since existing curricula rarely include this type of materials, so teachers' collaboration in this direction is crucial. Existing literature indicates the facilitative role of teachers' participation in communities in their professional development and more specifically when introducing educational innovations (Lerman & Zehetmeier, 2008) such as the one targeted by Mascil. However, how teachers collaborate to design and implement innovative tasks linking IBL and workplace is an area that requires further study. Under this perspective, in this paper, we explore the factors that influence the design/implementation of IBL tasks relating mathematics and workplace contexts during collaboration among teachers from different schools.

## Theoretical framework

The integration of IBL into the mathematics classrooms presupposes the transformation of the traditional teaching practices. Doorman (2011) indicates the shift in the teachers' roles from telling to supporting, scaffolding and fostering students' reasoning, while students pose questions, inquire, explain, extend, evaluate and collaborate. He highlights the need for new classroom norms and learning environments incorporating open problems with multiple solutions through the use of tools and resources. Artigue and Blomjoi (2013) also indicate the 'authenticity' of questions and students' activity that requires connection with real life and out-of-school questions fostering the experimental dimension of mathematics. Inquiry can take three forms (ibid): (a) Structured: The teacher provides the students with the appropriate method/materials to solve the given problem. (b) Guided: The teacher provides the students with the necessary materials and the students have to find the appropriate strategies for solving the given problem. (c) Open: The students have to find problems or questions they would like to solve and chose the methods/materials for solving them.

The crucial role of resources has been discussed in terms of the documentational approach of didactics (DAD, Gueudet & Trouche, 2009) focusing on teachers' interactions with resources as a way to capture teachers' professional development. The term *resource* describes a variety of artifacts such as a textbook, a piece of software, discussions with colleagues etc. An integral part of teachers' professional activity is the search, selection and modification of resources as well as their implementation in class working individually or collectively with colleagues. This process – called documentational genesis - results to a document after classroom implementations. Through a class of professional situations and teachers' experience, the existing resources are modified as documents that can be further transformed to new documents over time. The process of gathering, creating and sharing resources to achieve the teaching goals in the context of a community is called *community documentational genesis* resulting in the *community documentation* composed of "the shared repertoire of resources and shared associated knowledge (what teachers learn from conceiving, implementing, discussing resources)" (Gueudet & Trouche, 2012, p. 309). In this study, the group of participating teachers is considered as a community of practice (Wenger, 1998) characterized by mutual engagement (collaborative norms and relationships), joint enterprise (common goals) and shared repertoire (collaborative production of resources). IBL has been studied in relation to different theoretical frameworks (Artigue & Blomjoi, 2013) but not in relation to DAD and community documentation. Linking IBL and workplace contexts is a challenging research area (Triantafillou et al., 2016) in terms of the factors that influence teachers' collaborative work. In this paper, we explore the community documentation aiming to link IBL and workplace contexts by the following research question: Which factors influence the forms of inquiry integrated in community documentation of IBL tasks relating mathematics and workplace contexts?

## Methodology

### Context and participants

The implementation of Mascil in Greece included thirteen groups of 8-12 practicing teachers from mathematics, science and technology. In each group, the teachers collaborated in two cycles of design→implementation→hetero-observation→redesign with reflection as a core element, for a school year with the support of a teacher educator. PD meetings took place before and after each implementation. In the first two PD meetings, teachers were introduced to the project rationale as well as to the main principles of IBL and workplace mathematics. In the next meetings, teachers collaborated in subgroups to transform existing Mascil tasks or design new ones in the same spirit and reflect on their experiences from the implementations. In the end of the project, the participating teachers were interviewed so as to address the impact of the PD program on their professional development. In this paper, we focus on a subgroup of four qualified mathematics teachers: tA (PhD), tB (Master degree, teacher educator), tC (Master degree) and tD (teacher educator). They collaborated in the design and implementation of two tasks (*Stairs* and *Kites*). Here we study the case of *Kites*. Implementation took place in different grades (Figure 1) in four public secondary schools of Athens. tB's school is experimental, i.e. it is connected to the university, it supports experimental teaching methods and students are selected after examination. Hetero-observation informed the subsequent transformations of the initially shared resources.

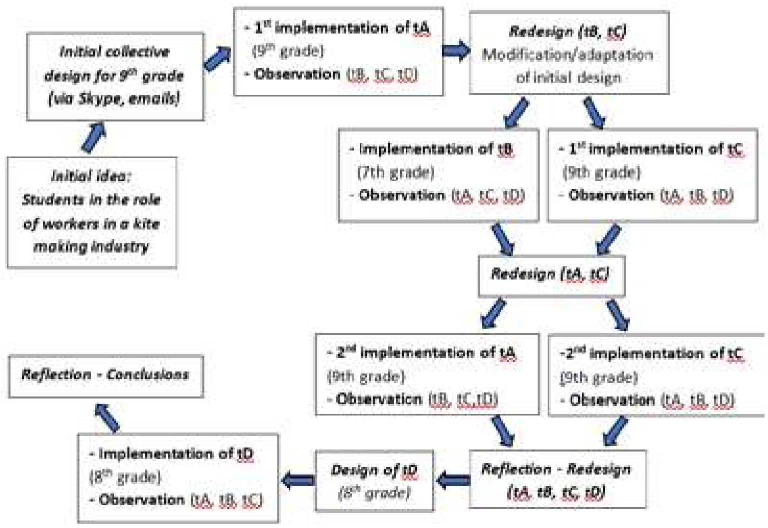


**Figure 1: The process of implementation**

**The task**

The task was formulated gradually through discussions among the four teachers having as a starting point the idea of kite making. Factors that influenced the task design were the multiplicity of issues inherent in the construction of kites, the targeted mathematical content and its connection to the curriculum as well as the need to leave space for the students to become partners in the formulation of the problem. The initial common assumptions were: *(i) The initial idea*: Students' adopt the role of employees in a small kite making industry aiming to optimize the amount of paper used. *(ii) Preparing the implementation*: Students work in groups, search for the following information in real-life sources and upload it in an e-class folder: (a) the kind of paper used for kites (dimensions, package, cost); (b) shapes/dimensions of kites with a focus on: regular hexagon of diameters 80cm, 100cm and 120cm respectively that can be consisted of 6 equilateral triangles, or 2 isosceles trapezoids, or 4 right trapezoids with different colors; regular octagon of diameters 80cm, 100cm and 120cm respectively consisted of 8 isosceles triangles; and rhomboid. *(iii) Integrating IBL:* Based on the students' collected information, the regular octagon and the rhomboid were excluded.

**Data collection and analysis**

The collected data consisted of: (1) teachers' e-mail messages; (2) transcripts of PD meetings; (3) teachers' resources (lesson plans, worksheets, ppts, digital files); (4) teachers' notes from hetero-observation; (5) teachers' activity reports; (6) teachers' interviews; (7) videos from classroom implementations. Following DAD methodology (Gueudet & Trouche, 2012): (a) we analyze teachers' work in time periods in and out-of-class (reflexive investigation principle); (b) we address their decisions taken in order to formulate their design through its use (design-in-use principle); and (c) we consider their work embedded in and influenced by different collectives such as the PD meetings (collective principle). We used data from different time periods: excerpts from their hetero-observation notes that took place before and after their own implementation, the developed materials and their transformations over time including the arguments with which they documented their options and presented in the PD meetings and in their activity reports.

## Results

### Preparation and initial designs

In this phase, the teachers worked on the initial idea aiming to formulate and orient the problem. Their documents included preparatory activities for hexagonal kites of 80cm diameter (since they were the most common type) aiming to help students understand that: (1) this kind of kite cannot be constructed by a single rectangular sheet of paper, thus the hexagon has to be divided in smaller different geometrical shapes; (2) hemming is necessary (i.e. each sheet edge is creased and rolled over onto itself) in order to make the kite robust; (3) companies save paper by using the remnants for the tail; (4) paper cost depends on the type of package and paper size. The initial document of tA for the 9th grade included an open task and it was further refined collectively by the group of teachers as follows: *"We are workers in a kite making company. Our director suggests reducing the waste of paper. So today we will work in groups focusing on the case of regular hexagon kites of 80cm diameter to explore the ways by which a hexagonal kite can be composed by different geometrical figures and the amount of paper needed. The dimensions of the paper we use are 70×100, the package includes 46 pieces and costs 23€. In order to reduce the paper, our company decided to construct the kite tail with newspapers instead of the remnants (as we used to do). Thus, our aim is to minimize the remnants of the paper by exploring how to combine different geometrical figures. These figures can be: (a) 2 isosceles trapeziums; (b) 4 rectangular trapeziums; (c) 3 rhombs; (d) 2 rectangles and 2 isosceles triangles; (e) 6 equilateral triangles. For gluing the internal figures, we use adhesive tape while for the external segments comprising the perimeter of the hexagon we use hemming. The hem is an isosceles trapezium of 4cm height"*. Students work in groups and study one of the above five cases. Below, we provide a brief description of the corresponding activities: (1) Students are given an 8×3 table (Figure 2). The first row includes two rectangles representing respectively a real sheet of paper (with known dimensions) and a 1:4 scale one. The students have to compute the dimensions of the 1:4 scale rectangle. The next six rows correspond to the above mentioned five combinations by which hexagonal kites can be constructed. In the second and the third column, regular hexagons are given representing respectively a real kite and a 1:4 scale one. For each combination of shapes the

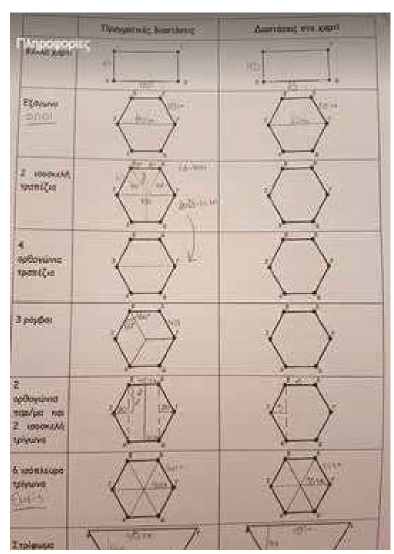

**Figure 2: A completed worksheet of activity 1**

students have to divide the given regular hexagon in corresponding parts and compute both the dimensions of the real kite and the ones of the 1:4 scale. In the last row, the students are asked to compute the dimensions of two given isosceles trapezoids representing the hems of the real kite and of the 1:4 scale respectively. (2) Each group announces the results to the whole class. (3) In a millimeter paper and in a 1:4 scale each group represents the real rectangular sheet and explores if a regular hexagon with peripheral hems fits within it. Then, they confirm that this construction is not possible through the use of a given Geogebra file. (4) The students cut with scissors the geometrical figures they worked on in the first activity (including their peripheral hems), put them down on millimeter papers (1:4 scale of the real paper) and glue them when achieving the best coverage (minimizing gaps). In this activity, students experiment to achieve the best coverage by combining their own geometrical figures with other groups' figures. They confirm their findings through the Geogebra file. (5) The groups exchange their cut geometrical figures to optimize their coverage.

The computations mentioned above, required the knowledge of properties of regular polygons, Pythagorean Theorem and trigonometry taught in 8th grade. The four teachers planned to implement the task as follows: tA and tC in 9th grade, tB in a 7th grade and tD in 8th grade. So, they adapted the initial design taking into account the respective curriculum and their different contexts. tC, for instance, whose class included many low achievers, expressed her concerns about her students' required knowledge. Thus, she designed a preparatory lesson aiming to remind the properties of the basic geometrical figures embedded in the kite construction as well as the steps for constructing a regular hexagon with ruler/compass. Then, the students were asked to divide given regular hexagons in corresponding parts (see a filled worksheet in Figure 3). At the same time, tB excluded the computations of the given figures' dimensions, since the 7th grade students do not have the required knowledge. His emphasis was on the concepts of scale and symmetry. In his design the regular hexagons and their geometrical components (both with peripheral hems) were given (Figure 4) and the students were asked to reproduce them in a sheet of paper (1:4 scale) and cut their components with scissors. He targeted the exchange of geometrical figures among the groups and the exploration of the best coverage with different combinations by using more than one sheets of papers. Finally, tD exploited the initial collective design for teaching the properties of regular hexagons in the 8th grade. Summarizing, community documentation in this phase revolved around a common body of activities of the initial design. Besides, they concentrate mainly on adaptations related to the curriculum and to their students' mathematical abilities/weaknesses.

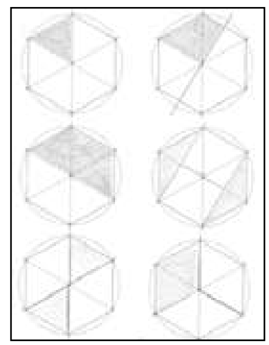

**Figure 3: Dividing a regular hexagon**

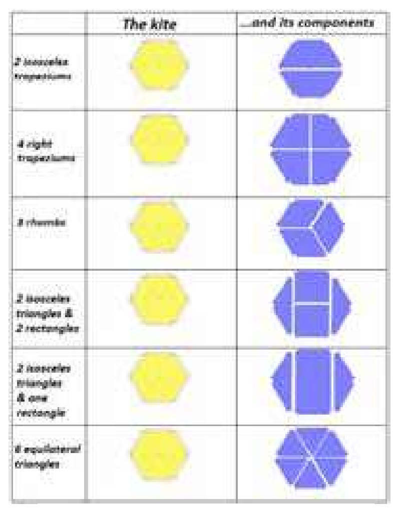


**Figure 4: Hems and geometrical components**

**Implementations and redesigns**

The first implementation took place in a 9th grade classroom of tA. Despite the fact that the whole process fascinated the students, but it was time consuming since: (1) Mistakes in the calculations and wrong application of geometric properties were noticed. (2) Each group undertook the calculations of a specific combination of geometrical components and communicated the results to the others. This process has been time consuming and students of one group may did not understand the calculations of the other groups. (3) The construction of the 1:4 scale hexagon on a millimeter paper and the cutting of its components based on the previous calculations were also time consuming. These difficulties resulted in the limitation of students' exploratory activity (e.g., there was no time left for exchanging the cut geometrical figures across the groups) and the failure to complete the lesson in the foreseen two hours.

**Figure 5: the combination of two groups**

The process of hetero-observation allowed the rest of the teachers to modify their initial designs by giving students ready-made materials (i.e. rectangles 17.5×25cm representing the real sheet for kites and regular hexagons of diameter 20cm with hems representing a real kite, all in a 1:4 scale) so as to have more time for exploration. The lesson of tB in a 7th grade classroom of his experimental school (where exploration and group work constitute common practices) embodied elements of both guided and open inquiry. He prioritized the use of manipulatives and digital representations (Geogebra) as a main element of IBL. After experimentation with different combinations for covering the 17.5×25 sheet, two groups of students decided that the optimal coverage of a sheet was achieved by the use of 2 rhombs and 2 equilateral triangles (Figure 5). Thus, 3 sheets include 6 rhombs and 6 equilateral triangles and allow the construction of 3 kites (2 kites with rhombs, 1 kite with equilateral triangles). This way the numbers of equilateral triangles and rhombs needed should be multiples of 6 and 3 respectively. Since each package includes 46 pieces, the students reached the notion of Least Common Multiple (LCM) as appropriate for finding the best solution (we note that LCM was not explicitly part of the teachers' design). Therefore, with three packages (138 sheets=LCM (3, 6, 46)) they could construct 138 kites (46 with equilateral triangles and 92 with rhombs).This finding provided an incentive for other groups to optimize the final solution. The lesson closed with a whole class discussion where the best solution was chosen and the connections to the realistic context (kite making industry) became explicit.

The emergence of the notion of LCM was not taken into account in the initial design and challenged further the teachers' reflection. Thus, the second implementations of tA and tC were characterized by a more conscious attempt of the two teachers to bring to the fore the notion of LCM. However, while tA's students' exploration resulted in better solutions in terms of paper save and diversity of combinations, tC's students faced a lot of difficulties. For instance, one group of tA used 4 sheets to construct 4 kites with three different combinations (i.e. 2 kites with trapeziums, 1 with equilateral triangles, and 1 with 2 isosceles triangles and 2 rectangles). Through the use of LCM, these students concluded that with the use of 92 sheets they can construct 92 kites of the above three different combinations. During her implementation (9th grade), tC faced the difficulties of tA's first lesson and also many students did not understand the role of hemming. As a result, the notion of LCM was not exploited to offer multiple solutions. tC in her activity report mentioned "I should have engaged them in the construction of a real kite with the use of paper and wooden straws". A comparison of the two implementations shows that tA followed an open form of inquiry taking more time than expected (3 teaching hours instead of 2), while tC - due to her students' low mathematical abilities - followed a structured form of inquiry downgrading the demands of the task.

The last implementation was carried out by tD in a 7th grade classroom. By observing the other teachers' teaching, she realized that many problems stemmed not only from the calculations but mainly by obstacles in the process of groups' exchange of geometrical components. Thus, she decided to support the communication among the groups by taking the role of 'crier', 'circulating' around and showing different geometrical figures so as to facilitate the choice of other groups' figures that minimize the free space in their sheets (i.e. she showed specific groups' solutions to the other groups highlighting the free parts of the sheet and motivating them to conceive its best coverage or its reconstruction by using some of their own geometrical figures). Besides, this choice was also related to the fact that the students' initial attempts left uncovered big parts of the sheet. In her activity report, tD considers that "A real kite construction in the classroom would have revealed to the students that a large amount of paper is lost". Her implementation embodied both guided and open inquiry: the first part of the worksheet offered guidance to the students in the introduction of the corresponding notions in the realistic context while in the second part of the lesson tD emphasized the importance of communication among the groups keeping a mediating role for her.

## Discussion and concluding remarks

In the preparation and initial design phase, the main issue was the orientation of the task consisted of: (i) the openness of the task; (ii) the embodied mathematical concepts; (iii) the specificities of constructing a real kite (kinds, dimensions, hemming, paper packages, costs); (iv) the complexity of mathematical exploration (geometrical figures and their properties, multiplicity of solutions); (v) the connection to the workplace context (cost, constrains in the use of paper) and the need of students' engagement in collecting relevant information; (vi) the roles of teachers and students; and (vii) the teaching management. The data collected by the students revealed the specificities of kite construction and the teachers' documents were adapted to their different school grades/contexts.

In the implementation phase, the initial open task took different forms of inquiry. The initial open approach of tA, brought to the fore issues that took time from inquiry. The open inquiry increased the level of uncertainty and resulted in not having a solution by the end of the first lesson. The process of hetero-observation was critical since: (1) tB adopted guided and open inquiry bypassing the time-consuming elements of tA's teaching and including ready-made materials to ensure more time for inquiry. This choice facilitated the emergence of concepts not initially anticipated by the teachers (i.e. LCM) and left space for unexpected solutions to emerge. (2) tA (in her second lesson) and tD followed guided inquiry aiming to facilitate students to approach gradually the notion of LCM and help them to find as many solutions as possible. They used ready-made manipulatives and their teaching management was aligned to support IBL and communication between the groups (i.e. tD acting as crier) targeting intentionally the emergence of different solutions through the notion of LCM. This kind of activity strengthened the connections to the workplace context and led to the extension of the mathematical inquiry (i.e. multiple solutions from tA's students). Finally, the tC's belief concerning her students' low mathematical abilities resulted in a structured form of inquiry. She tended to decrease the level of uncertainty by providing resources aiming to remind students the properties of the basic geometrical figures embedded in the kite construction. However, the low mathematical abilities of tC's and tD's students were again a factor that limited the emergence of multiple solutions. Another factor was students' difficulties in understanding the role of specific parts of kite construction (e.g. hemming). This was indicated in tC's and tD's reflections where they stressed the importance of engaging students in the construction of a real kite.

Summarizing, the factors that influenced the adopted forms of inquiry were: teachers' conceptions of their students' mathematical knowledge; the different school contexts (e.g., differences in students' prior mathematical experience, classroom norms, familiarization with IBL approaches); and the range of the prerequisite knowledge. The connection to the workplace context favored the emergence of mathematical concepts but the limited available time led teachers to structured or guided forms of inquiry. Hetero-observation facilitated the sharing of teachers' associated knowledge by promoting awareness of the above factors and became the incentive for subsequent modifications of the shared repertoire of resources, favoring shifts from one form of inquiry to another. In conclusion, these factors influenced the community documentation indicating hetero-observation as a catalyst mediating in a dynamic way the evolution of design and implementation.

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