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Investigating two trainee teacher educators' transformations of the same resources in technology enhanced mathematics

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We address the documentational work of two trainee teacher educators in the context of their practicum in an in-service program concerning the use of digital tools in mathematics. Since they drew upon the same existing resources, we investigated the operational invariants – i.e., implicit aspects of their knowledge and epistemology underlying their designs – that influenced and differentiated their documentational work. The identified operational invariants were (a) their focal points during the observation of other teacher educators' classrooms, (b) the constrains and opportunities provided by the educational context, (c) their epistemologies regarding the role of technology in the teaching of mathematics as well as their conceptions of trainee teachers either 'as students' or 'of students'.

Keywords: Documentational approach, teacher educators, digital tools.

INTRODUCTION

In this paper, we study the didactical design and corresponding material developed by trainee teacher educators in teaching mathematics with the use of digital tools. The trainees drew upon existing resources as they started to teach in teacher education classrooms during their practicum and created their own documents. Our focus is on the factors that influence the development of the trainees' documents based on the same existing resources and the classroom implementation of their design.

The study took place in the context of an in-service program adopting reform-oriented perspectives to train teacher educators into the use of digital tools in the classroom of mathematics. The aim of the program was to provide the participants with methods, knowledge and experience in in-service teacher education and to educate them in the pedagogical uses of digital technologies for the teaching and learning of mathematics. One of the reform aspects of the approach for teacher education (see Kynigos & Kalogeria, 2012) concerned teacher educators' and teachers' active engagement in creating their own didactical design and material as coherent part of their professional development. Taking into account that teacher educators have very few resources to draw on directly (Zaslavsky, 2008), it was critical for the trainees to get used to developing their own material. In this course, the trainees were engaged in designing and generating resources in the form of microworlds and scenarios (i.e., structured activity plans addressing critical aspects of a pedagogically sound use of technology for the teaching and learning of mathematics). A structure [1] for addressing these aspects was developed by Educational Technology Lab (http://etl.ppp.uoa.gr), which participated in the design of the course and the corresponding material. The training program took place in specialised University Centres (UC) for 350 hours. The participants were experienced qualified mathematics teachers but the majority of them had no previous experience in the pedagogical use of digital tools. The plan was to employ the newly trained teacher educators in wide-scale 96h courses to groups of teachers in specific Centres for Teacher Education Support (CTES). The trainees in UC were given material by the trainers after each lesson and an official document containing theory and a set of twelve generic scenarios as a basis for organizing their subsequent teaching in CTES. During the course, the trainee educators gained significant experience with the pedagogical use of five categories of digital media: Computer Algebra Systems, Dynamic Geometry

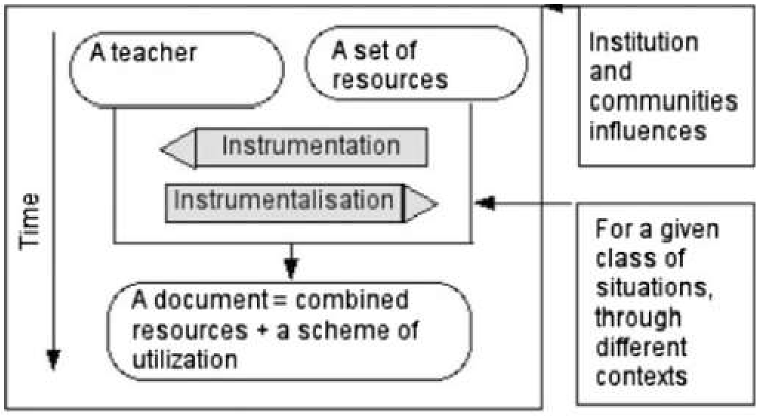


Figure 1: Documentational genesis

Systems, Programmable software, Simulations and Data Handling tools. By the end of the course the trainees had to have developed one scenario for each of these categories as well as scenarios for the practicum. Practicum was part of UC official structure provided shortly before the end of the course, so as to engage trainees in field activities and give them the experience of implementing their design in real classroom conditions and reflecting on it. Practicum took place in 30 hours and it was divided in two parts: teaching in school and observation – teaching in CTES. Here in focus is the second part consisted of (a) observation of other teacher educators' teaching in CTES, (b) design of a 3-hour lesson for teachers in CTES under the supervision of a mentor, (c) implementation in the classroom, (d) presentation of design and implementation in whole class special reflective sessions, (e) activity report by the trainees.

THEORETICAL FRAMEWORK

Over the last years a number of researchers have indicated that the study of resources in practice and context constitutes an important theme in mathematics teacher education and deserves a focus of attention (Adler, 2000). We adopt the documentational approach of didactics according to which the teacher's work is developed with and on resources in a dialectic process where design and enactment are intertwined (Gueudet & Trouche, 2009). An implication of this approach is that curriculum material is not conceived as a static body of resources that guides instruction but rather as a set of objects amenable to changes depending on the teacher's didactical design. Gueudet and Trouche (2009) use the term resources to describe a variety of artifacts such as a textbook, a piece of software, a student's sheet, discussions with colleagues etc. Through a class of professional situations and teachers' experience, the existing resources are transformed into documents according to the formula: *Document = Resources + Schemes of Utilization.*

A scheme of utilisation of a set of resources incorporates "*practice* (how to use selected resources for teaching a given subject) and *knowledge* (on mathematics, on mathematics teaching, on students, on technology)" (Gueudet & Trouche, 2011, p. 401). Practice entails observable parts of teachers' stable behaviour for a given class of situations (called usages). On the other hand, knowledge embodied in resources is implicit and can be inferred from the usages. This knowledge is intertwined with teachers' beliefs and it is difficult to distinguish one independently of the other (Thompson, 1992). The constituent elements of this kind of knowledge are the *operational invariants* built through different contexts of using the resources.

Creation of documents is considered as unfolding through a dual process of *instrumentation* (the resources act on the teachers and influence their activity) and *instrumentalization* (teachers act upon these resources as they appropriate and reshape them) (Gueudet & Trouche, 2009). This process is named documentational genesis (ibid, see Figure 1) and gives birth to a new entity: a document, which can be further transformed to a new document over time.

The importance of interrelating knowledge and epistemology, the difficulty to distinguish them, as well as their influence on the everyday teaching practice has been stressed by many researchers (Nespor, 1987; Thompson, 1992; Ernest, 1994). However, the relation between teachers' epistemologies and practices is complicated and not linear as it is strongly affected by constraints and opportunities afforded by social context (Ernest, 1994). For example, a teacher's conceptions regarding the teaching of mathematics may be rooted in the epistemological paradigm of absolutism while her/his own practices might be closer to the paradigm of fallibilism and vice versa. Thus, interconnections between teacher's practices and epistemologies formulate the following different roles in her/his own teaching (ibid): instructor (i.e., targeting skills' mastery and correct performance), explainer (i.e., targeting conceptual understanding with unified knowledge) and facilitator (i.e., targeting problem posing and solving). At the same time each role signifies a different stance towards the curriculum and the corresponding teaching material and leads to three patterns of curriculum use (ibid): the strict following of a text; the modification of the textbook approach, enriched with additional problems and activities; and the construction of the mathematics curriculum by the teachers themselves. As we will analyze later in the paper, the trainee teacher educators of our study – despite the fact that they were given the same initial resources – adopted different roles and corresponding didactical designs when they started teacher education themselves in the context of their practicum. We consider that these differences were determined by a combination of operational invariants that was unique for each one of them. Thus, in order to explain the differences between trainees' design and implementation, it was our choice to investigate the underlying operational invariants both in the design of documents and their usages in real educational contexts. Existing research on teachers' documentational work highlights the need to identify categories of operational invariants permitting refinement of the analysis of schemes of utilisation (Gueudet & Trouche, 2009).

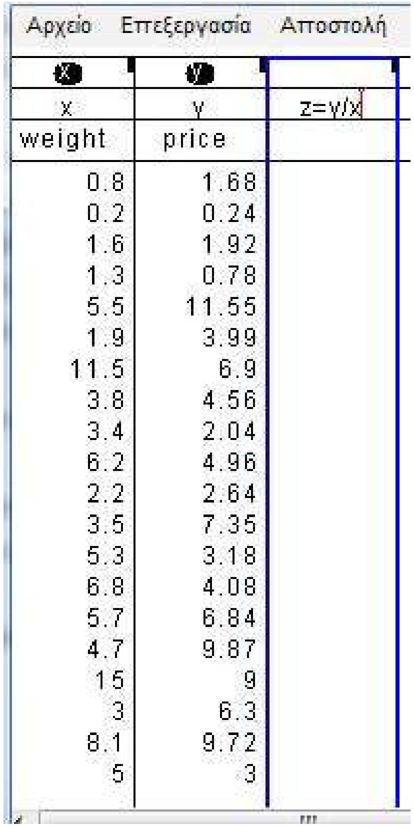
In this study, our aim is to investigate how the formula *Document = Resources + Schemes of Utilization* works when different teacher educators build the development of their documents upon the same existing resources. Thus, we investigate the underlying factors (i.e., operational invariants) that influence trainee teacher educators' design. We looked for these operational invariants within the space where the processes of instrumentation and instrumentalisation take place.

METHOD

The UC class of trainee teacher educators that we analyse here consisted of 16 qualified mathematics teachers (five of them had a doctoral degree, one was a PhD student and the rest held a Master degree). The data we analysed consisted of: (1) verbatim transcription of the discussions that took place during the 30 hours of the practicum in the UC classrooms, (2) the material constituting the trainees' designs for their lessons in CTES (scenarios, worksheets, ppts, etc.) and (3) their activity reports. The activity reports were templates in which trainees had to insert text describing aspects of their designs and their experience from its implementation. From the analysis of the 16 trainees' documentational work, in this paper, we present two cases with the aim to highlight differences in trainees' design of documents and implementation in teacher education classrooms. Particularly, we chose Ian and Jim as exemplary cases because their teaching in CTES was based on the same official scenario. This allowed us to view comparatively the documents they created and thus to address the underlying operational invariants. Our role as academic trainers and mentors in the practicum allowed us to capture the evolution of their documentational work in all phases of the practicum. In resonance with Gueudet and Trouche's (2011) principles regarding methodological aspects of research on documentational genesis we chose to (a) analyze Ian's and Jim's work in time periods in and out-of-class (reflexive investigation principle), (b) address their decisions taken in order to formulate their design through its use (design-in-use principle), and (c) consider their work embedded in and influenced by different collectives (e.g., teachers in CTES) (collective principle). We used data from different time periods: excerpts from their observations in CTES that took place before their design during the reflective sessions, the official material and its transformations by Ian and Jim including the arguments with which they documented their options and presented in the reflective sessions and finally Ian's and Jim's activity reports. In the analysis we adopted a data grounded approach (Strauss & Corbin, 1998). Initially, each one of us (the two authors) worked separately for coding trainees' work and identifying operational invariants. After reaching a common consensus, we jointly completed the analysis.

ANALYSIS

Ian's and Jim's lessons in CTES concerned the teaching of linear functions ($y = ax$) with Function Probe (FP) [2]. Their teaching was based on one of the twelve official scenarios provided for the course. The problem


Αρχείο Επεξεργασία Αποστολή

x	y	z=y/x
weight	price	
0.8	1.68	
0.2	0.24	
1.6	1.92	
1.3	0.78	
5.5	11.55	
1.9	3.99	
11.5	6.9	
3.8	4.56	
3.4	2.04	
6.2	4.96	
2.2	2.64	
3.5	7.35	
5.3	3.18	
6.8	4.08	
5.7	6.84	
4.7	9.87	
15	9	
3	6.3	
8.1	9.72	
5	3	

Figure 2

included in this scenario was: "A salesclerk sales three products A, B, C of different prices. After every sale he records the quantity x (in kg) and the amount of money y received. When he had completed twenty sales he passed the values in two columns in the Table window of FP so as to check: (a) how many sales were made by each product and (b) if there had been mistaken sales. In how many ways can he conduct the check?". The indicative design of the official scenario suggested the following teaching sequence in three phases.

<u>Phase 1</u> *(Exploring regularities – relations between different groups of proportional amounts):* It is provided a ready-made table containing 20 pairs (weight, price) of 20 sales in two corresponding columns. Teacher can chose or discuss with the students the possibility to fill in one more column defined by the ratio $z = y/x$ (Figure 2). In this case, students are expected to notice that this ratio takes three particular values (0.6, 1.2, 2.1) except one. These values can be an indication for the existence of linear relations $y = ax$. Then the students send the points (x, y) to the Graph window (Figure 3) and they are expected to notice that the three values of the ratios correspond to three different groups of collinear points. Also they are suggested to construct the graph of $y = x$ and stretch it dynamically with the mouse so as to coincide to each of the existent groups of points (Figure 4). The corresponding formula for each transformation of the graph appears in the upper right corner of the Graph window. The students are expected to explore the problem either algebraically (i.e., by linking the corresponding ratios to linear functions) or geometrically (i.e., by dynamically manipulating the graph of $y = x$) and to connect the three ratios that encountered in the table to the three values of the coefficient a of $y = ax$ respectively.

<u>Phase 2</u> *(Testing the formulas of proportional amounts through different representations):* The students can be engaged in generating the exact prices of each product in three different ways: (a) In the Table window: they can construct one new column corresponding to the weight of each product (use of command 'Fill in' with values from 0 to 10 and step 0.1). Then, they are asked to construct three new columns so as to calculate the exact prices of each these products using the formulas $y = 0.6t$, $u = 1.2t$, $z = 2.1t$. This way the salesclerk will be able to know the exact price for each product until the weight of 10 Kg without calculating it. (b) In the

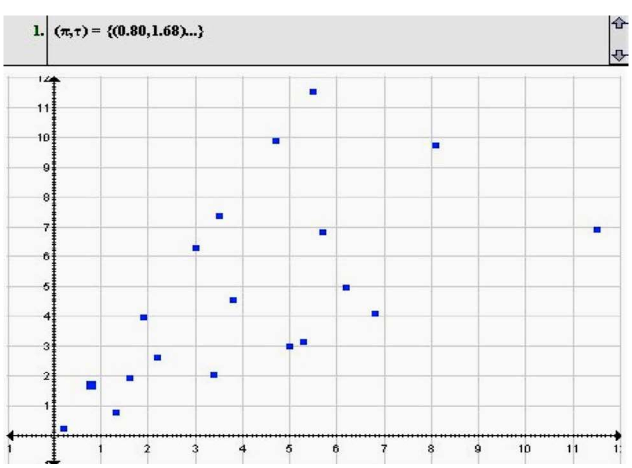


Figure 3: Sending points to the Graph

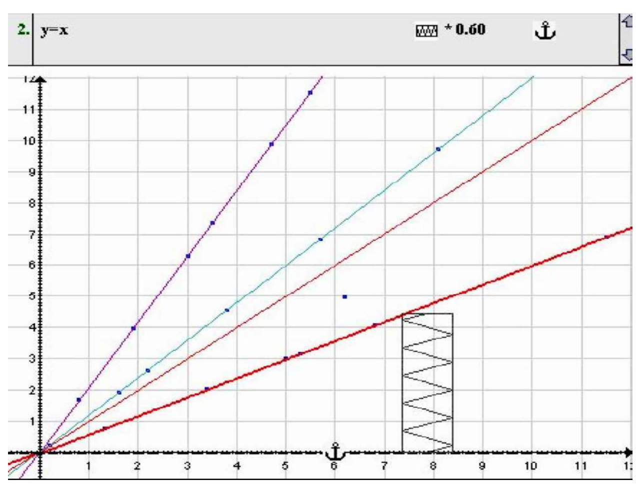


Figure 4: Stretching $y = x$

Calculator window: they can create three buttons (see [2]) for calculating all prices of the products through a functional relation. (c) From the Graph to the Table window: they can take points from the graph and send them to two columns in the Table standing for x and y respectively.

<u>Phase 3</u> *(y=ax+b as a transformation of y=ax):* $y = ax + b$ can be investigated through vertically stretching $y = ax$ at the Graph window and constructing new columns in the Table window defined by functional relations (e.g., $w = 0.6t + 0.5$).

Ian's documentational work

During his observation in CTES Ian, in both his activity report and the reflective sessions, concluded that in order to design his own lesson in CTES had to take into account the following aspects: (a) the different levels of teachers' familiarization with digital technologies, (b) difficulties with particular tools/functionalities of FP (e.g., creation of buttons in the Calculator), (c) difficulties in conceiving the links between the different representations of FP. Thus, he started his lesson in CTES by demonstrating particular functionalities of FP in a whole class session through the use of an interactive whiteboard. Then, he gave to the teachers a worksheet with the ready-made table of the official scenario. The worksheet covered the three phases of the official scenario and it was structured in the form of small steps-instructions insuring the correct use of FP tools for the requested activities. For instance, instructions concerning the first phase were the following: "Table window: Fill in the 3rd column with the ratio *y/x* and press enter", "Write down the resulting values and explain what they show", "Table window: Send points to the Graph", "Graph window: Graph → Graph choices → Check 'Show transformations' → Click on the icon '*y*=…' to create the graph of $y = x$". In the second phase, Ian provided a detailed account of the three ways by which teachers could have the prices of each product through the use of the Table, the Graph and the Calculator window respectively.

In his activity report Ian describes his choices in the design as shown in the following excerpt: "The development of my design follows the existing official material of CTES with a differentiation as regards the worksheet which is more instructional due to a better time control … Though the worksheet should provide learners with freedom to pose and answer their own questions, I chose to make it more instructive – probably more than needed – so as to support teachers' familiarization with the available tools while solving the problem … Another reason for choosing a more instructive perspective stemmed from the fact that the trainer of this particular CTES classroom had a similar approach in his own teaching. Thus, I targeted a more smooth transition from teachers' trainer to myself …". During the reflective sessions in UC, Ian again justified his choices in the design of the worksheet after being asked by another trainee if he had been satisfied by his teaching in CTES and if he would change something in it: "I would prefer a less instructive design. But I was anxious about time. Through my choice, the worksheet was completed by all teachers in time. They liked the environment and seemed to have learned better the FP functionalities".

Teachers' and students' engagement in exploratory activities with digital tools was a central idea underlying the design of the UC course. Ian seemed to have shared this view, but he finally designed a rather instructive lesson in CTES. Thus, Ian seems to consider teachers mainly 'as students' and his design targets the development of two kinds of knowledge: (a) about technical aspects of software and (b) about the ways by which the available tools can be interrelated to subject knowledge. For instance, he provides detailed instructions on how to approach the targeted functional relation through the different FP windows (Table, Graph, Calculator).

The operational invariants underlying his design were related to the following factors: (a) his emphasis on constrains and opportunities afforded by the context (e.g., time restrictions, technological environment) which led him to follow instructive design and practices respectively, regardless of his (possibly different) epistemological conceptions for the teaching and learning of mathematics; (b) his observation in CTES concerning teachers' difficulties with FP and the previous teaching model adopted by the trainer in CTES; (c) his conception of trainees mainly 'as students' who need detailed instruction in order to overcome their difficulties. All these elements influenced the instrumentation/instrumentalisation interplay in his design leading to the integration of small-step instructions in the worksheets.

Jim's documentational work

Jim observed another trainer's lessons in CTES that preceded his own lesson and included teachers' initial familiarization with FP tools apart from the stretch tool. Thus, he started his lesson by demonstrating in the interactive whiteboard technical issues related to the use of FP with an emphasis on the use of the stretch tool. His worksheet included the following open tasks for the teachers (without any kind of instructions, even for aspects of the software): 1) The teachers were asked to work in groups of two and provide both an algebraic and a geometrical solution for the given problem. Construction of tables of values for the three products (in the Table window) or creation of buttons (in the Calculator window) is suggested. 2) Group discussions for the potential findings of a group of students and ways to approach them. 3) Whole-class discussion of the above findings. 4) Whole-class discussion for the added value of FP. 5) Design of indicative questions for the students by each group of teachers. 6) Presentation of groups' design in the classroom.

Describing his lesson in one of the reflective sessions in UC, Jim mentions: "I gave to the teachers a table with two columns corresponding to the weights and the prices for 20 sales of the three products. There was also a third empty column. I did not tell them to calculate a ratio or to send the two columns to the Graph. I was curious to see how many groups would be able to find them … I asked them to solve the problem algebraically and geometrically. I recommended the creation of columns in the Table window for the values of the different products as well as the creation of three buttons in the Calculator … I organized the class in groups of two following the model of their trainer … All groups found at least one solution. I passed through the groups and encouraged them to find another one … They discussed in their groups, they explored, I showed them how to stretch the graph of $y = x$. I was stressed by the possibility of not being able to use the FP tools. Thus, I had prepared another worksheet including more detailed instructions for the tool use. Finally, I didn't give it to them. If someone had faced problems with FP I would have given it to him. I believe that the level of instruction should be gradually decreased. When we first introduce new software we need to prepare a more instructive worksheet. But in the next lessons the worksheets should be as open as possible. The aim is to support teachers' thinking. You cannot tell them continuously 'press this' or 'press that'….".

Jim's design is closer to the ideas favored by the UC course. He took the risk to organize his lesson around an open worksheet and to confront potential teachers' difficulties with FP tools on the spot. In his design he initially considered teachers 'as students' and through a social process which evolved in six phases he came to view them as 'teachers of students'. Particularly, in phase one, he engaged teachers in exploring the problem (geometrically and/or algebraically), while in all subsequent phases, the teachers were asked to adopt the role of 'teachers of students' through a series of activities. At the instrumentation level, he took into account the model of teaching adopted by the official trainer in CTES as well as the fact that the main functionalities of FP had already been taught in previous lessons. Besides, his conception of the nature of worksheets regarding the degree of instruction led him – at the level of instrumentalisation – to incorporate open tasks. Facing the challenge to balance the correct use of technological tools and their integration in creating meaningful mathematical representations, Jim gives priority to the second. Jim is not interested in technological skills per se but how these would be incorporated in teachers' didactical design. Jim's choices reveal that he targets trainees' more complicated forms of knowledge as his design intertwines linear functions and its pedagogy with technology (approaching $y = ax$ algebraically or geometrically through ratios and dynamic manipulation of $y = x$). In terms of Ernest (1994), he adopts the role of facilitator who targets problem posing and solving and favors the development of meaningful material by the teachers themselves.

The operational invariants underlying the instrumentation/instrumentalisation processes were related to the following factors: (a) his epistemological conceptions of the ways mathematical knowledge should be approached though the use of technological tools; (b) his observation concerning the teaching agenda of the official trainer in CTES; (c) his conception of trainees initially 'as students' who work in groups to explore different solutions of the given problem with the use of tools and subsequently as 'teachers of students' who are facilitated by his mediation to exploit the tools and transform them into didactical instruments for mathematics teaching at the professional level.

CONCLUSION

Our analysis revealed the development of two kinds of documents: an instructive one and an open one. Taking into account the formula *Document = Resources+Schemes of Utilization*, we conclude that differences in the nature of the two documents can be explained through corresponding differences in the trainees' schemes of utilisation, i.e., identification of different operational invariants for each one of them. Both trainees intended to continue the CTES trainer's teaching model and to address the potential teachers' difficulties with FP. Ian's instructive approach seemed to have also been influenced by his concerns of differences in teachers' familiarisation with FP tools as well as by the time restrictions. While Ian aims to control all the above factors through an instructive document, Jim's design emphasises teamwork and appropriate teaching interventions by himself.

Thus, in the context of teacher education in technology enhanced mathematics, operational invariants that influence trainee teacher educators' documentational work in the context of their practicum seem to be closely connected: (a) to their experiences from the observation in CTES classrooms (focusing either in the trainee teachers' difficulties or the model adopted by the official trainers of the respective classrooms); (b) to the importance they attribute to the constrains and opportunities provided by the wider educational context; and (c) to their epistemologies regarding the role of technology in the teaching of mathematics and the ways they conceive trainee teachers, for example, emphasis on technology per se favouring the conception of teachers mainly 'as students' or on the integration of technology as a coherent part of teachers' designs favouring the transition of trainees from 'teachers as students' to 'teachers of students'. The first two operational invariants are directly linked to the teaching practice taking place in a real educational context and it is useful to be taken into account in every teacher education course. The third operational invariant reveals parts of the trainee teacher educators' beliefs that seem to influence the ways they design and use resources for teachers' meaningful integration of technology. Thus, the third seems to be a catalyst in favouring less instructive approaches by the trainee teacher educators and at the same time it indicates a domain for interventions in the design and further enrichment of the course. A potential suggestion for reinforcing the reform aspect of the course (or similar courses at the level of educating teacher educators) could be an earlier introduction of practicum. According to our findings, practicum provides a context dense of opportunities for challenging and questioning trainees' beliefs as they externalise them through the cycle 'design-implementation-reflection'. Thus, an early introduction of practicum in conjunction with appropriate mentoring could help trainees to negotiate and redefine their beliefs towards a pedagogically sound use of technology in their teaching.

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ENDNOTES

1. 1. Title, 2. Scenario's identity (author, subject area, topic), 3. Rationale (innovations, added value by the use of technology, students' learning problems addressed), 4. Context of implementation (grade, duration, location, prerequisite knowledge, social orchestration of the classroom, goals), 5. Phases of implementation (sequence of activities, roles of the

participants, anticipated teaching/learning processes), 6. Possible extension, 7. References.2

2. FP is a multi-representational software with three windows: Table, Graph and Calculator. Function graphs can be produced in a number of different ways, for example, inserting a formula for the function, "receiving" ordered pairs (x, y) from a table ("x" and "y" columns can be generated). Particular icons allow horizontal and vertical transformations of functions (translations, reflections and stretches) through direct actions on the graph. Stretching is carried out with the stretch tool that allows mouse-driven horizontal and vertical stretching. In the Calculator, the user can incorporate functional relations into buttons which reserve these relationships for future use.