Addressing Collective and Individual Aspects of Teacher Design with Digital Resources in Collaborative Settings

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Contents

Abstract

In this chapter, we address the interplay between collective and individual aspects of teacher design in collaborative settings in the light of the new opportunities that digital resources (DRs) offer. Taking into account the growing research interest into the collective dimension of teachers' design-work, our aim is to answer what are the forms, conditions, and products of teachers' collective design-work with DRs as well as what and how the individual teacher learns by participating in collaborative work with DRs. We also aim to answer what theoretical and analytical perspectives are used by researchers to study teachers' collective

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design-work with DRs. To answer these questions, we conducted a systematic literature study leading finally to 36 peer-reviewed publications. Our first thematic analysis resulted in two main themes (the process of teacher collaboration, the impact of teacher collaboration on teacher professional learning) and corresponding subthemes. The next step of our analysis focused on: the context, the product, the purpose, and the processes of the design-work; the theoretical and methodological approaches by which it was studied; and the main findings. The final synthesis indicates that (a) teachers' collaborative design-work has usually positive learning outcomes for individual teachers, (b) the role of DRs in the collaboration depends on their affordances, and (c) the collective-individual interplay has been studied mainly by a focus on the effects of the collaboration on the individual teacher and not the other way around. Areas of further research are also discussed.

Introduction

Recently, research has started to focus on mathematics teacher design as a central issue (Brown 2009). Despite the novelty of the concept, its meaning has evolved during the last decade primarily due to the introduction of digital resources (DRs) into mathematics education (Pepin et al. 2016, 2017a). In these recent conceptualizations of teacher design, we see three significant shifts: (1) the role of the individual has shifted, (2) the role of teacher collaboration has become more prominent, and (3) the definition and the role of the curriculum materials have changed. We now outline this evolution.

In one of the earliest conceptualizations, Brown (2009) characterizes teaching as a design activity. He argues that teachers are engaging in design – whether or not they intend to do – when they "use materials as tools [e.g. curriculum resources] to transform a classroom episode from an existing state to a desired one" (p. 23). Building on a cultural-historical perspective, Brown focuses on the actions mediated by the tools and the ways these actions afford and constrain the teacher's instructional decisions, planning, and enactment of teaching. Brown distinguishes between curriculum resources and teacher resources where the latter is a teacher's knowledge, beliefs, and goals (understood as intentions). These resources are conceived as idiosyncratic characteristics of the single teacher (and not as resources per se), while curriculum resources represent artifacts in a Vygotskian sense, such as representations of tasks, instruction, and scripts that are intended for enactment. The central element in Brown's (2009) approach is the dialectic interplay between the mediated actions of curriculum resources and the individual teacher's capacity to engage with these actions.

Rather than focusing on the individual teacher, newer approaches to teacher design emphasize the importance of collaborative work and also the new opportunities that DRs offer (Pepin et al. 2017a). These approaches often build on a broad conceptualization of resources as "that" a teacher uses to re-source their classroom teaching including material and non-material resources, such as their knowledge or talk with colleagues (Adler 2000). The general understandings of what constitutes a resource and what their roles are have thus been broadened within the research field.

Even though, teacher design is often used "casually with little understanding of [its] different facets" (Pepin et al. 2019, p. 140), there are attempts at providing nuanced and deeper conceptualizations. This applies to the study of Pepin et al. (2017b), which define teacher design capacity as consisting of three components: (1) an orientation, a goal, or points of reference for the design, (2) a set of firm, but flexible design principles, and (3) "reflection-in-action" type of implicit understandings. The third component is often called *design-in-use*, which refers to the teacher's (often implicit) adaptation of the resources during enactment of teaching. It is in line with Brown's (2009) approach to include enactment of resources in the notion of teacher design.

However, later Pepin et al. (2019) claim that the above definition provided "an ambitious, idealized image of teacher design capacity" (p. 125). They propose a new model for teacher design activities, in which teachers' *modes of design* are stretched by two dimensions: *the teaming dimension* (from working alone to working in a collective) and *the use dimension* (from own use to generic use). This model results in a matrix with four different design-modes: d, D/d, d/D, and D, where d marks a narrower scope and D a broader scope (see Table 1).

In contrast to Brown's approach, the "collective" plays a prominent role in this conceptualization both in terms of local or broader collaborative work and in terms of the intended use of the design. The expanding provision of DRs has partially fueled this focus on the collective dimension of teachers' design-work, in particular, by offering new opportunities for collaboration across time and space. For example, to share resources on digital platforms and participate in MOOCs (Massive Open Online Courses) and online communication (Huang et al. 2021; Pepin et al. 2017a; Pepin et al. 2017b).

The shift from perceiving teacher design as predominantly an activity for the individual teacher related to their own classrooms to conceiving it as a highly

Table 1 Four modes of teacher design (Pepin et al. 2019, p. 128)

Use/teaming	Working/designing alone	Working/designing in a team
Designing for own use/teaching	A teacher designing on their own for their own teaching (e.g., lesson preparation at home).	Teachers designing in a team (e.g., of colleagues in the same school) for their own teaching.
	d	D/d
Designing for use by others	Teachers designing on their own/alone for use by others (e.g., expert teachers, professional designers).	Teams of teachers/experts designing for use by others (e.g., teams of professional designers).
	d/D	D

collective activity often fueled by new DRs calls for investigations of the changed conditions and forms of the interplay between individual and collective aspects. As teachers around the world engage more and more frequently in collective design-work, it becomes important to gain insight into how individual teacher's learning is supported by the new conditions and forms of collaborative design-work involving or facilitated by DRs and vice versa. Furthermore, it is essential to know more about how this can be investigated theoretically and what the possible blind spots are. Hence, we seek to answer the following research questions:

1. What are the forms, conditions, and products of teachers' collective design-work with DRs?
2. What do individual teachers learn by participating in collective design-work with DRs and/or facilitated by DRs?
3. What theoretical and analytical perspectives/approaches are used to address the above two questions?

Methodology

To address our research questions, we conducted a systematic literature study. We anticipated that too few research publications would include all of our four keywords (or their synonyms): individual aspects, collective aspects, DRs, teacher design. Therefore, we decided to focus on two of them – collective aspects and digital resources – under the assumption that they would more often imply a focus on the last two keywords – individual aspects and teacher design – than the other way around. Inspired by our preliminary reading, we jointly elaborated the keywords, which we describe below with examples of their most central synonyms:

> Collective aspects: (teacher) collaboration, collective work, collaborative work, work in groups, community, learning community, community of practice.
> Digital resources: digital technologies, digital tools, ICT, ICT environments, digital environments, scenario design, lesson sketch, pandemic.

Furthermore, we developed four criteria to help us select publications (for full text reading in particular):

1. Both collective aspects and DRs must be substantial parts of the study. That is, the two keywords or their synonyms should be included in the title, abstract, or keywords of the publication.
2. The collaboration shall be between teachers, teachers and researchers, or between researchers who work together as teachers.
3. The publication shall focus on in-service teachers (from preschool to university level) (or researchers as described above), not on prospective teachers as they are often involved in asymmetric collaboration of a different nature.

4. DRs in terms of video recording must be embedded in or used in connection with other DRs.

Based on our previous work and knowledge within the area, we initially selected relevant books and conference proceedings such as Borko and Potari (2020), Gueudet et al. (2012), and Trouche et al. (2019b). In particular, the proceedings edited by Borko and Potari (2020) was relevant as it included papers from the recent ICMI Study on teacher collaboration in which one theme (out of four) was "Tools and resources used/designed for teacher collaboration and resulting from teacher collaboration." Next, we searched the major journals on mathematics education (e.g., *Zentralblatt für Didaktik der Mathematik-ZDM*, *Journal for Research in Mathematics Education-JRME*, *Educational Studies in Mathematics-ESM*, *Journal of Mathematics Teacher Education-JMTE*, *Journal of Mathematical Behavior-JMB*) and more specific journals (e.g., *International Journal of Lesson and Learning Studies-IJLLS*, *Digital Experiences in Mathematics Education-DEME*, *Mathematical Thinking and Learning-MTL*, *Mathematics Education Research Journal-MERJ*, *International Journal of Science and Mathematics Education-IJSME*). Finally, we conducted a systematic search in the database ERIC using three increasingly elaborated search strings primarily based on ERIC's keywords. The most elaborated string was:

((((((DE "Computers") OR (DE "Video Technology")) OR (DE "Handheld Devices")) OR (DE "Video Technology")) OR (DE "Virtual Classrooms")) OR (DE "Program Design")) OR (DE "Program Development") AND (((((DE "Teacher Collaboration") OR (DE "Cooperation")) OR (DE "Team Teaching")) OR (DE "Community")) OR (DE "Communities of Practice")) OR (DE "Discourse Communities") AND "mathematics teachers"

Our database search had a dual purpose. On the one hand, we wanted to include PhD dissertations and articles and papers published in journals and proceedings different from the ones we had selected. On the other hand, we wanted to ensure that we had found the relevant publications during our initial search in the journals by obtaining some replica.

The database search resulted in 101 hits, which we reduced to 18 publications by scanning their title, abstract, and keywords based on the above-mentioned criteria. When searching the journals, books, and proceedings, we found 140 publications, which we similarly reduced to 45. Checking for replications and scanning full texts left us with five publications from the database search and 37 publications from the other searches, in total 42 publications to be included in our study.

To analyze these 42 publications in relation to our research focus, we constructed a table with five categories: (1) theoretical approach, (2) focus, research question, and methodology, (3) kind of DRs and their use, (4) collective versus individual aspects, and (5) main results. We distributed the publications among the three of us and analyzed the papers thoroughly. Based on these systematics analyses, we omitted further six publications (either they did not sufficiently fulfil the criteria or

they were review papers) and categorized the remaining 36 publications in relation to their focus and type of outcome (category 2 and 5). This resulted in two main themes: the process of teacher collaboration and the impact of teacher collaboration on teacher professional learning, which we divided into three and four subthemes respectively (the number in the parentheses is the corresponding number of publications):

1. The process of teacher collaboration
   - Collective and individual interplay (9)
   - Genesis of a community of practice (CoP) (3)
   - Emerging tensions (3)
2. The impact of teacher collaboration on teacher professional learning
   - Teacher knowledge (6)
   - Teacher expertise (3)
   - Learning as/through participation (4)
   - Design practice (11)

For each of the seven subthemes, we conducted a second analysis of the included publications. This analysis was inspired by Pepin et al.'s (2019) approach to teacher design, and it focused on: (1) the context of the study (participants, place, duration, modes of teachers' design); (2) what the actors designed (product); (3) why they designed (purpose); and (4) how they design (design approaches, conditions, DRs' role) and how the researchers investigated this (theoretical and analytical approaches) as well as the main results of the study. There were some overlaps between the first and second analysis, especially as regards theoretical considerations and main results. Furthermore, we analyzed three of the publications, Hansen et al. (2016), Johnson et al. (2016), and Robutti et al. (2021), under both main themes. Based on the second analysis, we made a synthesis for each subtheme, which we present in the following parts. The syntheses do not strictly follow the structure of the analytical lens.

The Process of Teacher Collaboration

Collective and Individual Interplay

The nine studies that address the collective and individual interplay take place mainly in a setting where a small number of teachers collaborate in the school context for planning and enacting everyday lessons (Gueudet et al. 2013; Rocha 2019; Trouche et al. 2019a) (design-mode: D/d). Three of the nine studies focus on teacher collective design of teaching resources for others such as online resources at national level (Gueudet and Trouche 2012), distance university courses (Borba et al. 2018), and activities involving DRs for students of borderland schools (Penteado and Skovsmose 2009) (design-mode: D). These three studies refer to the collaboration of different stakeholders in larger design groups. Two of them, Pepin and

Gueudet (2020) and Hansen et al. (2016), include design of resources for others and actual implementation in teachers' own classrooms (design-mode: D and D/d). The last study (Anderson 2020) reports the interaction between the 1738 members of a public Facebook group (tailored to mathematics education) as they share and discuss artifacts (e.g., activities) from their practice (design-mode: d and D/d).

The purpose and the product of designing everyday lessons involve teaching materials such as lesson plans and criteria for providing feedback to students and parents (Gueudet et al. 2013) and DRs to teach algorithmic thinking and programming (a new curriculum topic) (Rocha 2019; Trouche et al. 2019a). The collective work for designing for others aims to develop national resources such as e-textbooks (Pepin and Gueudet 2020) and online resources (Gueudet and Trouche 2012), open university distance courses (Borba et al. 2018), and activities with DRs (e.g., Dynamic Geometry) to support students' social inclusion (Penteado and Skovsmose 2009). In most of these studies, DRs appear to be the outcome of teacher design while in few cases a digital environment facilitates their design-work. For instance, in Hansen et al. (2016) a virtual manipulative targeting students' learning of fractions, *Fraction Lab* was shared as an artifact on which the teachers were asked to co-design and improve its initial design. At the same time, they reflected on the role Fraction Lab could have on their own teaching. In Anderson (2020) the teachers interacted asynchronously through Facebook by sharing artifacts from members' practice that provided all members access to real classroom situations.

Most of the studies that address the collective and individual interplay use the Documentational Approach to Didactics (DAD, Trouche et al. 2019b) and the Communities of Practice Theory (CoP, Wenger 1998). In particular, the focus is on the relationship between teacher's individual schemes and their participation in online or face-to-face design communities. In some cases, the researchers analyze the collective work of teachers looking for evidence of this relationship. For example, Pepin and Gueudet (2020) report that individual schemes intervene in the collective work when teachers engage collectively in an activity with a "usual" goal (i.e., they already developed an individual scheme). However, in cases of designing with new goals for which the teachers have not yet developed a scheme (i.e., designing a new e-textbook), they can develop shared schemes and operational invariants that influence their engagement in the collective work. In other cases, the researchers focus on the role of shared resources and different agents involved in the documentation work, that is selecting, modifying, and creating new resources. For instance, Gueudet et al. (2013) report that planning lessons together (choosing the textbook) and sharing resources (files on a computer, or a reference book) can evidence collective processes. They also identify conditions that seem to support collective work such as the provision of space and time for collaboration, common perceptions and beliefs about the teaching of mathematics with other teachers, shared professional needs, support by the school authorities. In some studies, the researchers' focus is on the individual teacher while their participation in collectives is in the background. For instance, Rocha (2019) uses the notion of documentational trajectory to identify the role of a teacher's (Anna) participation in collectives when designing teaching activities. Specifically, she links Anna's participation in

collectives and the evolution of her resource system. In the study of Trouche et al. (2019a) two teachers' collective work is in the foreground and it is studied through DAD, the Anthropological Theory of the Didactic (ATD, Chevallard 1999), and CHAT (Engeström 2014). DAD draws attention to the whole resource environments of the two teachers. ATD focuses on the knowledge at stake (i.e., didactics and algorithmics) in teachers' interactions and the contextual constraints (e.g., level of school, curriculum, society). CHAT draws attention to the evolution of the aim and objects of the teachers' activity and the emerging conflicts (e.g., between natural language and/or programming rules).

The role of different agents in teachers' collective design-work is studied in three of the reviewed studies under different theoretical lenses. Borba et al. (2018) use the constructs of interactive didactical digital materials and poly-teaching to capture the complex relation between the individual and the collective when different stake-holders (teacher, tutors, students) are engaged in the design-process of a Virtual Learning Environment. The main theoretical position is that the production of knowledge is collective and includes human and non-human actors. Hansen et al. (2016) highlight that the involvement of highly motivated teachers who perceive themselves as agents of change in the domain area is one factor that supports a successful co-design of virtual manipulatives in a CoP. Penteado and Skovsmose (2009) report that the collaborative dimension in the work of different agents (i.e., practicing teachers, prospective teachers, researchers) has an important role in providing socially relevant opportunities for their students.

One study (Anderson 2020) addresses how collaborative interactions in a Facebook group, where individuals share real classroom situations (e.g., requests for help, ideas), support teacher professional learning. The analysis reveals four different discourse structures: commenters providing desired support (Desired); commenters offering different ideas (Reframe); commenters challenging requested support (Challenge); and commenters working together to build a new understanding (Generate). The results show that the group members were able to increase their access to knowledge for mathematics teaching.

Summarizing, the individual and collective interplay takes place in face-to-face or online communities consisting mainly of a small number of teachers working to develop or use DRs for their own teaching. In that context DRs include classroom materials, assessment tools, and lesson plans. In the few cases, where teachers design for others, DRs are e-textbooks and online resources. The individual-collective interplay is studied through different theoretical lenses. DAD provides a way to see this interplay in relation to individual schemes and shared schemes. Other theoretical lenses (e.g., CoP, CHAT) emphasize the role of different agents, communities, and contextual conditions in the collective-individual interplay.

Genesis of a CoP

The three studies that address the genesis of a CoP extend the previous research on teacher collaboration as the actual collaboration and its role in the genesis of a

community get a central role. These studies take place in online or face-to-face PD settings where groups of teachers collaborate with researchers/instructors around the design and enactment of tasks in their classrooms (Bakogianni and Potari 2019; Hodges and Cady 2013) (design-mode: D/d). For instance, in a face-to-face PD setting at the university, 11 secondary mathematics teachers and two researchers collaborated in the study of Bakogianni and Potari (2019) to plan and enact tasks involving the use of DRs in the classroom. In the study of Hodges and Cady (2013) a school-based lesson study group of six mathematics teachers collaborated with instructors in the context of a two-year blended-format (online synchronous using Centra software, online asynchronous using Blackboard, and face to face) PD program to design and implement lessons in their classes. In the last study, a larger group of 21 teachers collaborated asynchronously on a mathematics task in the context of an online collaborative and content-focused PD course (Matranga and Silverman 2022).

The purpose and the product of teacher design are oriented by the aims of the emerging CoPs posed by the involved researchers or project stakeholders. For instance, in Bakogianni and Potari (2019), teachers planned and enacted lessons based on the use of the statistical simulation software Fathom in order to support the development of students' statistical thinking and reasoning. Here the role of Fathom is to be integrated in teaching through teacher collaboration. In Hodges and Cady (2013), the teachers collaborated to plan, implement, and reflect upon multiple lessons involving the use of DRs (Geometer's Sketchpad, virtual manipulatives, interactive websites such as NCTM's Illuminations) so as to make decisions based upon evidence of student learning. This was in line with the project's aim to improve teachers' content and pedagogical content knowledge. In Matranga and Silverman (2022), a mathematical task including interactive applets was given to teachers by PD course coordinators to focus on reasoning about functions and communicating mathematical ideas. The design aspect is implicit in teachers' activity – it concerned mainly teachers' revisions of their initial solutions after taking feedback by other teachers and collective reflection on the learning process. In the last two studies, the role of DRs – as communication platforms – is to frame and facilitate collaboration among teachers.

In all three reviewed studies, CoP is used together with other theoretical constructs to highlight specific elements of teachers' interaction in the community as well as the conditions for the formation of a CoP. In particular, the analysis of teachers' participation and interactions in the community as well as the development of the final outcome are used as evidence to address the evolution of teachers' collective activity. For example, Bakogianni and Potari (2019) use Adler's (2000) construct of "re-sourcing" to analyze the integration of Fathom in the community's shared repertoire. The resourcing process was characterized by three phases: emergence (no interaction with the DR), exploration (the teachers used Fathom to design lessons and related it to the learning potentials and the features of teaching), and immersion (the teachers had enacted the lessons and developed awareness of Fathom's potential in teaching and learning). Hodges and Cady (2013) studied to what extent the use of lesson study and the Cognitively Guided Instruction model in

school-based CoPs supported teachers to develop a joint enterprise (developing and refining the lesson at hand) and a collective way of reasoning (making decisions based upon evidence of student learning). The results revealed the critical role of schools and their expectations for mathematics instruction in the formation of a CoP. Thus, teachers from a specific school, who perceived significant agency in determining "best" practices saw a joint enterprise toward improving mathematics teaching and learning, whereas the teachers at another school perceived little control over how mathematics instruction took place. In Matranga and Silverman (2022), the focus is on both the individual participation in the group (mode of participation) and the participants' interaction in the community (the content of their interaction). The authors use constructs from social practice theory (e.g., core, intermediate, and periphery participation) and social network analysis to measure network cohesion by quantifying the extent to which mathematics teachers interact. The analysis identified an emergent divide between participation in the core and periphery during initial weeks of the PD and then a reduced divide and emergence of a social network that resembles a community.

Summarizing, the studies reported in this subsection emphasize the genesis of a CoP around the use of DRs (synchronous and asynchronous software, statistical simulation software) mainly for classroom use. The construct of CoP is combined with other frameworks to address individual participation in the group with the collective practice of the community. The results indicate positive outcomes for the CoP participants such as teachers' awareness of DRs' potential in teaching and learning. Also, they bring to the fore the critical role of school context, mode of teacher participation, and content of interaction in the genesis of a CoP.

Emerging Tensions

Collaborative work in communities involves tensions that affect the way that teachers interact and the outcome. Three of the reviewed studies focus on this issue. They mainly involve large number of teachers collaborating with researchers (Gueudet et al. 2016; Robutti et al. 2021) or other stakeholders (Johnson et al. 2016). In two of the studies, the participants design for use by others: the co-designing of task-based professional development in high-school algebra in a large, diverse urban school district in USA (Johnson et al. 2016) or a grade 10 e-textbook in the context of the French Sésamath teacher association (Gueudet et al. 2016) (design-mode: D). In the study of Robutti et al. (2021), a group of Australian in-service secondary school mathematics teachers design a new kind of digital item through the use of GeoGebra – called Mathematical Equivalence Reusable Learning Objects (MERLO) – for their own teaching in the context of PD course as part of a joint project in Italy and Australia (design-mode: D/d). Earlier, the Australian teachers were engaged in reflecting on a list of MERLO items developed by in-service mathematics teachers in Italy and link them to mathematics curriculum in Australia.

The purpose and the product of teacher design in all three papers are oriented by developing mathematical content – including tasks – to engage students in rich

mathematical activity through the use of different kinds of DRs. In Johnson et al. (2016), the collective work involves co-designing, analyzing, modifying, and evaluating mathematics tasks targeting innovative approaches to the creation and adaptation of digital STEM curricula. The DRs are in the background while the quality of the tasks is the main purpose of the co-design in the PD. Similarly, in the study of Robutti et al. (2021), the teachers design digital items (MERLO) aiming to engage students in deep reasoning, exploring, and arguing about mathematical concepts. In the study of Gueudet et al. (2016) the teachers design the functions chapter of a new e-textbook. Beyond the mathematical and didactical coherence of the chapter, the teachers aim to take into account in the design new possibilities offered by technology in terms of new potential structures of the textbook and new interactions with users. The purpose of the design in this study was constantly developing over the years due to changes in the synthesis of the team and the aims of the Sésamath board. In the last two studies the DRs are in the foreground and constitute the outcomes of the design. In addition, in Gueudet et al. (2016) DRs in the form of communication platform (Sésamath) also facilitate collaboration among teachers.

All studies report tensions in the process of design. Using the Tatar's design tensions framework (2007), Johnson et al. (2016) indicate that even when partners share a common vision and generally agree upon an approach, tensions stemming from the lack of design consensus affect their participation and learning. The authors characterize these tensions and conclude that handling them is possible but it requires balancing goals, values, and resources. By combining DAD and CHAT, Gueudet et al. (2016) report that the choices of teachers were influenced by the emergence of tensions among the group in three specific moments. The first tension concerned the object of the activity "designing a 'toolkit'-type textbook", and a belief shared by the members "not all paths are relevant" (p. 201). During the second moment, the object of the activity (design of interactive exercises) was new for the teachers and led to the development of a common document and combination of beliefs. During the third moment, another tension occurred between the different beliefs of the members as regards the progression of the chapter in sections. Through discussion a consensus was reached and all the different beliefs were respected. Under a boundary crossing perspective (Akkerman and Bakker 2011), the study shows that the learning objects (MERLO items) acted as boundary objects. Two kinds of boundaries are reported: (a) institutional boundaries between school systems in Australia and Italy and (b) boundaries of practice between static and dynamic representations inside a single community (i.e., teachers in Australia).

In summary, collaboration between teachers in designing or using DRs is a rather complex process. It involves tensions that are related to the way that the participants conceptualize the designed product as well as the institutional and social conditions. In particular, the products that involve DRs such as e-textbooks, digital items, and digital communication platforms seemed to be new for the teachers so it was difficult to reach design consensus. The theories of DAD, CHAT, and boundary crossing have been used to study the emergence of the tensions and the way they were handled.

The Impact of Teacher Collaboration on Teacher Professional Learning

Teacher Knowledge

We identified six studies that focus on teacher knowledge as an outcome of teacher collaboration. Across these studies, DRs play three different roles, one for each pair of studies. The first role of DRs is to facilitate teachers' PD by providing an online environment for self-directed and collaborative learning, a so-called MOOC. Taranto et al. (2020) investigate the *Math MOOC UniTO* (used in Italy since 2015) in which 278 and 358 in-service teachers inscribed for two specific modules in 2016/17. Hollebrands and Lee (2020) examine teachers' learning, when they enroll in different MOOCs for self-study and additionally are invited to participate in joint activities (e.g., forum discussions). The study draws on data gathered from 14 course offerings (2015–2019) with 4429 unique enrollees across three MOOCs in North Carolina (USA).

The second role of DRs is related to the first, but rather than framing teachers' PD, this role is to facilitate collaboration among teachers, with a minimum of external support from teacher educators. Segal et al. (2020) investigate a digital platform (RAMZOR) that facilitates collaboration among teachers in Israel by offering a common environment to preserve, share, and jointly improve materials for teaching and learning. In Alqahtani and Powell (2017), seven middle and high school American teachers collaborate in the *Virtual Math Teams with GeoGebra (VMTwG)* (a collaborative DGE) in small groups to solve open-ended geometric problems and report weekly to the whole class.

The third role of DRs is to be integrated in teaching through teacher collaboration. Clark-Wilson and Hoyles (2019) focus on a sample of 60 teachers from a large three-year project with 209 teachers from 48 London secondary schools. The authors examine the gaps between the teachers' planned and actual enactments of a Cornerstone Maths curriculum module including digital mathematical technology. Hansen et al. (2016) investigate what 23 specialist primary teachers in England learned by participating in the co-design of the previously mentioned virtual manipulative Factions Lab.

The purpose and the product of the design activities in all the above studies are closely related to the role of the DRs. When DRs facilitate teacher PD or teacher collaboration, there are two purposes, which are common to all four studies. The first purpose is that teams of researchers designed the digital environments to support PD and collaboration for a wide range of teachers from different demographic areas, several of whom normally have limited resources for PD (design-mode D). The second purpose is, that these digital environments require the participating teachers to engage in design activities themselves such as designing tasks (Taranto et al. 2020), lesson plans, teaching programs and assessment tasks (Segal et al. 2020), and teaching units (Alqahtani and Powell 2017). Across the four studies, teachers are encouraged to collaborate on their designs, but they are designing for their own teaching (design-mode d or D/d). In the two last studies, the teachers plan and enact

lessons based on specific DRs. The purpose is to provoke their rethinking of the mathematics or their previously held ideas by engaging them with DRs that offer new representations allowing new actions and dynamic ways to engage with mathematics. Clark-Wilson and Hoyles (2019) use a lesson-study approach to teacher collaboration (design-mode D/d) while Hansen et al. (2016) engage teachers in co-designing Function Lab to ensure the design of a fit-for-purpose product (design-mode D).

The six studies use four different theoretical approaches, networking Meta-Didactical Transpositions with connectivism (Taranto et al. 2020), the interconnected Model of Professional Growth (Hollebrands and Lee 2020), the Instrumental Genesis (Alqahtani and Powell 2017), and variations of Teacher Pedagogical and Content Knowledge (TPACK, Mishra and Koehler 2006) (Clark-Wilson and Hoyles 2019; Hansen et al. 2016).

All studies indicate that teacher collaboration is important for teacher learning, and five of them demonstrate positive learning outcomes in terms of teachers' increased knowledge. Taranto et al. (2020) and Hollebrands and Lee (2020) both emphasize the asynchronous affordances of the MOOCs, such as online discussion forum and communication message boards. Taranto et al. (2020) introduce the term *explosive learning* in contrast to linear learning as the numbers of interactions between participants can expand in an (unpredicted) exponential way in an asynchronous setting, while it can only increase in a linear way in a synchronous setting. Hollebrands and Lee (2020) conclude that geographically dispersed teachers were able to notice students' mathematical thinking from videos and discuss it within online forums. They also stress that expert panel videos tended to spark deep discussions among the participants. Hansen et al. (2016) demonstrate that the process of becoming co-designers of Faction Lab allowed teachers to engage deeply with its affordances. Especially its potential of multiple representations and different kinds of tasks than those on pencil-and-paper challenged the teachers' thinking and development of TPACK. Clark-Wilson and Hoyles (2019) show that teachers' intended enactments were not generally realized in the classroom, which applied particularly to their intentions to interact with DRs during whole-class teaching. Rather, they tended to just display the software. The authors claim that a change of this situation requires broader collaborative support in the school setting.

In summary, the studies vary greatly in terms of theoretical and methodological approaches (i.e., from small-scale studies to large-scale studies) and the roles of DRs. The studies, in which DRs are used to facilitate teacher PD or teacher collaboration, show tremendous and promising possibilities for offering individual teachers new learning opportunities, which are independent of time and demographic conditions. These studies show gains in participating teachers' knowledge and highlight in particular the asynchronous affordances of the MOOCs, videos of expert panels, videos of students' thinking, and online forums as central for getting access into aspects that are crucial for classroom teaching. Only one study used data from teachers' classroom teaching. This study focused on teachers' integration of DRs based on lesson study and it showed only limited changes as regards the teacher's knowledge and use of DRs.

Teacher Expertise

In the area of studies that focus on teacher collaboration around the use of DRs and its impact on their professional learning, the construct of mathematics teacher expertise has been considered by three studies. In particular these studies attempt to define teacher expertise and identify ways of its development. The studies take place in different collectives inside and outside school contexts where teachers design resources for use by others or by themselves. Specifically, the collectives are: the Teaching Research Group (TRG), a widely spread collective schoolwork unit for Chinese teachers since 1952 (Wang 2018; Pepin et al. 2017c); AeP in France, an association of French schools linked to the French Institute of Education (Wang 2018); the French Sésamath teacher association (Rocha et al. 2017); school-based teacher collaboration on a daily basis (Pepin et al. 2017c; Rocha et al. 2017). The design-mode for all studies is D/d, while Rocha et al. (2017) also address design for use by others (D) as well as individual design (d).

The purpose and the product of teacher design in all three studies concern the design of lessons for everyday teaching. Specifically, the teachers select, organize, modify, adapt, create, and share resources to plan, enact, and reflect on lessons for their own classrooms. In Wang (2018), DRs exist in the background as one kind of teaching resources. In the studies of Pepin et al. (2017c) and Rocha et al. (2017), DRs are used to develop teaching materials such as a new e-texbook or Sésamath online resources. Moreover, the DRs in the form of platforms (Sésamath) or software application for sharing content (Padlet) act as a means for supporting teacher collaboration.

All studies view teachers' interaction with resources as a way to address teacher design expertise. They attempt to identify the dimensions of such expertise and how it develops in collectives. Wang (2018) defines Design Expertise in relation to the schemes developed in a teacher's interaction with resources. Pepin et al. (2017c) relate teacher expertise to Schoenfeld and Kilpatrick (2008)'s dimensions of mathematics teaching proficiency that include knowing of school mathematics and students as thinkers, teacher reflection on practice, creative teaching approaches to support mathematical understanding, and noticing of students' thinking. Rocha et al. (2017) define the notion of a teacher's *documentation expertise* as "the abilities and related knowledge to deal with the whole process of interacting ... with resources" (p. 209).

As regards the development of teacher design expertise, Wang (2018) reports three conditions promoting such development: (a) conflicts in understanding or ideas are the entry points to see the influences on each other; (b) agreements and complements could reinforce teachers' common ideas or enrich the current solutions; (c) questions and answers are a direct way to benefit from each other, especially for something unknown to the other. Pepin et al. (2017c) study three individual teachers' resource systems to trace the development of teacher design expertise. They identified that DRs (e.g., websites, digital forums, online materials) were part of the teachers' resource system used together with text resources as information for and materials in/for instruction. Rocha et al. (2017) point out the role of the

experience of working with resources (documentational experience) in collectives, in the development of teachers' expertise. For instance, documentational experience of individual teachers in collectives not only brings resources to them, but also some thought styles that guide their resource design. They develop expertise through trying to adapt those resources and thought styles into their new work. All the studies use DAD and its constructs (resources system, documentational experience) to identify teacher expertise and activity theory for tracing the influence of collective work on its development.

Summarizing, teacher design expertise and its development have been studied mainly through the lens of DAD. It is mainly related to knowledge needed for interacting with DRs and it develops as the teachers attempt to adapt the resources in the design of lessons for everyday teaching. The experience of individual teachers with DRs and the conditions of collective work promoted the development of teacher design expertise through conflicts, questions, and agreements among the participants.

Learning As/Through Participation

We have identified four studies that focus on teachers' learning as or through participation in different kinds of collaborative work. Dalby (2021) and Sinclair et al. (2020) both examine the collaboration between few teachers and researchers, while they explore how to use iPads to facilitate assessment processes in three English schools, respectively, how to use a novel touchscreen application, *TouchTimes,* to teach multiplication in Canadian classrooms. Both studies focus on contextual aspects of the collaborative settings. The other two studies, Chieu et al. (2011) and Suh et al. (2021), investigate what American teachers notice, when they watch teaching practices represented in an animation, respectively, video-clips from their own classroom, embedded in an online environment. Chieu et al. (2011) compare nine in-service teachers with eight novice teachers, which participate in the same experiment, while Suh et al. (2021) focus on 30 teachers and six coaches participating in a PD focused on video-based lesson study.

The purpose and the product of the design activities vary across the four studies. In Sinclair et al. (2020), the purpose of the teachers' design-work is to use the new models of multiplication provided by *TouchTimes* to plan and enact teaching experiments on their own (design-mode d). The purpose of the design work in Dalby (2021) is that lesson-study groups plan lessons together and enact them in classrooms (design-mode D/d). The purpose in Chieu et al. (2011) is to improve the researchers' design of a web-based interactive rich-media technology to be used with many teachers (design-mode D). The researchers, in Suh et al. (2021), have a similar purpose with their design of video-based lesson study (design-mode D), while the teachers engage in collaborative designs of tasks in which each teacher has to select one high leverage practice recommended by NCTM as their personal research goal (design-mode (D or d)/d).

Three different theoretical approaches are used to study teachers' learning as or through participation: boundary objects (Sinclair et al. 2020), CoP (Chieu et al. 2011; Dalby 2021), and teacher noticing (Chieu et al. 2011; Suh et al. 2021). Both Dalby (2021) and Sinclair et al. (2020) demonstrate that teacher learning is not an automatic consequence of participating in collective design-work and that individual teacher's learning is highly influenced by contextual aspects. Dalby (2021) identifies three chracteristcts that explain (at least partly) the shown differences in individual learning gains: the boundaries provided by the selected research focus, knowledge priorities (teachers with less knowledge of the research focus, gained lesser), and division of responsibility (teachers with strong technical knowledge were more center to the project than teachers with less). Sinclair et al. (2020) show that the collaboration consisted of ongoing emergent disruptions (all related to *TouchTimes*' new models of multiplication), which seems to be the norm rather than stability. The authors show how complex it is for teachers to adopt new DRs that challenge normative meanings and how their development of new understandings is highly affected by contextual aspects, such as DRs, textbooks, colleagues, social norms, their previous experience, concern for students' learning, and researchers. Both Chieu et al. (2011) and Suh et al. (2021) conclude that the teachers' noticing was improved by their participation in the digital enviroments. Chieu et al. (2011) focus on teachers' actions in relation to different affordances of the DRs and demonstrate for instance that teachers' discussions were more in-depth, when they referred to the embedded animation, and that these animations served as a common point of reference, helping both novice and in-service teachers to effectively notice and discuss noteworthy events. Suh et al. (2021) show that teachers notice their own shortcomings and the ways they hope to improve their practice in the future, which the authors stress as promising for changing future practice. They conclude that the collegial exchanges on the video annotation tool allowed the teachers to push one another toward ambitious teaching.

In summary, half of the studies investigate how teachers' participation in face-to-face collective design-work support them in learning to use DRs in mathematics classroom, while the other half examine how teachers can learn to notice by participating in collective design-work faciliteted by DRs. The first half focuses on contextual aspects of the settings, showing that individual teacher learning is highly influenced by such aspects, which can support one teacher's learning and hinder another's within the same group. The second half shows that working with digital representations of classroom practices can improve teacher noticing.

Design Practices

We identified 11 studies addressing the impact of teacher collaboration on their learning in terms of teacher design practices. These studies take place in online or face-to-face PD settings where groups of teachers collaborate with researchers, teacher educators, and/or other stakeholders on the planning and enactment of tasks involving the use of digital tools. In six studies the teachers collaborate mainly

in face-to-face settings (Cooper et al. 2020; Cusi et al. 2020; Elbaum-Cohen and Tabach 2020; Chen et al. 2020; Johnson et al. 2016; Misfeldt and Zacho 2016) while in one study teachers collaborate through the use of an online platform (Robutti et al. 2021). The other four studies combine the two settings. For instance, in Rocha (2018), Rocha and Trouche (2017), and Trouche et al. (2020) one teacher participates in different collectives outside her school either online or in person and uses part of the designed resources in her teaching. Also, Patahuddin (2013) reports an ethnographic study of teacher–researcher collaboration in Australia combining numerous visits in school and regular online communication. Concerning the design-mode, in the majority of these studies teachers design with others for their own teaching (design-mode: D/d). There are three studies where teachers work collaboratively with other stakeholders designing resources for use by others and by themselves: Rocha (2018) analyzes the design-work of a teacher while participating in a collective with 33 members that design resources for a new textbook in the context of a teacher association in France; Rocha and Trouche (2017) and Trouche et al. (2020) analyze the design-work of a teacher in different collectives in France (e.g., associations of mathematics teachers, educational institutes) (design-mode: D/d and D). Johnson et al. (2016) report on a research–practice partnership where district leaders, teachers, web engineers, and researchers collaborate for designing a digital STEM curricula (design-mode: D). There is one study (Elbaum-Cohen and Tabach 2020) that focuses on the individual work of a teacher (Yosef) in Israel who designs lessons with the use of DRs for his own teaching in the context of a school-based PD course (design-mode: d).

The purpose and the product of teacher collaboration in terms of design practice are oriented by the aims of the corresponding projects and research studies. Thus, the main design-mode, the collective work for designing for teachers' own use, aims to: design lessons that are sensitive to the issue of productive talk in the classroom (Chen et al. 2020); select from the internet and transform collaboratively tasks through the use of a dashboard that supports filtering tasks by metadata (Cooper et al. 2020); develop and share open-ended scenarios that support and include creative ideas for mathematics teaching and learning (e.g., developing games) (Misfeldt and Zacho 2016); design, reflect, and redesign scenarios by representing classroom interactions through ordered sets of scenes/cartoons in Lesson Sketch (Cusi et al. 2020); develop skills and expertise in exploiting the internet (online resources, websites) in mathematics teaching (Patahuddin 2013); design and enact a new kind of item (MERLO) through the use of GeoGebra with the aim to promote students' mathematical reasoning (Robutti et al. 2021). The collective work for designing for use by others aims to develop: tools/criteria (e.g., rubrics) for evaluating the quality of mathematical tasks (Johnson et al. 2016); textbook resources relevant to a new reform (Rocha 2018); resources supporting the creation of new resources and favoring a reflection on teacher's own documentation work (Rocha and Trouche 2017; Trouche et al. 2020). In the above studies, DRs are mainly used with the aim to be integrated in the design and selection of tasks for classroom teaching or PD purposes. In two studies, DRs in the form of platform frame the collaboration of teachers (Chen et al. 2020; Johnson et al. 2016).

Seven of the eleven studies address changes in teachers' lesson planning, three studies address changes in actual teaching, and one study addresses both types of practices. The seven studies that focus on lesson planning examine the interrelation between the co-designed product and the participants' professional learning. The role of the collective work as well as the mediating role of the used DRs for teachers' professional learning are highlighted differently in the reviewed studies and are addressed under diverse theoretical and methodological perspectives.

Lesson planning: the role of DRs Four studies consider collective design in conjunction with the use of specially designed DRs as a way to address the impact of teachers' collaboration on their design practice and professional learning. For instance, through the use of the instrumental approach (Guin et al. 2005), DAD, and the analytical concepts of domain and frame, Misfeldt and Zacho (2016) describe how two groups of teachers used various knowledge domains (i.e., situations or practices such as schooling, mathematics, games) in their co-design of teaching scenarios in GeoGebra through open-ended approaches. Teachers' learning is described as instrumental development in relation to creative and project-oriented mathematics teaching indicated by their: increased competence with GeoGebra; didactical reflection on the students' use of technology; and appropriation of creativity for teaching mathematics. Also, Cooper et al. (2020) focus on the development of teachers' discourse of curricular design through the use of a dashboard for navigating a collection of tagged learning resources with prescribed categories of didactic metadata. The analysis focuses on the way that the groups of teachers used the dashboard (e.g., filters used in their search). Two theoretical lenses were used: commognition (Sfard 2008) and boundary crossing. The collective element seems to relate to the use of boundary crossing between the practices of the teachers and the researchers (as expressed by the latter in the design of the tool). Commognition was used to identify teachers' discourse of curriculum design through the identification of four facets, that is typical keywords, visual mediators, narratives, and recurring routines. These facets were also used to identify boundaries between the teachers' and the researchers' discourses and respective boundary crossings that indicated teachers' professional learning.

In one study (Cusi et al. 2020), the used DR provides innovative representations of mathematics teaching practices through cartoons (Lesson Sketch). The DR allows teachers to focus on and discuss the various possibilities in which a classroom interaction might evolve. Teachers were engaged in designing and redesigning a scenario in Lesson Sketch and using it as script for classroom enactment and further reflection. Under the framework of Meta-Didactical Transposition (Arzarello et al. 2014), the authors highlight the evolution of teachers' praxeologies in parallel to the corresponding evolution of the scenario designed collaboratively by teachers. The shared reflections and the provided feedback about the intervention during the collaborative work on scenario design promoted teachers' learning about adopting new practices (e.g., questioning themselves as a way to address decisions on scenario design).

In the study of Johnson et al. (2016), the DRs exist in the background and the emphasis is on the tensions that arise among the participating stakeholders in the

teams around the design of tasks maintaining cognitive demand and promoting equity. The tensions involved commenting on rubrics to codify various uses and interpretations of language in tasks and rating tasks (e.g., for DRs). Under the design tensions framework (Tatar 2007), the authors consider different tensions that emerged in the collective work and conclude that tensions stemming from the lack of design consensus affect participation and learning. Professional learning in terms of teacher design is implied through the participants' alignment around the quality of mathematical tasks and their understanding of the role of design choices in balancing multiple values and goals.

Lesson planning and professional learning: teachers' trajectories Individual teachers' interaction with DRs over long periods of time was prioritized by three studies aiming to address the impact of teacher collaboration on teachers' design practice (Rocha 2018; Rocha and Trouche 2017; Trouche et al. 2020). Under a DAD perspective combined sometimes with other frameworks (i.e., Structuring Features of Classroom Practice – Gulay and Ruthven 2015, thought collective – Fleck 1981/ 1934, CoP, boundary crossing), the authors of these papers propose two central notions: *resource system* for studying teachers' activity with resources as a whole and *documentational trajectory* for addressing the influences of teachers' collective activity on their documentation work over the years. Rocha's (2018) analysis of each of Anna's and Sonia's documentational trajectories indicates that the work of different collectives plays an important role in justifying their resource design choices and uses. Sésames is mentioned as an example of such a collective and MET (Mise en train, organizing teacher's work at the beginning of the lesson including task launching) as an example of an activity emerging in it. MET is discussed as a resource that had a structuring role in Anna's way to create resources for the teaching as well as for use by others (Rocha and Trouche 2017). By analyzing further Anna's collective work with a colleague (Cindy/documentation-working mate) Trouche et al. (2020) report that: first, the DRs played an essential role in spurring these teachers' collective work with resources, not only during their practical work "on the spot", but also over time, providing the means for constituting a shared collective resource system; second, the DRs opened up space for renewing these teachers' resource systems; third, the face-to-face interactions between the two teachers around a complex issue (i.e., teaching algorithmics) led them to critically analyze possible contributions to the available resources (i.e., Scratch). These authors conclude that professional learning is likely to be more beneficial in groups as teachers develop a common resource system that is sourced by several professional experiences and trajectories. The notion of resource system has helped them to develop deeper understandings of teachers' use/design of digital or non-digital resources in/for their teaching. Also, it provided a window into the ways collaboration shapes their professional work and learning over time.

Lesson enactment and professional learning As regards the three studies that address the impact of teacher collaboration on lesson enactment, the focus is again on the relationship between teachers' participation in communities and aspects of their classroom teaching. In most cases, the researchers analyze the planning and enactment of lessons looking for evidence of this relationship. Two studies focus on

changes in teachers' discourse. Chen et al. (2020) focus on changes in one teacher's talk in the classroom because of her participation in a video-based PD that aimed to facilitate teacher learning of "academically productive talk" (Resnick et al. 2010). In the PD, video-clips of teachers' classroom teaching were coded according to the framework of academically productive talk, and the PD prompted teachers to reason about one another's discourse data and to plan for improving their classroom discourse. The PD empowered evidence-based discussions and collaborative learning, and the case teacher improved her discourse practices considerably.

The study of Elbaum-Cohen and Tabach (2020) focuses also on teachers' discourse practice as they participate in a reflection-encouraging PD on the integration of DRs in mathematics teaching. The authors use the commognitive theory of learning with particular emphasis on professional identity. A teacher's professional identity is defined as the collection of all reified (i.e., what is this person), meaningful (i.e., depicting critical characteristics), and endorsable (i.e., support the story with evidence) stories that are told about that person. The community of peers in the PD seemed to play a significant role in the development of the case teacher's professional identity in dialectic way: the teacher contributed to this community and the community contributed to his practice through reflecting on the stories of the peers. The teacher's professional identity "took a turn from an impotent teacher who has no impact on his students' learning, to a teacher who is sensitive to his students' needs and is able to attend to those needs with pedagogical and technological means" (p. 124).

In another study, Patahuddin (2013) uses zone theory (Goos 2005) to examine closely the teaching practice of one teacher, Jack. The focus is on Jack's learning processes in terms of integrating the use of online resources over time in his everyday teaching. Acting as a critical ethnographer (i.e., refusing to support Jack), Patahuddin analyzes Jack's tensions when integrating online resources in his classes (e.g., allowing students to work independently at the computers versus the need for continual assistance and time constraints). The researcher's effort in promoting the use of internet created a better alignment between Jack's zone of proximal development (ZPD) that represents teacher knowledge and beliefs and zone of free movement (ZFM) that represents the professional context, which defines the teaching actions allowed. This intersection held promises for the development of the teacher. This may indicate that working with teachers who have larger compatibility between ZPD and ZFM would promise a more productive impact.

Planning and enacting lessons: professional learning through boundary crossing One study (Robutti et al. 2021) addresses the impact of teacher collaboration on both planning and enacting lessons. The authors report two boundaries and their crossings. In the first crossing, the teachers in Australia engaged in the new practice of designing MERLO items relevant for their context (e.g., national curriculum). In the second crossing, the same teachers engaged in the new practice of lesson planning through the use of dynamic representations rather than static ones. The teachers recognized the utility of the dynamic feature of the items and welcomed it in their lessons, transforming their teaching practice with a hybridization process that, in some cases, crystallized (in the terms of Akkerman and Bakker 2011). The above

crossings were possible due to the use of dynamic geometry software (GeoGebra) that supported a modification in the design of the item. The analysis shows boundary crossing as a process of transformation that can influence a modification (more or less stable) in the practices of the participating teachers.

Summarizing, under diverse theoretical and methodological approaches as well as kinds of DRs the studies reported in this subsection focus on lesson planning and/or enactment indicating rich learning outcomes in terms of teacher design and teaching practices. Specifically, some studies show the critical role of DRs such as GeoGebra, a dashboard, and innovative representations of classroom interactions in the evolution of teachers' competences and creativity in designing teaching resources as well as in making research-informed curricular decisions. Some other studies reveal transformation of teaching practice evidenced in teacher discourse and lesson enactment as well as in the development of professional identity. Few studies that focus on teacher collective work with DRs over long periods of time reveal enormous learning outcomes for teachers such as developing shared resources, renewing their resource systems, and critically analyzing the contribution of DRs in student learning. In all these studies teachers designed DRs in different collectives and, generally, they designed with others for their own teaching.

Discussion

Our literature study attempted to shed light onto the interplay between collective and individual aspects of teacher design-work around the use of DRs. For this, we focused on the context, the product, the purpose, and the processes of teachers' design-work and its impact on their professional learning under different theoretical approaches. We addressed the above issues by analyzing thematically the 36 selected papers. The two main themes that emerged concern the process of teacher collaboration and the impact of teacher collaboration on teacher professional learning. The first theme addresses our first research question (forms, conditions, products), while the second theme addresses our second research question (the individual teacher's learning). Our third research question, theoretical and analytical approaches, is addressed within each theme.

As regards the process of teacher collaboration, most of the studies take place in the school context where a small number of teachers collaborate to plan and enact everyday lessons. Some studies address the emergence of a CoP where teacher groups collaborate with researchers in online or face-to-face PD settings. Few studies involve large numbers of teachers collaborating with researchers or other stakeholders where emerging tensions are studied as well as the role of different agents in the process of collaboration. In most of these studies, DRs are the product of teacher design while in a few cases DRs facilitate their design-work in the form of a communication platform or an environment for designing tasks and lessons. When teachers work in an online platform, their collaboration is facilitated by the available infrastructure. Our study shows that the goals of teacher collaboration are often oriented by researchers or project stakeholders. Teachers are expected to fulfil these

goals both when designing for others or for their own teaching and perceiving themselves as agents of change. In a few cases, teachers are self-motivated and take the initiative to collaborate exploiting the affordances offered for instance by social media (e.g., Facebook) to share design experiences or to develop teaching resources. The conditions that support or inhibit teacher collaboration have been reported by only few studies. The provision of space and time, the development of common perspectives, and the school expectations are considered important for supporting teacher collaboration while institutional and classroom practice boundaries create tensions that need to be overcome.

DAD and CoP are the main theoretical perspectives used to study the process of teacher collaboration involving the use of DRs. DAD, especially combined with CoP or CHAT, offers constructs to address the relationship between teachers' individual work and their participation in collective design-work. In these cases researchers look for evidence of the interplay between collective and individual by analyzing individual teaching and searching for links between teachers' individual schemes and collective experiences. CoP helps researchers to relate teachers' participation and interactions in a community for the evolution of collective design through negotiation and reification. It also allows researchers to study the critical role of context in relation to the individual teacher's participation in a CoP and the reified products. CHAT, ATD, and boundary crossing frameworks have been used to address conditions and contextual constraints that frame teacher collaboration as well as the collective and individual interplay. Main take-aways of these studies include: the complexity of teacher collaboration and the difficulty to reach design consensus; the interrelation between individual and collective schemes; the need to make more explicit the mediational role of different DRs in the process of teacher collaboration related, for instance, to space for autonomous work and motivation; and the need for further networking of different theoretical approaches to address subtle issues underlying the process of teacher collaboration.

As regards the impact of teacher collaboration on teacher professional learning, it has been addressed in three different manners: first, by identifying dimensions of teacher knowledge in collective design-work including teachers and often researchers/teacher educators; second, by using teacher design expertise to analyze in-depth the character of teacher knowledge related to their use of resources; and third, by applying a social practice approach to address teacher learning as/through participation in design-collectives. The studies prioritize how DRs mediate such learning and the conditions that frame it.

The studies show that specific kinds of DRs (e.g., online environments, digital teaching materials, navigation tools) shape teachers' collective design-work and influence what individual teachers can learn. For instance, the novelty of a specific DR challenges teachers' understanding of the design practice (lesson planning and enactment) while teachers' familiarity with a DR can lead to adaptations in actual classroom use. Also, researchers' principles for designing a digital navigation tool can act as a boundary object allowing coordination of teachers' and the researchers' discourses as regards task selection. DRs that facilitate communication such as online discussion forums or communication message boards provide realizations

of professional learning beyond a linear progression. Co-designing and interacting with a DR (e.g., a virtual manipulative, an e-textbook) promote awareness of the potential of these artifacts in mathematics teaching and learning. Representations of teaching through specially designed DRs that simulate classroom interactions (i.e., cartoons) can act as a common point of reference for teachers leading to deeper discussions about classroom phenomena.

Most studies within the second theme indicate positive learning outcomes for the individual teacher. However, several studies report a complex picture as regards contextual aspects that can affect the individual outcome of collective design-work. In particular, conditions that promote (or hinder) such learning are emerging conflicts, tensions and disruptions among the participants, peer queries, and feedback. Also, new understandings can be affected by the established social norms, teachers' previous experience, and the role of the researchers. Very few studies take a detailed view on the dialectic relationship between individual teacher's learning and peer interaction. In addition to DAD and boundary crossing as frameworks to study teachers' professional learning, also CoP, Meta-Didactical Transpositions, TPACK, teacher noticing, commognition, and zone theory have been used.

We will highlight five main findings regarding the impact of teacher collaboration on individual teacher's learning:

1. Using DRs to facilitate teachers' collective design-work (e.g., MOOCs, online forums) seems to have a positive impact on individual teacher learning.
2. Digital representations of classroom practices can support the development of teacher noticing.
3. Teachers' collective design-work focused on the use of DRs in mathematics classroom can influence the individual teacher learning in terms of contextual aspects and familiarity with DRs.
4. Individual teacher learning-gains are related to the transformation of design and teaching practices (but only few studies investigate such learning gains over time and in relation to students' learning).
5. Diverse theoretical and analytical approaches have been used to address aspects of teacher professional learning.

Our synthesis of the literature study indicates the state-of-the-art of the interplay between collective and individual aspects of teachers' design-work with DRs and stresses existing gaps and areas for further research. Specifically, teacher collective design-work facilitated by DRs has particularities stemming from the affordances of the DRs that provide new opportunites for communication and co-design. Moreover, it seems that teachers' design-work and the collective-individual interplay become more complex as they involve the co-existence of different perspectives, norms, and identities. The existing attempts to develop theoretical and analyical frameworks to study this interplay have focused on the direction from teachers' collective design-work to individual design and learning with DRs. However, there is a need to further elaborate frameworks and network theories to address this direction in more details and also to address the other direction: How individual teachers influence collective

design-work. Our literature study also shows that the specific affordances of DRs play a central role for teachers' collective design-work and their learning. Further research is needed to define more explicitly these specificities of DRs and relating them to their mediating role in teachers' design-work. Researchers using existing analytical constructs (e.g., teacher design expertise) need to take into account these affordances of DRs when analyzing the complexity of the individual-collective interplay.

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