Networking constructionism and social semiotics in order to investigate students' bodily engagement with tasks in three-dimensional space

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Abstract

In this paper we aim to address the problem of fragmentation of theoretical frameworks within the field of mathematics education with technology while exploring the potential of turtle metaphor for students' meaningful engagement with angle in three-dimensional (3d) space. We developed cross-case analyses of two experiments: the one took place in the Greek context under a constructionist theoretical perspective and the other one was conducted in the UK context under a social semiotic perspective. The analysis indicates that the aforementioned method enhances our efficiency to capture tacit aspects of theoretical frames which nevertheless have an important bearing on analyses and knowledge emerging from the research experiments.

Keywords

Turtle metaphor, angle, 3d space, constructionism, social semiotics.

Introduction

The study of 3d objects (e.g. angles, 3d shapes) is known to be an obstacle for many students beginning to study 3d geometry. This is an area of mathematics in which students' informal ways of experiencing shape and motion within the three-dimensional world around them are excluded by the current teaching approaches in schools. Common approaches within the school curriculum provide relatively limited forms of experience, often relying almost exclusively on paper-based 2d representations of 3d objects. In particular, we note issues identified by research in relation to identification and operation with angles (Clements & Battista, 1992) and coordination of the various facets of the angle embedded in the physical contexts of corner, turn and slope (Mitchelmore & White, 2000). The development of new modes of representation within specially designed technological tools has generated further interest in the area of 3d geometry. As for angle, one of the prime affordances of such tools is the multiple linked representations designed to make different aspects of angular relations more accessible and meaningful to students. For instance, Logo-based computational settings integrate dynamic aspects of angle as turn with symbolic registers provided by Logo within a differential geometrical system (Papert, 1980). Existing research suggests that interaction with 2d Logo-based computational environments can be a fruitful context to challenge students' intuitions and ideas about angle as turn come into play through the turtle metaphor (Magina & Hoyles, 1997, Kynigos, 1997). More specifically, these studies seemed to adopt a body-syntonic approach to mathematics learning according to which construction of mathematical meaning could be considered as interrelated with students' sense and knowledge about their own bodies (Papert, 1980). This connection to personal bodily knowledge is operationalised through ‘playing turtle’, either literally by walking along a path or metaphorically in the imagination. In the research reported in this paper the students worked with a digital medium called *MaLT (MachineLab Turtleworlds)* which integrates a 3D Turtle Geometry, driven by a specially designed version of Logo with variation tools for dynamic manipulation of graphically represented mathematical objects (Kynigos et al., 2009).

Our main research aim was to explore the potential of MaLT as a context to investigate students’ construction of meanings for angle in relation to 3d geometry. However, extension of Logo to 3d space, which is close to our physical experience of the world, raises new issues related to the extent to which the ‘playing turtle’ metaphor can be adaptable and relevant in this context. Thus, it can be seen as an opportunity to reconsider the role of bodily engagement in mathematics teaching and learning, taking into account the recent research interest in the use of gestures in mathematics education. Much of this has focused on the gestures used by students, analysing the contribution made by gesture to learning and mathematical meaning making (e.g. Radford, 2009). In considering gestures used by teachers, studies have shown teachers and students making shared use of gestures initiated by student communication efforts (Arzarello et al., 2009; Maschietto & Bartolini Bussi, 2009) and teachers using deictic gestures as mediating resources (Bjuland et al., 2009). Taking into account that bodily engagement in general and gesture in particular constitute an interesting characteristic when considering learnability of mathematics, here we report research aiming at shedding light on the potential of turtle metaphor through the use of MaLT to facilitate students’ meaningful engagement with angle in 3d space. Yet, it seems difficult to really appreciate this potential, since it is needed to take into account the visions provided by specific theoretical frameworks in technology enhanced mathematics, and because of the fragmented character of these frameworks (Artigue, 2009).

Our aim in this paper is to combat fragmentation, trying to connect visions based on different theoretical perspectives. We have chosen the metaphor of networking theoretical frameworks and the idea of combining and coordinating frameworks “for the sake of a practical problem” (Prediger et al., 2008 p.172). We draw on data from an experimental teaching programme, conducted as part of the ReMath project (Representing Mathematics with Digital Media, European Commission FP 6, IST4-26751). More in particular, we reflect on the role played by theoretical frames in two teaching experiments designed and implemented with MaLT by two research teams working in different national and didactic contexts under different theoretical orientations. The first one was conducted in the Greek context by the University of Athens Educational Technology Lab (ETL) project partners under a constructionist theoretical perspective. The second one took place in the UK context by the Institute of Education (IoE) project partners through a social semiotic approach. The question we aim to tackle is: what new insight about the potential of turtle metaphor through the use of MaLT might be gained from contrasting different research studies carried out by researchers working in different research and didactic contexts under different theoretical perspectives? As a way to combine/coordinate the two frameworks we were engaged in the task of developing cross-case analyses of the conducted pairs of experiments, i.e. a unified associative/comparative description of two studies by way of constructionism and social semiotics. We introduce briefly the main ideas of the two frameworks and then we report on the teaching experiments. In the analysis, we first highlight the elements of an analysis from each theoretical perspective. Then, due to space availability, we demonstrate a comparative commentary on the two analyses only from a constructionist point of view.

Constructionism and multimodal social semiotics

The main theoretical framework adopted by the ETL team is constructionism (Papert, 1980, Harel & Papert, 1991). A fundamental construct of ETL's constructionist perspective is *situated abstraction* (Noss & Hoyles, 1996) that addresses the nature of concepts and the way in which they are formed**.** According to this theoretical tool, abstraction is seen as a process of layering meanings on each other, rather than as a way of replacing one kind of meaning (concrete, referential) with another (abstract, decontextualised). The idea is that students could web their own thinking by communicating with and through the computational tools and shaping them to fit their own purposes, including the need to communicate with others. In the ETL view, situated abstraction can be seen as a physical/intellectual context providing new resources for the learners to (re)think-in-progress while exploiting the available tools to move the focus of their attention onto new objects and relationships within the setting, while maintaining their connections with existing ones (Noss et al., 1997). ETL used MaLT as a means to engage students in making connections between static and dynamic contexts for experiencing angle in 3d space.

The primary theoretical framework adopted by the IoE team is multimodal social semiotics (Kress et al., 2001). Although originating in linguistics, this theoretical framework challenges the primacy of language as a means of communication and meaning making, highlighting the different potentials for meaning offered by different modes of communication and various available semiotic systems (O'Halloran, 2005). Multi-modal and multi-semiotic environments allow participants many opportunities for making meanings with the representations available to them and choices about the most apt representations to employ in order to communicate their desired meanings. As for mathematical communication, IoE team adopts a perspective which recognises the multimodal nature of communication and the importance of studying the contributions made by different modes of communication and representation. Kress et al.'s (2001) multimodal analysis of communication in science classrooms shows teachers and students making use of a "complex ensemble" of modes, including gesture alongside speech, writing, images, etc. In this vein, IoE team views learning mathematics as learning to participate in specialised mathematical forms of discourse which includes recognition of how the specialised discourse is distinct from others, including the everyday. The objective guiding the IoE research was to investigate the meanings students made for angle in relation to 3d geometry through their semiotic activity in the context of working with multiple modalities of resources.

The computer environment

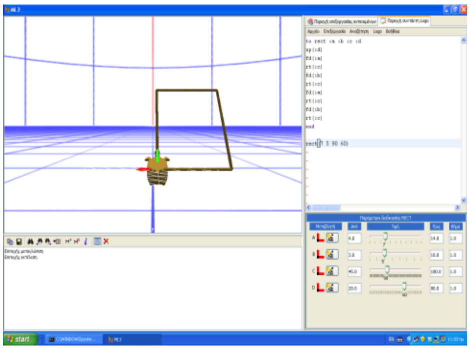

Figure 1: A rectangle in MaLT.

MaLT (Kynigos et al., 2009) is a programmable environment designed to extend Turtle Geometry to 3d. It consists of three interconnected components (see Figure 1): the Turtle Scene (TS), the Logo Editor (LE) and the Variation Tools. The available version of Logo provides an extension of Logo commands in 3d space including two new types of turtle turns: 'UPPITCH/DOWNPITCH n degrees' ('up/dp n') which pitches the turtle's nose up and down on a plane perpendicular to the one defined by right-left turns and 'LEFTROLL/RIGHTROLL n degrees' ('lr/rr n') which moves the turtle around its own axis. MaLT provides variation tools which afford dynamic manipulation of variable values. Figure 1 shows the Uni-dimensional Variation Tool (1dVT) whose main part consists of 'number-line'-like sliders (see Figure 1), each corresponding to one of the variables used in a Logo procedure.

Methodology

The study was conducted as part of a programme of 'cross-experimentation' (Artigue, 2009) in which the designers of the software (in this case ETL) and another 'alien' research team (in this case the IoE team) designed and conducted separate teaching experiments in their local contexts.

The ETL experiment took place in a secondary school with one class of twenty 7th grade students (13 year-old) and one experimenting teacher who acted also as a researcher. The class had eighteen teaching sessions in all with the experimenting teacher over two months. The pedagogical plan aimed at engaging students in knowledge building activity with some degree of autonomy independently of the standard curriculum and, indeed, deliberately by-passing traditional teaching approaches. Task 1 engaged students in developing their 3d sense of motion (i.e. simulating the take-off and the landing of an aircraft in MaLT with the use of a concrete model of an aeroplane). In the next three tasks angle was approached through the simulation of 3d geometrical objects which involve turning often encountered in everyday physical angle situations. In particular, in task 2 students were asked to construct rectangles in at least two different planes of the TS simulating the windows of a virtual room. In tasks 3 and 4 students were asked to develop or to correct parametric procedures so as to simulate the opening and closing of a door (task 3) and a revolving door (task 4). In classroom observation a participant observation methodology was adopted. The main corpus of data included video-recorded observational data, researchers' observational notes as well as the sorting and archiving of the corpus of students' work on and off computer. In analyzing the data we looked for episodes where meanings related to the visualisation and conceptualisation of the notion of angle in the simulated 3d geometrical space were expressed by the students. In most cases (e.g. episodes involving actors' bodily engagement), we base our analysis on the joint study of the transcribed interactions with the available video recordings.

The IoE experiment was conducted in a state secondary school in London with a Year 8 class (aged 12-13 years). The students had no previous experience with MaLT or other forms of Logo. The IoE pedagogical plan was designed to engage students with representations of 3d shape through designing and constructing a virtual building that would be of use to the school or wider community (e.g. a sports hall). The IoE tasks were similar to those developed by ETL but there were three distinct contextual differences: (a) the IoE team used a range of both traditional and innovative representations (e.g. students' use of multilink to reconstruct buildings from isometric drawings or to construct buildings through the use of plans and elevations); (b) the educational goals of the tasks remained clearly within the standard curriculum; (c) the students involved were marginalised within the school and broader educational system and had many difficulties – both social and mathematical – engaging with the planned activities. A sequence of nine lessons was taught collaboratively by the class teacher, the researchers and a student teacher attached to the class. In each lesson a video record was made, focusing on the teacher or researcher during whole class interaction and on a selected student or group of students during individual or group tasks. The video aimed to capture gestures and the various visual and physical resources available, including the computer screen when in use. Episodes in which use of multiple semiotic modes was evident were selected for transcription (see Morgan & Alshwaikh, 2011 for more details).

Bodily engagement through gestures

One common theme of the two analyses concerned the students' and/or teachers'/researchers' use of gestures that emerged during the implementation of tasks in the classroom. One significant type of gesture was a set of stereotyped hand and arm movements, often associated with use of the terms *turn*, *pitch* and *roll* and the associated Logo instructions (see Figure 2). They may be considered *iconic* gestures (Roth, 2001), in that each bears a visual resemblance to the anticipated trajectory of an object moving in 3d space.

ETL analysis: Gestures scaffolding meaning generation for angle in 3d space

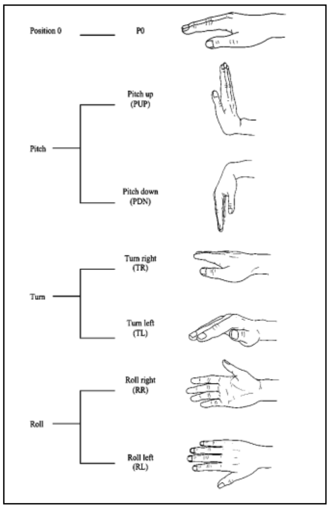

Figure 2. 3d turn gestures.

Bodily engagement in the Athens experiment was related to the students' informal or spontaneous use of iconic gestures. It thus constituted one aspect of the ETL analysis which emerged as a coherent part of the students' construction of meanings for angle in 3d space interrelated with their attempts to describe the turtle's navigation as well as to conceptualise the role of 2d representations in forming angular relationships in 3d space. In the next episode the students' use of gestures appeared as part of their struggle to understand the ways by which the combination of the new turning commands in the Logo language could affect the manipulation of a 2d geometrical figure so as to construct the simulation of a door. We note that the episode took place before students were asked by the researchers to construct a door simulation which constitutes one activity of the ETL pedagogical plan (task 3). Initially Group B students constructed a rectangle with three variables on the horizontal 'ground plane' (Table 1, Procedure 1). Having recognised the way in which the up(90) command affected the position and the orientation of the turtle, they inserted the command up(90) at the beginning of the respective procedure and constructed the same rectangle on the 'screen plane' which -in mathematical terms- is perpendicular to the 'ground plane' (Table 1, Procedure 2).

to rect :a :b :c	to rect :a :b :c	to rect :a :b :c :d
fd(:a)	up(90)	up(:d)
rt(:c)	fd(:a)	fd(:a)
fd(:b)	rt(:c)	rt(:c)
rt(:c)	fd(:b)	fd(:b)
fd(:a)	rt(:c)	rt(:c)
rt(:c)	fd(:a)	fd(:a)
fd(:b)	rt(:c)	rt(:c)
rt(:c)	fd(:b)	fd(:b)
end	rt(:c)	rt(:c)
	end	end
Procedure 1	Procedure 2	Procedure 3

Table 1: Logo procedures for rectangles.

When trying to concretise the new position of the rectangle in 3d space one student used her hands so as to mimic the movement of the turtle from the surface to the 'screen plane' (Figure 3).

R: What happened to the turtle with up(90)?

S1: [Whole rt arm horizontal P0, hand moves up PUP 90°] It [i.e. the turtle] took it [i.e. the rectangle] that way.

R: If we put 45, what would have happened?

S1: [rt hand moves up PUP 45°] It [i.e. the rectangle] would be nearly in the middle.

R: If we put 50;

S1: Ok, not in the middle. [rt hand moves up PUP a bit more] A bit more than that.

The dynamic character of student's bodily engagement in the simulation challenged both of them to try to visualize it on the screen. S1's iconic gesture here signified the actual move of the recrangle in 3d space. At that time, this kind of gesture seemed to provide a basis for S1 to make sense of the type of turtle move in 3d space, to consider it as varied and to link it to the relevant Logo command (uppitch). So, S1 afterwards had the idea to replace the value 90 in the command up(90) (Table 1, Procedure 2) with a new variable :d to see what would happen. Then, dragging on the slider of the variable (:d) in 1dVT had the effect of the figure dynamically moving upwards – downwards visualising in that way the dynamic move of the rectangle in different planesas well as the preceding uppitch-downpitch gestures made by S1.

Figure 3

S1: [Moving the slider (:d)] Look! If we move it [i.e. the slider (:d)] upwards it [i.e. the rectangle] raises ... If we move it [i.e. the slider] downwards it descends.

In the evolution of the episode the students had the idea to insert in the procedure a roll command so as to simulate the continuous move of a door. The sequence of what happened next is as follows. Initially one of the students substituted the command up(:d) (Procedure 3) with one of the roll commands (rr :d). Moving the slider (:d) then she realized that the direction of the axis of rotation was perpendicular to the screen plane (‘*it turns as a wheel*’ she said) (Fig. 4, on the left).

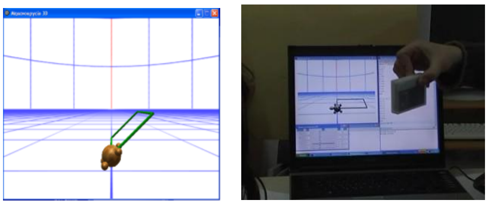

Figure 4: A ‘rolling’ rectangle (left) and a video-cassette as ‘rectangle’ (right).

At this phase students continued to ‘play turtle’ to identify the type and the sequence of the turtle turns which would result in the desired simulation. In doing so, they faced difficulties in imagining -and thus mimicking- in which way the turtle ‘moves’ the rectangle in different positions and directions in 3d space. They found efficient to rehearse the move of the rectangle with the use of a concrete 3d object, in this case a video-cassette, so as to visualise the change of planes of the rectangle as a result of the change of the initial position of the turtle in 3d space (Fig. 4, on the right). So, students realized that initially the rectangle needs to be raised up and then turned (‘rolled’) on the right. Modifying accordingly the Procedure 3 students used one more variable in the command rr(:e) that was inserted after the initial command up(:d). They subsequently achieved to simulate the ‘opening-closing’ door (Fig. 1) by dragging the slider (:e) on the 1dVT after selecting the value 90 for :d.

The above episodes indicate the conceptualisation of angle as a spatial visualisation entity interrelated with move through different planes ‘inside’ the TS. We see a dynamic aspect in students’ bodily engagement in these episodes. While ‘playing turtle’ with the use of hands and/or the cassette, the students defined the dynamic manipulation of the rectangle by using position and heading of the turtle which seemed to ‘coincide’ with the rectangle (i.e. the turtle appears in some way to ‘carry’ the rectangle). Actually, the students oscillated between two frames of reference: (a) the world frame: defined in terms of the fixed directions ‘up’ and ‘down’ and (b) the vehicle frame: typically associated with the orientation of a moving entity, here the turtle. In the initial construction on the ‘ground plane’ the vehicle frame of reference coincides with the world frame of reference. In other words, the ‘up’ in relation to the turtle’s position coincides with the ‘up’ of the simulated 3d space. Thus the students’ gestures at that time integrated both iconic and deictic features: they command the turtle to move ‘upwards’ to the ‘screen plane’ and indicated this through S1’s gesture showing also the ‘up’ direction in the everyday world. Here the desired drawing concerning the transition of the designed rectangle to the ‘screen plane’ coincides to the required type of turtle turn. At the same time, the students enacted certain situated abstractions concerning the position of the rectangle in 3d space in relation to the turtle’s continuous turning and finally they were able to express these dynamic movements/turns with the use of variables. Thus, students were able to coordinate turtle’s/rectangle’s turning in the 3d space with the formal (Logo) notation and the dynamic manipulation provided by the available tools. We highlight the episodes in order to show the evolution of students’ purposeful use of the available tools as situated in a larger process of abstracting angle as a spatial visualization concept *within* the setting by making connections between existing and emergent views of angle in 3d space.

IoE analysis: Imag(in)ing 3d movement with gesture

As we started to view the video data collected during the London experiment, it was noticeable that the teachers and researchers made extensive use of iconic gestures in an apparent attempt to support students' planning and execution of constructions. This set of gestures constituted a new semiotic system, linked with, but not identical to, both the linguistic description of movement and the symbolic system of Logo. Students also made use of these and other gestures to support their communication about turtle movement. Although the students used 'these' gestures to indicate that their hand and arm movements resembled those used by the teachers/researchers, we believe, as will become apparent, that the students made use of them in different ways, thus construing different meanings.

***Episode 1*:** In the introductory session with MaLT, one of the research team introduced the notion of turtle movement using a toy aeroplane (i.e. simulation of the plane taking off). While navigating the aeroplane, the teacher accompanied the physical movement of the hand/aeroplane with a verbal description, using and stressing the terms pitch, roll and turn in synchrony with the associated gestures. In a later lesson, recognising that some students were still having difficulty distinguishing between these different kinds of turn, the class teacher used her arm and hand to act out the role of the turtle drawing a 'door' under instruction from the class while a researcher entered the Logo instructions into a computer, displaying the resulting turtle path on a large screen. The teacher was careful to follow the conventions of the gesture system in order to emphasise the relative nature of turtle movement. Thus, for example, she turned her hand in a down pitch gesture when given the instruction to go down, even though this resulted in her hand pointing horizontally as in Figure 5. This resulted in conflict for students between their intended outcome and the visual feedback provided, leading to rapid self-correction of the Logo instructions.

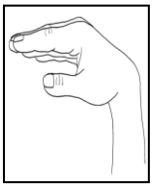

Figure 5: down pitch.

Episode 2. Student T, having constructed one rectangular wall, was trying to construct a second wall perpendicular to the first. She explained what she was trying to draw using language and gesture.

1	here	*whole rt arm vertical P0, palm facing away from body, moves up in direction of fingers*	
2	turn here	*TR, arm moved in direction of fingers (maintaining TR position)*	
3	turn here	*attempt to move rt hand TR again (too difficult?)*	
4		*switch to lt hand, arm horizontal pointing rt, hand PDN (fingers pointing down)*	
5	turn here	*moves forearm clockwise, hand still PDN (fingers pointing left)*	
6	but I want it to come forward	*turns arm (awkwardly) so that, hand still in PDN position, fingers point towards body*	

Table 2: T imagines a wall.

The switch (lines 3-4) between use of right and left hands appears to be a response to the physical difficulty of achieving the desired position with the right hand. We consider what remains the same and what is changed with this switch of hand. The switch allows T to maintain the direction in which the fingers are pointing (down). This may be taken to represent the turtle heading within the vertical plane parallel to the screen. However, in switching arms, she changes the relationship between arm and hand from a *turn* gesture to a *pitch* gesture. We use *turn* and *pitch* within the conventions set up by the teachers/researchers and the Logo language, not to suggest that T associates her gestures with these terms. On the contrary, she does not appear to attach any significance to the distinction, focusing solely on the position of her hand and the direction in which her fingers are pointing in order to describe the intended turtle movement. While she is to some extent 'playing turtle' with her hand, she is defining the turtle's movements by using position and heading at the corners of her imaginary wall rather than by using turn and distance as required by the Logo language. The use of the turn and pitch gestures is thus not supporting her move into using Logo code and may indeed have made her communication with teachers/researchers less effective.

In considering the difference between the ways in which teachers/researchers and students were using the 'same' gestures, we distinguish between the two notions of *imaging* and *imagining*. We define *imaging* as using an iconic gesture to create an image of the construction of the turtle path. The movement of the hand mimics the movement of the turtle: the forearm is held parallel to the current heading of the turtle and the hand is moved to define the next heading. Thus the gesture indicating 'up pitch' is always relative to the current heading of the turtle. In episode 1, the teacher/researcher gestures were imaging the process of construction of the turtle path. In contrast, in episode 2 student T used apparently similar hand movements to construct a very different effect. For her, the relationship between forearm and hand did not appear to have significance, as she was willing to substitute a pitch down gesture with her left hand for a turn right gesture with her right hand. We characterise her use of gesture as *imagining*, referring to her mental image of the desired outcome of turtle drawing. Such use appears to have both iconic and deictic characteristics. In this episode, as in several other episodes of student gesture within the data set, the gesture points to the desired direction of movement in order to draw the desired outcome, rather than mimicking the required type of turn. Thus, for example, a movement in the 'up' direction (within the plane of the screen) might be indicated by use of the spoken word up accompanied by a 'pitch down' gesture. While it might appear at first sight that students adopted the specialised gestures employed by the teachers/researchers, the students' use and interpretation of these gestures may be closer to the resources of everyday discourse than to those of the MaLT microworld (Morgan & Alshwaikh, in press). The extra leap of imagination required to 'play turtle' as if in control of an acrobatic aircraft or perhaps in deep water with highly developed underwater manoeuvrability may be too great for genuine body syntonicity.

Commentary on the two analyses from a constructionist perspective

Comparing the ways in which both teams analysed the students' use and interpretation of gestures to support their communication about turtle movement reveals distinct differences. IoE interest in gesture arose from concerns about the ways in which students might make use of new semiotic resources offered to them by teachers/researchers and about the coordination of different semiotic systems. The IoE analysis highlighted differences in the meanings associated with the gestures by teachers/researchers and the students while 'playing turtle'. Teachers and researchers used specialised hand gestures to communicate with students about 3d movement. Students used the 'same' gestures but to communicate different meanings in relation to turtle movement. Whereas the *imaging* by teachers/researchers mimicked turtle movement in a kind of 'playing turtle' action, student use of gesture to *imagine* the outcome of the movement seems closer to deixis, pointing in the direction of movement from a viewpoint outside the turtle. Thus, IoE researchers focused on the relationships between the formal set of gestures related to Logo terms as used by teachers/researchers and by students while the relevance and importance of students' informal or spontaneous use of gesture has not been a focus of their attention in their study. Although it is apparent from the IoE analysis that the student engaging in the task did generate meanings about the notion of angle in 3d space, IoE chose to focus on the distance –actual and conceptual- between the students and teacher/researcher use of hand gestures and that the 'playing turtle' metaphor did not easily transfer into 3d context.

The ETL team's interest in gesture emerged as part of the teams' research focus on the students' use of the available representations of MaLT to construct meanings for angle in 3d space. Under this perspective the ETL team aimed to address the relevance and importance of students' informal and/or spontaneous use of gesture as part of their attempts to achieve their goals in the given setting (e.g. simulating the opening-closing of a door). Thus, the team focused on how the available representations in MaLT served as a resourse for the students to challenge their engagement with the tasks –which involved the use of their bodies or other objects- to construct meanings for angle in 3d space. The meaning generation process in both the IoE and ETL experiment is perceived by ETL as being in close relation to the students' hand gestures which most of the times were mimicking turtle movement in a kind of 'playing turtle' action. From the ETL perspective the episode 2 provided by the IoE team would have beeen analysed as part of the students' attempts to conceptualise angle through a specific geometrical construction (i.e. vertical 'walls'). For analysing the same episode, the ETL team would have been interested in identifying what was visualised on the screen and in which ways gesturing affected the subsquent students' experimentation to complete or explore the current geometrical construction with the available tools as well as meaning generation for angle.

Conclusion

At the level of networking constructionism and social semiotics, the above comparative description of two analyses shows how such a process may reveal tacit aspects of theoretical frames which nevertheless have an important bearing on analyses and knowledge emerging from the respective research experiments. How for instance the two teams differing perceptions of the students' active engagement in gesturing influenced the resulting analyses concerning meaning generation. A general overview of the commentary reveals that while the constructionist approach of ETL seems to illustrate the students' abstractions providing insight into the mutual shaping of student/computer interaction (involving communication between the participants), the social semiotics perspective of IoE seems to illustrate the complexity of communication patterns that may affect the construction of meaning. This brings evidence that by combining constructionism with social semiotics, cross-analysis captures more efficiently the potential of MaLT as compared to the use of a single framework specific to a particular experiment. This is clearly a first step towards coordinating these approaches in order to get an integrated framework to analyse the potential of turtle metaphor into 3d context.

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