



Designing and modifying artifacts through actual implementation in mathematics classrooms

Elissavet Kalogeria, ekaloger@ppp.uoa.gr

Educational Technology Lab, School of Philosophy, University of Athens, Greece

Giorgos Psycharis, gpsych@math.uoa.gr

Department of Mathematics, University of Athens, Greece

Kalliopi Ardavani, popiardv@hotmail.com

3rd High School of Glyfada, Greece

Abstract

This paper reports research aiming to highlight how the design of artifacts with the use of digital tools in mathematics can be informed by the experience of their implementation in real classroom settings. Under a constructionist theoretical perspective, a group of two researchers and a newly trained teacher educator collaborated to explore the distance between design of tasks with digital tools and their actual implementation in the classroom of mathematics. A series of activities related to the Thales Theorem were created and implemented in different classes with the use of Sketchpad. The implementation led to significant changes in both the artifacts used (worksheets, microworlds) and the teaching management. The constructionist consideration of the artifacts as transformable objects facilitated the development of a scenario based on elements that emerged in actual teaching.

Keywords

Transformable artifacts, scenario design, communities of inquiry.

Introduction

Our general aim in this paper is to explore the contribution of constructionism as a design framework in addressing the potential of artifacts with technological tools in mathematics classrooms. The term artifact here describes tasks – and the correspondent worksheets and microworlds – designed as coherent parts of scenarios with the use of technology for the teaching and learning of mathematics. Constructionism as an epistemological paradigm and as a learning theory has been extensively used for designing expressive digital artifacts and studying students' generation of mathematical meanings individually and collaboratively. Over the years constructionism has also been used as a design framework in order to inform research and practice in a range of contexts such as the integration of digital tools in mathematics classrooms and the design and implementation of artifacts and activities by teachers as means for reflection and professional development (Noss & Hoyles, 1996, Kynigos, 2007). A common feature of all these approaches is that they were based on the use of expressive media designed under fundamental constructionist principles (e.g. construction of mathematical meaning through collective bricolage with artifacts, Papert, 1980). However, due to the current proliferation of digital tools for the teaching and learning of mathematics as well as the recently highlighted need for connecting different theoretical frameworks in order to address more efficiently the potential of artifacts for mathematics learning (Artigue, 2009), it seems to be useful to investigate the



contribution of constructionist approaches to learning as design in a range of different contexts and computational environments. For example: how a constructionist approach to development and modification of artifacts with dynamic geometry systems (DGS) might influence the evolution of teachers'/researchers' didactical design? This paper reports research aiming to highlight how the design of artifacts with the use of technological tools in mathematics can be informed by the experience of their implementation in real classroom settings. We adopted a broadly constructionist framework in designing and modifying artifacts with DGS (as a kind of 'bricolage') in a collective reflection context involving teachers and researchers. In this perspective, artifacts were considered as malleable objects transformed after reflection on the practices developed when implemented in actual classroom settings. Thus, the reported research indicates a novel approach to developing scenarios for technology enhanced mathematics: from practice to theory.

Theoretical Framework

The study was based on the assumption that the use of digital tools into the educational process requires teachers to undertake an active role in establishing new, enriched curricula, while making meaningful decisions for their students (Budin, 1991). This assumption implies the study and redefinition of the relationship between teachers and mathematics curriculum. When curriculum is referred to, we have to distinguish between the formal, intended curriculum (that which resides in state frameworks, guides, textbooks, and in teachers' minds as they plan what they will do) and what it appears to be, which is curriculum as enacted by teachers in the classroom and curriculum as experienced by students (Gehrke et al., 1992). The teacher - curriculum relationship can be approached through various theoretical frameworks (Remilliard, 2005), the most traditional of which, conceives the teacher as an "implementer" of the formal curriculum. Through this perspective, the clarity and the detailed instructions of curriculum texts aim at a closer guidance of the teacher and his/her dependence on them. However, existing research has shown (Remilliard, 2005) that there is a mismatch between the formal, intended curriculum and the enacted one. This finding emphasizes the importance of the context of implementation and recognizes the significant role of the teacher as mediator on what is planned and what really takes place in the classroom. Very important factors that influence teachers' mediation are their knowledge, beliefs and experiences. From this point of view, the teacher can be perceived as the "interpreter" of the curriculum, a person who has the ability to adjust or reconstruct it, according to his/her students and the learning conditions of each classroom. This "participative" relationship can be achieved through a process of design, during which, the teacher perceives and interprets existing resources and sources of the curriculum, evaluates the constraints of the classroom settings, balance trade-offs and comes up with strategies (Brown & Edelson, 2003). This process also involves enacting the respective plans in the classroom with students and redesign after each implementation.

The so-called "lesson plans" constituted the first step for the teacher's involvement in the design process. They particularly include: the material that has to be taught, the teaching method, the material required, the teaching time and the worksheets. The introduction of new material and theoretical frameworks brought to light new elements related to the learning environment that had to be included in the design, such as students' groupwork, spatial ergonomics, availability of other sources etc. As a result, the lesson plans yielded their position to the "activity plans". The notion of activity, that appears highly upgraded in the current mathematics textbooks of Greek High school, includes students' action upon the development of a specific mathematical concept (Stein et al., 1998).



However, the activity plans do not address the complexity of the learning environment, since they are mainly teacher-centred and seem to focus on the didactical sequence of the designed activities rather than on the expected learning processes and the students' expected actions. Over the last years, researchers in technology enhanced mathematics have suggested the educational scenarios as the means to fill this gap. From one point of view, scenarios can be considered as further developments of activity plans. However, current research in the field suggests that scenario development constitutes an in depth penetration into the teaching practice since scenarios include not only the usual topics of activity plans but also references to interrelated activities, as well as teaching, learning, administrative and socio-cultural processes (Laborde, 2001).

Teachers' engagement in scenario design with the use of technology brings in the foreground another parameter of the already complex teaching and learning environment: the technology. This new parameter, perturbs the existing balance of the system student-teacher-subject to be taught (Laborde, 2001), thus the teacher is supposed to determine through a series of decisions if, where and how technology would be used. Since many mathematics curricula have been enriched with activities that include the use of technology, the teachers can chose to implement them as given without making any changes or to modify them or even create their own ones. The way that teachers use technology is closely connected to their beliefs about the role of mathematics' teaching and the way they treat the curriculum's commitments. A scenario that includes the use of technology in mathematics has to provide a documentation concerning a number of crucial parameters of the teaching and learning process such as the students' difficulties with the targeted mathematical concepts and how these difficulties are addressed. Most importantly, a scenario has to describe the way and the form under which technology is going to be integrated. For instance, we consider the idea of half-baked microworlds (Kynigos, 2007). The term was introduced under a constructionist perspective to describe microworlds that incorporate an interesting idea but they are incomplete by design so as to invite students to deconstruct them, build on their parts, customize and change them, eventually constructing a new artefact through the modification of the original one. In designing a half-baked microworld the teacher has to make choices as to what the students are going to do with the available tools and the types of meanings that they are expected to construct through its use. In the present study we found relevant to export the idea of half-baked microworlds to a collective reflection context involving teachers and researchers. In this context, we used the idea of 'half-baked' to refer to artifacts which are considered by teachers/researchers as amenable to changes by themselves after implementation in the classroom through a cyclic process of 'design-implementation-redesign'. Other parameters that also have to be included in the scenario's design are the social orchestration of the class, the time and space aspects of the environment and the teaching management of the activities. In this sense, the process of developing scenarios signifies a change from the teacher as implementer to someone who actively constructs the practice of schooling and strengthens the teacher's professionalism (Carlgren, 1999). The processing of the teacher's decisions during the design (which is the subject of the present study) is considered as an integral part of his/her professional development. The recent teacher educators' training programs in Greece concerning the use of the digital technology in the teaching of mathematics, have led to the development of a series of notable scenarios. However, the majority of these scenarios has been developed theoretically, without being implemented in the classroom. As we see, at least in our country, there is a gap between the design of artifacts for mathematics teaching with the use of technology and their potential transformation if they had been applied in real educational settings. In the present study we took a constructionist perspective in order to investigate the distance between designed and actual implementation of activities with technological tools in the classroom of mathematics. More in particular, we considered scenarios and its accompanied material (i.e. microworlds, students'



worksheets) as questionable, malleable and extensible objects rather than as static models that had simply to be strictly implemented by the teachers (Kynigos, 2007). Thus, we chose to develop our study around the creation and further modification of a scenario after its implementation in real classroom conditions. The multiplicity of our roles in this process (designers, facilitators of the teaching process, researchers) has necessitated the need to be engaged in collaborative activities around communities of inquiry (CoI) (Jaworski, 2003).

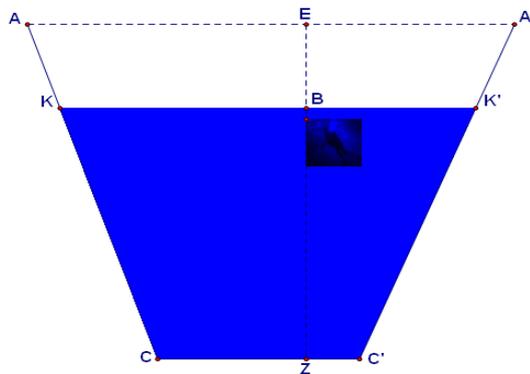
Teachers' CoIs can be an indispensable mechanism of support for the application of new, innovative actions, such as the use of technology in the teaching process. According to Jaworski (2003), the participants of a CoI related to mathematics classroom might be beginning, practicing, in-service teachers or even teacher educators. They -at all levels- are learners. Learning occurs while developing inquiring approaches on their practices (learning-on-practice) and rethinking about their actions and their artifacts. While starting wondering about issues of their teaching, by determining problems and complexities, they are approaching even more a situation of rethinking on action (reflection-on-action), during which they are able to take a thorough critical look over the facts (since they have already happened). This stage according to Jaworski (1998) constitutes an indication of meta-cognitive awareness. Simultaneously, this process provide the participants with the opportunity to enhance their knowledge, which includes –according to Shulman (1986)- the knowledge of mathematical content, the pedagogical one as well as the curricular knowledge. However, the theoretical knowledge that derives from the academic field is of different nature from the school-bound knowledge of the teachers (Jaworski, 2004). CoIs have come to normalize this difference, by enabling the implementation of theories and suggestions that are afterwards likely to become objects of reflection. In this context, the process of communally developing scenarios with the use of digital tools in mathematics, may contribute to the creation of a detailed, informal curriculum, enriched, well-documented with inquiry methods and has the potential to redefine the teacher-curriculum relationship. The specific community we refer to in the present paper had the purpose of operating as a link between members of the academic community, teachers and teacher educators.

The research

Context and aims: The current study is the outcome of the collaboration between a newly trained teacher educator for the pedagogical use of digital tools in the teaching of mathematics and two researchers from the academic field. At the beginning of the study the teacher educator had just completed the attendance of a 350-hour course at the University of Athens (i.e. in University Centres, UC) as part of a large scale professional development program of the Ministry of Education concerning teachers' familiarization with the use of digital technologies for teaching and learning of mathematics. The aim of the program was to provide the participants who were selected experienced mathematics teachers with methods, knowledge and experience in in-service teacher education and to educate them in the pedagogical uses of expressive digital media and communication technologies for the teaching and learning of mathematics. During their participation in the course the trainee teacher educators had developed a number of scenarios for different mathematics topics with the use of different categories of technological tools (e.g. DGS, Logo microworlds). However, the program at that time had not incorporated any process of implementation of the developed scenarios in real classroom setting. Consequently, the trainees had not had an opportunity to 'test' if and how their scenarios 'work' in real classroom conditions. In the present study we created a CoI as a means to address the transformations of didactical design through implementation of activities in mathematics classrooms. In this community, the teacher educator operated as secondary teacher since the designed tasks were



implemented in some of her classes. A basic principle underlying this community was to create links between theory and practice by communal design of artifacts and further transformation of them after tested in real classrooms. In other words, we propose a cyclic process of developing artifacts for mathematics: successive phases of implementation inform further transformation of the previous versions of the artifacts. We started from the idea of approaching Thales Theorem with the use of Sketchpad for the 3rd grade of High-school through a relevant problematic situation. The introductory problem was:



On your screen you can see the bed of an irrigation channel. The length of the side AC is 10m. This side is graded throughout its length. When the channel was full of water, a diver measured the height of the water level vertically and found it 9.50m.

If today the water level is in position K of the side AC that corresponds to the value 7.50m, what is the vertical height of the channels' water level? Should the person responsible for the channel send the diver again?

Figure 1. The introductory problem and the model of the Sketchpad microworld.

We created the corresponding material (worksheets and microworlds) and we tested them in two successive cycles of implementation (2 classes of 20 students, 4 two-hour lessons in each one of them). We were not that interested on the originality of the idea, but on the process of its implementation. We focused on the students' actions during the implementation of the activities. We discussed the findings that emerged during the lessons and we accordingly modified the current versions of the designed artifacts. By the end of this process we aimed at being able to fill in the seven units of a particular scenario structure that was developed by Educational Technology Lab: 1. Title, 2. Scenario's identity (writer, mathematics topic), 3. Rationale of the task (innovations, added value, teaching and learning problems related to a specific mathematical concept), 4. Context of implementation (grade, duration, location, prerequisite knowledge, necessary material and tools, social orchestration of the classroom, goals), 5. Analysis of activities / presentation of the implementation's phases (sequence of activities, roles of the participants, anticipated teaching and learning processes), 6. Extension of the scenario, 7. Bibliography. Our research focuses on the evolution of the modifications we did in the artifacts while gaining insight into their actual use in the classroom as well as from our evolving collaborative work.

Research questions: Through our study we aimed to investigate the following questions: How the designed artifacts were modified through their implementation in the classroom? Were there any unexpected incidents during the actual implementation that affected these modifications?

Research method: The present qualitative research deals with engineering a particular form of learning (teaching of Thales Theorem with the use of Sketchpad), and systematically studying this form of learning within the context defined by the means of supporting it. It has an interventionist nature, as well as prospective and reflective faces implemented through iterative design, thus it constitutes a design experiment (Cobb et al, 2003).

Data collection: a) Observation notes regarding all of the groups, the class as a whole, and every activity included in the worksheets, b) diary of what happened in every part, what impressed us, what could be done differently, c) videotaping of 4 groups of students' movements on the



computer screen (CamStudio), d) worksheets completed by the students.

Data analysis: After each lesson and before the next one, a discussion between the members of the team followed. A comparison of the data acquired through the aforementioned means was conducted for every group of students. We then had to compare them with our goals and the anticipated ones. The conclusions of these discussions initially led to four categories of modifications: a) the worksheets b) the microworlds given to the students for investigation c) the teaching management. By the successive implementations, we further elaborated sub-categories of modifications within the above categories. In the results section of this paper we emphasize on these modifications rather than on the preceding interactions within the community members.

Results - Discussion

In the beginning all groups of students were given the first worksheet and a ready-made file with a Sketchpad microworld. The file consisted of 2 pages. On the first page, a ready model of the introductory problem was given. We decided that in this phase we were not interested in the process of modeling the problem by the students, as much as in their capability to explore it. On the 2nd page, a geometrical shape of Thales Theorem generalization through dynamic manipulation was given. Through a series of activities and with the use of Sketchpad tools (measurements, computing ratios, tabulation, dynamic manipulation of K – corresponding movement of the diver) the students were led to discovering the Thales Theorem and its implementations. The three kinds of modifications which emerged during the consecutive implementations in the laboratory will be described below.

1. Modifications to the worksheets

- 1a. *Integrating the model in the worksheet:* One of our main goals was the comprehension of the problem, thus, some of the straight line segments were given in the worksheet and the students were asked to fill in their given values or replace them with a variable, according to the data required. Simultaneously, a Sketchpad microworld which represented a model of the problem was given to the students. But it was noted that the students tended to use the measurements of the software, even in cases where we did not want them to. For instance, the students used these measurements for identifying the unknown segments of the problem, instead of representing them with a variable. The modification we did was the integration of the model in the worksheet, so that the students could focus only on the worksheet and thus having more possibilities to understand the problem at hand (i.e. to identify the variable and the constant quantities). This finding led us to recommend the use of the Sketchpad microworlds to take place later on. The recommendation was added in the 5th unit of the scenario structure.

- 1b. *Rewording the text:* In the cases where the students were not able to understand the text of the worksheet, we rephrased it through verbal modifications of the activities. Particularly, this happened only at the points where we judged that the text could have been formulated more clearly. Examples: i) Some questions were abstract, like “what happens when the water level changes?” The question had to be formulated more specifically, so that the students could focus on the variation of ratios, initially by moving K on AC. We additionally added a button to the Sketchpad file (in the form of animated graphics) for changing the water level when pressed. The simultaneous observation of the values of the ratio in every position of K offered a combination of representations which involved the dragging of K manually on AC and the animated graphics of the fluctuation of the water level with the update of the measurements. ii) After the students have experimented with the software and have discovered – formulated the Thales Theorem as well as the relation $AK/KC=EB/BZ$ through the movement of K, they were asked to “give an



explanation for this relation”. What we had in mind was to engage students in identifying that this relation results by simply switching the means of Thales ratio. But no group of students understood what exactly we asked for. We thus rephrased the question as “how is this relation connected to the Thales Theorem?”. iii) As mentioned earlier, in the 1st activity some straight line segments were given and the students were asked to fill in the requested data, in order to separate the known from the unknown ones and represent the latter with the help of variables. We determined though, that this was not understood by the students. So, we created a table with 2 columns (known/unknown segments) and asked the students to classify the given segments respectively and to either fill in their length or express them with the use of variables.

- *1c. Removing of “noisy” information from the activities:* In some cases, particular activities involved additional information. For instance, in order for the students to identify the equality of the ratios involved in the model (e.g. $AK/EB=KC/BZ$) 5 ratios, 2 products and 3 sums were given. The students had to measure and put them into a table. Then, through dynamic manipulation of K they were expected to observe the resulting measurements and distinguish the equal ratios.. But in practice, it appeared that this amount of information confused the students and disorientated them from the discovery of the requested relations. Part of this “noisy” information was withdrawn and this knowledge was taken into account when writing the 5th unit of our scenario.

- *1d. Integrating intermediary activities:* In cases where it was judged that there had to be a connection with previous knowledge (terms, concepts) and that connection had not been achieved with the existing activities or was not adequately taken into account in the planning, we added activities, either with further use of the software, or without it. Example: For the formulation of Thales Theorem, we had designed a set of activities. However, the students found it hard to characterize two segments as “proportional” or to use the word “proportionality” for the equality of two ratios. This difficulty was related to the fact that the students could not easily discern that the nominators of two equal ratios could belong to different lines. In order to support them overcome this difficulty, we added an activity which helped them relate the nominators with line AC and the denominators with line EZ. This experience was utilized in the 3rd unit of our scenario pertaining to the learning problems the students face with the particular mathematical concept.

- *1e. Performing visual modifications:* This type of modifications surprised us: In an activity where the students were asked to measure 6 segments, 5 of which were in a single line of the text and the 6th in the next one. We observed that the majority of students did not see and thus did not measure the 6th segment. We performed a visual modification in order for all the segments to be positioned in the same line of the text.

- *1f. Integrating new activities – Extending the scenario:* We left this type of modification for last, as the answers of 2 groups of students gave us the idea to extend the scenario to algebra and trigonometry. The students had already discovered Thales Theorem, its various forms and how could be used in the problem at hand. They had answered questions such as “compute the height of the water level when $KC=6.5m$ ” or “find the sidebar gradation when the height of the water level is 5m”. Our next goal concerned the generalized form of Thales Theorem’s use. Thus, we designed the following task: “The man responsible for maintaining the channel wants to know the height of the water level for every notch of the side AC. Find a relation that computes it”. We expected them to transform the relation $AC/EZ=KC/BZ$ as $BZ=KC*EZ/AC$. What gave us the idea of extending the activities to algebra and trigonometry was the answers of the two groups of students, who not only found the requested relation, but elaborated it further. Particularly, they



replaced EZ/AC with the result of the measurements, which was 0.95, and so they express BZ as $BZ=0.95 \cdot KC$. Actually, they identified and expressed the linear dependence between the two segments. Having this stimulus from the students, we asked them insert the measures of BZ and KC in a table as well as the corresponding ratio. After taking nearly 10 measures of these segments for different positions of K, the students were asked to represent the corresponding pairs (KC, BZ) graphically. This way Thales Theorem was connected with the graph of proportional amounts (1st grade of High-school). Finally, we added a 3rd page – microworld to the file (Figure 2).

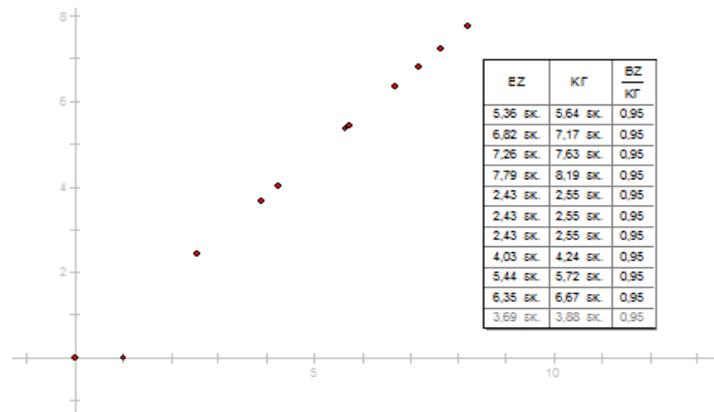


Figure 2. Graphical and tabular representation of the proportional relation.

In addition, we modified the model of the 1st page of the Sketchpad microworld, so that, by moving AC and EZ the students could detect that 0.95 is constant for the particular position of BZ, KC and if those are moved the value of their ratios changes. Finally, the slope of the straight line of the graph, led us to trigonometric numbers, which we utilized to prove the Thales Theorem in its general form for two random side lines.

2. Modifications to the given microworlds

- 2a. Inserting new buttons as another possibility of multiple representations, mainly by inserting movement in the microworld (example 1bi above).
- 2b. Inserting pages, in cases such as 1f that we described above.
- 2c. Reconstructing the geometrical representation of Thales Theorem: The 2nd page of the

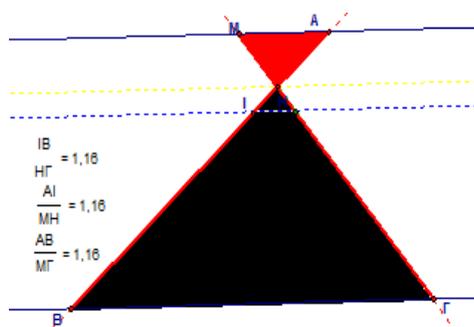


Figure 3. The final geometrical representation of Thales Theorem.

Sketchpad file contained three different versions of Thales Theorem (intersection of the side lines within the zone defined by the three parallels, outside the zone or on one of them). We reconstructed the geometrical representation so as to provide students the opportunity to explore all versions of Thales Theorem in a single unified figure. Example: In an activity for the generalization of the Thales Theorem, in the case where the side lines intersect within the zone of parallels, we detected that it would be better if the side lines could be moved so that the students can discover the Thales Theory in all cases. Consequently, the microworld of the

2nd page was modified so that by dragging A or M on the first of the 3 parallels, the students can see all the possible versions of the Thales Theorem by observing the measurements.



3. Modifications to the teaching management

Those modifications were incorporated in the scenario as concrete advice for the classroom management (the 5th unit of our scenario).

- *3a. Controlling the flow of the activity sequence in the classroom:* All groups of students were working with a high degree of autonomy. As the lesson progressed, we observed that the differences among the groups of the students in completing the activities of the worksheet were gradually growing. This of course means that the teacher has to provide more intensely support to the weaker groups and at the same time to organize frequently class discussions for sharing of ideas and approaches as well as institutionalization of the emergent knowledge.

- *3b. Integrating an intermediate lesson to the classroom:* We detected a need to elaborate further the knowledge gained in the laboratory in the traditional classroom (i.e. by solving exercises etc.).

Conclusions

Our general aim was to explore how constructionism as a design framework might contribute in addressing the potential of artifacts with technological tools in mathematics classrooms. We adopted a broadly constructionist framework in designing and modifying artifacts considering them as malleable objects transformed after reflection on the practices developed when implemented in real classroom settings. Three kinds of modifications emerged during the successive implementations of our designs in the classroom. These involved: modifications to the worksheets, to the given microworlds and to the teaching management. These modifications emerged as a result of incidents that took place during the implementation in the classroom. The most notable finding was the ease with which some students envisioned a geometrical relation in an algebraic way. This challenged the extension of our activities to involve linear functions and the notion of slope and thus making links between geometry, algebra and trigonometry. We also observed the ease with which the students used the measurements provided by the software which caused modifications leading to students to discern between measurement and computation. We detected difficulties that the students face in reading and comprehending the texts of the problems involved in the worksheets and developed ways to approach them (i.e. verbal and visual modifications to the worksheets). Management of the groups was also a case of special concern and we needed to revise our strategies several times. The functionalities of the microworlds that we provided to the students were realigned, as new cases we had not foreseen arose or the students gave us new ideas. The writing of the scenario was a result of this experience. The findings of our research show that this procedure enriched our knowledge (in Shulman's sense) and constituted a basis for developing a practically and theoretically documented scenario. It pinpointed features of the teaching and learning process whose inclusion in the scenario would be impossible if the practical implementation of the protoleia of our designs had not been preceded. Producing validated, well-documented material, in a pedagogical sense, has to be a main aim in training programs. This validation can occur through connecting training programs with teaching practice. Communities of inquiry can constitute the bridge for this connection and enrich academic knowledge with new research data which concern the implementation of digital technology. In summary, constructionist consideration of artifacts with digital tools as transformable objects in this study seemed to have created a context to forge connections between didactical design and classroom practice. This indicates also the productive nature of the challenge to situate constructionist approaches of design in a range of different contexts and computational environments in order to get an integrated framework to analyse the potential of artifacts with technological tools in mathematics classrooms.



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