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INTEGRATING RESEARCH TEAMS: THE TELMA APPROACH

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In the context of the Kaleidoscope Network of Excellence, six European research teams developed a methodology for integrating their research approaches. In this paper we present the methodology, based on a cross experiment, showing how it gave insight to the understanding of each team's research, and on the relationship between theoretical frameworks and experimental research.

INTRODUCTION

This contribution is about a research activity that is jointly carried out by six teams belonging to Kaleidoscope, a European Network of Excellence [1] that brings together many research teams in technology-enhanced learning. The aims are, on the one hand, to develop a rich and coherent theoretical and practical research foundation and, on the other hand, to develop new tools and methodologies for an interdisciplinary approach to research on learning with digital technologies at a European level (TELMA ERT 2006).

Within the activities of Kaleidoscope, a European Research Team (ERT) TELMA – Technology Enhanced Learning in Mathematics – has been established to focus on the improvements and changes that technology can bring to teaching and learning activities in Mathematics. TELMA ERT includes six teams [2] with a strong tradition in the field, and most of which have also been engaged in designing, developing, testing and integrating Interactive Learning Environments (ILE) for use in mathematics learning. TELMA first aim is to promote integration among such teams and to favour (a) the construction of a shared scientific vision, (b) the development of common projects and (c) the building of complementarities and common priorities in the area of digital technologies and mathematics education.

TELMA teams have brought with them different research questions, theoretical frameworks, work methodologies, cultural perspectives and views of the use of digital technologies for the teaching and learning of mathematics. So the teams started sharing knowledge, developing a common language and common topics of interest. This demanding task was addressed by analysing documents and some of the most significant papers provided by each team, focusing on topics considered as important for mutual knowledge and comparison among teams, such as digital technologies developed and used by the teams, theoretical frameworks and work methodologies, and contexts of digital technologies use. This work allowed identifying some common concerns (e.g., contextual, social and cultural dimensions of learning, instrumental issues, etc.), but it also put forward a diversity of ways to

deal with these common concerns which is due mainly to the variety of theoretical frameworks used by the teams (ibid.). For the sake of developing an integrated approach to the research on technology enhanced learning of mathematics, the need emerged to get a deeper insight on the role played by the theoretical frameworks each team uses in its own research. Aiming at finding some common perspectives, the teams decided to prepare a joint short-term project based on a cross-experimentation under which to look at the different teams' approaches concerning three interrelated topics: theoretical frameworks within which the teams face research in learning mathematics with technology, the role assigned to representations provided by technological tools, and the way in which each team plans and analyses the context in which the technology is employed.

This paper focuses on the teams' collaborative work aiming at *highlighting how specific theories may influence empirical research* as well as to *exhibit joint methodologies which can be used to compare, combine, integrate and complement different theoretical approaches*.

METHODOLOGY

TELMA teams' collaborative work is based on a cross-experimentation whose aims (among others) were to provide a better understanding of the ways theoretical frameworks influence (a) the analysis of a given educational software and of the potential it offers for the mathematics learning, (b) how this potential is exploited in a particular learning context, and (c) how the results of this exploitation are analysed and interpreted. Two main methodological tools were developed and used for achieving these goals: 1) the construct of *didactical functionality* of a tool; 2) a *cross-experimentation* framed by and developed together with collaboratively-produced guidelines.

The construct of Didactical Functionality

The construct of Didactical Functionality (DF) (Cerulli et al. 2005) was built with the aim of providing a common perspective, independent from specific theoretical frameworks, to address the variety of approaches (possibly depending on theoretical references) to the use of ILEs (as ICT tools) in mathematics education, and to link theoretical reflections and actual uses of ILEs in given contexts.

'With didactical functionalities we mean those properties (or characteristics) of a given ICT, and/or its (or their) modalities of employment, which may favor or enhance teaching/learning processes according to a specific educational goal.

The three key elements of the definition of the *didactical functionalities* of an ICT tool are: (1) a set of *features/characteristics* of the tool; (2) a *specific educational goal*; and (3) a set of *modalities of employing* the tool in a teaching/learning process referred to the chosen educational goal.' (ibidem, p.2)

These three dimensions are inter-related: although characteristics and features of the ILE itself can be identified through a priori inspection, these features only become functionally meaningful when understood in relation to the educational goal for which the ILE is being used and the modalities of its use. We would also point out that, when designing an ILE, designers necessarily have in mind some specific DFs, but these are not necessarily those which emerge when the tool is used. This may be especially the case when an ILE is used outside the control of its designers, according to different epistemological or educational perspectives, or in contexts different from those envisaged by the designers.

The notion of DF took a central and unifying role in the design and development of the cross-experimentation:

- on the one hand, the cross-experimentation aimed at exploring the DFs that the different teams would associate with an ILE they did not design;
- on the other hand, this notion was also used to structure the methodology for exploring the role played by theoretical frames in designing empirical research.

In fact, the three dimensions constituting the notion of DF are supposed to be always addressable, no matter what the theoretical assumptions of the research which is being analysed are.

The cross-experimentation

The cross-experimentation was intended to enhance integration among the teams, by addressing a shared set of research questions derived from the three key themes of interest of the project: contexts, representations, and theoretical frameworks. On the one hand the investigation of these themes constitutes a first level of integration among TELMA teams, at least in terms of addressing shared issues. On the other hand such themes are wide and open the space for a huge number of possible research questions: the need emerged to restrict a feasible smaller number of questions. In general, the choice of specific questions to address may depend on one's interests, on possible theoretical frameworks of reference, or on other constrains. This potentially constituted a sort of centrifugal force among the teams which could contrast with the aims of the cross-experimentation itself. Thus, common questions were chosen according to a specific methodology, as detailed in the next paragraph.

One principal characteristic of the cross-experimentation was the request for each experimenting team to design and implement a teaching experiment making use of an ILE developed by another TELMA team. This decision was expected to induce deeper exchanges between the teams, and to make the influence of theoretical frames more visible through comparison of the DFs envisaged by the ILE designers and those identified by the experimenting teams. Table 1 summarises the ILEs chosen, the teams who developed the ILEs and the teams conducting the experimentation.

ILE	Developer's team	Experimenting team(s)
<i>Aplusix</i>	MeTAH	CNR-ITD, UNISI
<i>E-Slate</i>	ETL-NKUA	UNILON
<i>ARI-LAB 2</i>	CNR-ITD	MeTAH, DIDIREM, ETL-NKUA

Table 1: The tools employed by TELMA teams in the cross experiment

Finally, in order to allow as much comparability as possible between the research settings, it was also agreed to address common mathematical knowledge domains (fractions and algebra), with students between years 7 and 11 of schooling in experiments lasting approximately one month.

The Guidelines

The *Guidelines* is a document collaboratively produced during the cross-experimentation which includes the research questions to be answered by each designing and experimenting team in order to frame the process of cross-team communication, as well as the answers provided by the teams before, during and after the experiments. This document was meant to draw a framework of common questions providing a methodological tool for *comparing the theoretical basis of the individual studies, their methodologies and outcomes*. Thus the questions had to reflect on the one hand the shared objectives of the cross experiment and its constraints, and on the other hand, the specificities of each research team. Thus the Guidelines were jointly built according to the following procedure:

- Three researchers of the TELMA group, experts in the subjects, developed three documents (one for each of the three key themes addressed by TELMA) each consisting of a set of possible research questions to focus on.
- The teams reviewed such documents and jointly chose a small set of questions to be addressed. The choice followed the criteria of (a) relevance to teams' interests and (b) feasibility within the cross-experimentation constraints.
- *A priori*, *a posteriori* and *a priori/a posteriori* sets of questions were developed to be answered by the experimenting teams respectively before, after and both before and after the experiments.
- In addition, each team that produced a tool employed in the experiment was required to provide a description of the educational principles underlying the design of the tool, and to indicate possible DFs of the tool.

Two examples of questions concerning theoretical frameworks are the following:

Example 1 (theoretical frameworks - a priori):

What theoretical frame(s) do you use and what motivated your choice? How do you see their potential and eventually limitations for this project?

Example 2 (theoretical frameworks - a posteriori):

In your opinion, in which ways do your theoretical choices have influenced:

- the analysis of the software and the identification of its didactic functionalities?
- the conception of the experiment?
- the choices of the data and their analysis?
- the results you obtain and the conclusions you draw from these?

The cross-experimentation and the Guidelines

After the production of the first version of the Guidelines document containing a set of key questions to be addressed and identifying basic information to be provided by each team, the Guidelines became a key element around which the main phases of the cross-experimentation were developed:

1. Production of a pre-classroom experiment version, containing plans for each experiment and answers to some questions (a-priori questions).
2. Implementation of the classroom experiments.
3. Analysis of the experiments.
4. Production of the final version of the Guidelines containing answers to all the addressed questions (including a-posteriori questions).

The Guidelines may be considered both as a product and as a tool supporting TELMA collaborative work: a product in the sense that the final version contains questions and answers as well as plans, descriptions of the experiments and results, and a tool in the sense that the Guidelines structured each team's work by:

- providing research questions concerning contexts, representations, and theoretical frameworks;
- establishing the time when to address each question (e.g., before or after the classroom experiment, etc.);
- establishing common concerns to focus on when describing classroom experiments, on the basis of the definition of DFs;
- gathering under the same document, the answers provided by each team to the chosen questions, in a format which could possibly help comparisons.

In a sense, the Guidelines go both in the direction of investigating how to employ a given ILE in maths education and in the direction of integrating the work conducted by teams.

The Guidelines became also a tool for analyzing the role played by theoretical frameworks in the design, implementation and analysis of the experiments themselves and for comparing and possibly integrating different research approaches of the teams. In fact the process of building the Guidelines, and at the same time of using them as a reference for comparing teams' researches, contributed to:

- the investigation of the relationships between teams' assumed theoretical frameworks and the employed/defined DFs (and questioning the effectiveness of such DFs).
- the analysis of teams' classroom experiments design processes, and the explanation of the key choices characterising such processes, could they be depending on theoretical assumptions, institutional, cultural or other constrains.

Such objectives were addressed on the one hand, by comparing and questioning teams' answers to the questions contained in the guidelines, and on the other hand, by addressing extra questions, like the one of *example 3*, a preliminary question for preparing the terrain for answering the a posteriori question of the guidelines reported in *example 2*:

Example 3 (DF – extra question):

If you were to design a new experiment aiming at the same mathematical educational goal and employing the same ICT tool, which characteristics of the experiment would you keep unchanged? Which of these characteristics do you think, according to the theoretical framework you chose, are necessary conditions for the experiment to be successful?

This kind of questions bridges the DFs employed/defined by teams for their experiments, and the theoretical frameworks they assumed.

RESULTS

As specified in the previous paragraphs, different issues concerning the role of theoretical frameworks in the design of teaching experiments were explicitly addressed by the cross-experimentation. In what follows, we outline the most significant elements emerging from the compared analysis and discussion of many aspects of the experiments carried on by TELMA teams. We start with TELMA researchers' retrospective reflections on the methodological tool itself.

Making clear and communicating the implicit

The relationship between theoretical reflection and cases of practice is certainly one of the main issues that characterised the effectiveness of the cross-experiment either as a tool for comparing/integrating research approaches, or as a tool for investigating how to employ ILEs in mathematics education. In particular, researchers involved in the cross-experiment witnessed the importance of the request of conducting an explicit reflection on issues such as “research questions”, “theoretical frameworks”, “educational goals”, “analysis of the ILE”, and the relationships between them, which influence each other, and which remain often implicit. The request to communicate to the other teams how these issues influenced each other and how they influenced/determined the design, implementation and analysis of classroom

experiments, forced each team to address them explicitly, and to leave as less unexplained choices as possible.

The effort of making explicit the possible implicit factors when designing teaching experiments may not be new, however even when a researcher autonomously faces this task, s/he often deals with her/his own concerns, addresses self-posed questions. On the contrary, the reflection brought forward during the TELMA cross-experimentation required researchers to address (in practice, not only at a hypothetical level) also questions/issues raised and formulated by other researchers. As a consequence, each researcher was asked to cope with theoretical frameworks and with approaches to research in mathematics education that could possibly be not compatible with her/his own ones.

TELMA researchers share the common feeling that though highly demanding *the request of making clear and communicating resulted in a very useful effort both in terms of refining each team's investigation concerning ILE in maths education, and in terms of making the descriptions of the single classroom experiments as comparable as possible.*

The interaction between theoretical reflection and cases of practice

The cross-experiment gave insights on how cultures and theoretical frameworks influence deeply how researchers conceive, conduct and analyse experiments. In what follows, we report on some interesting results with this respect.

On the conception of the experiment. Contextual and representational issues were central aspects of the study developed within TELMA project together with issues related to the role of teacher, social interaction and so on; consequently these were central issues of the cross-experimentation as well. Nevertheless the research teams did not address such aspects in the same way: rather, the cross-experimentation shows that though addressing the same main issues, *different teams had different priorities when designing their experiments.*

Such priorities (and differences among teams' approaches) may be determined by cultural backgrounds, theoretical frameworks and ways of approaching and conceiving research in maths education. For instance, in the experiment carried out by the DIDIREM team, the main theoretical references were the *Theory of Didactic Situations* (Brousseau 1997) and the *Anthropological Theory of Didactics* (Chevallard 1992). As a result, major attention was paid to (a) a detailed organization of a (potentially) cognitively rich 'a-didactic milieu' and (b) a distance between the experimental and the usual institutional contexts, as well as the necessity to keep this distance manageable by the teacher. Consequently, other aspects, even if considered interesting, were less emphasized (e.g., students' collaborative work, teacher's role beyond the management of the devolution and institutionalization processes).

On the contrary, the CNR-ITD team mainly referring to *Socio-constructivism* and *Activity Theory* (Cole and Engeström 1993; Engeström 1991; Vygotsky 1978)

assigned a high priority to social construction of knowledge and to the role of the teacher. Therefore, the experiment was mainly focused on these issues and minor attention was paid to other aspects (e.g., detailed organization of the milieu), many choices were not set up by the experimenting team but left to teachers (e.g., specific tasks and orchestration of the work).

Finally, let us quote ETL-NKUA team's theory-driven choice of not defining a 'strictu sensu' didactical goal for its experiment. Mainly referring to theories on 'the generation of mathematical meanings' such as *Constructionism* (Harel & Papert 1991) and *Situated Abstraction* (Noss & Hoyles 1996), ETL-NKUA researchers paid emphasis not on 'closed didactical goals' but on pupils' active construction of meanings as they operationalize the use of the available tools while making judgments and taking decisions in the process of solving a problem.

We hypothesize that such priorities may remain implicit and act as hidden variables – out of one's control – when designing experiments. The request of making clear and communicating allows/makes these variables revealed.

What theoretical frameworks do not say. In the previous paragraph we cited a few examples of how theoretical frameworks may – implicitly or explicitly – drive the design of a teaching experiment. This is but a part of the story; in fact the cross-experimentation revealed that though a theoretical framework may influence/inspire an experiment at a global level, it may not address/define many specific relevant aspects for the actual set up of the experiment itself. There seems to be a sort of a gap between what a theoretical framework offers, and what is needed to put into practice (within a classroom experiment). Such a gap is at the core of the relationship between theoretical reflections and cases of practice, and it remains often implicit. In the case of the TELMA cross-experimentation, the gap was revealed through comparisons among the different teams' experiments.

With this respect, the comparison results inspiring between UNISI and ITD-CNR experiments and between MeTAH and DIDIREM ones.

UNISI and ITD-CNR teams referred to compatible theoretical frameworks – respectively the *Vygotsky's Theory* (as for the construction of higher psychological functions) and the *Activity Theory* – and centered their experiments on the use of the same ILE, namely Aplusix. Nevertheless, from the ILE analysis, they identified different educational aims for their experiments. This resulted in two teaching experiments, both consistent with the respective theoretical frames, but deeply contrasting between them for the role of the teacher, the kind of tasks given to pupils, the validation of pupils' work, the use and set up of the tool.

Similarly, MeTAH and DIDIREM teams shared the same theoretical background - *Theory of Didactical Situations, Anthropological Theory of Didactics* - and experimented the same ILE: AriLab2. But their experiments still differed (though less dramatically than UNISI and ITD-CNR experiments) for important aspects such

as: who/what is responsible for validating pupils' work? Does validation emerge as a social product? Does it rest with the teacher? Or the opposite, does it rest with the ILE? Are pupils allowed/obliged/forbidden to use systems of representations other than those provided by AriLab2 (e.g., paper and pencil)?

CONCLUSIONS

In this paper we exhibited the specific methodology followed by TELMA teams to address the question of investigating how specific theories may influence empirical research. We have reported on four main facets of the TELMA work: (a) the use of the construct of DF as a means to link theoretical reflections and actual uses of ILEs in given contexts; (b) the collaborative design and realisation of a cross-experimentation approach as a joint methodology to help different developing and experimenting teams to make explicit their assumptions and the set up of their experimental investigations; (c) the development of a methodological tool (i.e., the Guidelines) for comparing the theoretical basis of the individual studies, their methodologies and outcomes, and (d) the preliminary analysis of the experiments.

This preliminary analysis evidences two essential facts that contribute to the existence of a gap between the theoretical and the practical facets of an experiment:

- theoretical frames do not fully determine the design of situations aiming at an efficient use of an ILE. Many decisions taken in the design and the implementation of such situations engage other forms of rationality or are shaped by cultural and institutional habits and constraints.
- theoretical frames themselves often act as implicit and naturalized theories, more in terms of general underlying principles than of explicit operational constructs.

These issues certainly contribute to explain why the first step of the TELMA work based on the reading of published papers was only moderately productive. Making the role played by theoretical frames visible and not just invoked needed specific methodologies. With this respect, the results sketched above comfort the efficiency of the methodology developed within the TELMA project, but the exportability of the presented methodology cannot be taken for granted. Is it applicable to other research projects? What are the conditions for its applicability? Moreover, given that different forms of rationality are implicitly engaged in the design and implementation of teaching experiences, to what extent may such implicit factors be accessible to an explicit study? Finally, we believe that this kind of research is of particular importance in the European context where more and more teams are involved in cross-country projects. With this respect, the TELMA experience rises the question of what level of integration among different research teams is actually possible and what level of integration is desirable if one wants to preserve the richness of the teams' differences. Some of these questions are being addressed in ongoing work of TELMA, and in other projects involving TELMA teams.

NOTES

1. Kaleidoscope is an initiative founded by the European Community (IST-507838) under the VI Framework Programme. See www.noe-kaleidoscope.org.
2. The teams (whose acronym is indicated in brackets) belong to the following Institutions: Consiglio Nazionale delle Ricerche – Istituto Tecnologie Didattiche – Italy (CNR-ITD); Università di Siena – Dipartimento di Scienze Matematiche ed Informatiche – Italy (UNISI); University of Paris VII – France (DIDIREM); Grenoble University and CNRS – Leibniz Laboratory – France (MeTAH); University of London – Institute of Education – UK (UNILON); National Kapodistrian University of Athens – Educational Tecnology laboratory – Greece (ETL-NKUA).

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