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# Studying secondary mathematics teachers' attempts to integrate workplace into their teaching 

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This paper focuses on 3 cases of mathematics teachers' attempts to integrate the workplace into their teaching while participating in a professional development (PD) program. We draw on the work of 5 groups of mathematics and science teachers who collaborated for a school year to design and implement tasks related to workplace nonroutine situations. Teachers' activities are analysed under an Activity Theory (AT) perspective. The results indicate different forms of interaction between the activity system of workplace and the one of mathematics teaching.
Keywords: Workplace, teachers' goals and actions, Activity Theory.

## INTRODUCTION

There is a great deal of research supporting claims that workplace settings may offer pedagogical opportunities for teachers to introduce authentic situations into their school classrooms activities (e.g., Nicol, 2002; Wake, 2014). These pedagogical opportunities refer to making mathematics meaningful to students by preparing them to explore open and unstructured problems that connect mathematical knowledge as taught at school and as used out-of-school. Our understandings of the nature of mathematical activity in workplace are informed by a number of research studies (e.g., Hoyles \& Noss, 2001). These studies indicate that mathematical notions underlying professionals' practices in workplace settings are mostly hidden and embedded in the particulars of the situations. This makes any attempt to connect workplace and mathematics teaching highly demanding. A challenge for a teacher, in this case, is to connect situations, symbol systems, technological and workplace tools, contextual constraints/rules and personal and professional knowledge to help students make sense of work processes.
How teachers can use the workplace as a context for designing and using lesson activities in the classroom remains an open question that has only received partial answers from small scale studies mostly on prospective teachers. For instance, Nicol (2002) found that a teacher education program including visits to workplace sites helped prospective teachers to keep the mathematics contextualized when designing activities for their students. Frykholm and Glasson (2005) suggested that teacher education courses involving collaboration between science and mathematics prospective teachers provide a fertile ground for them to develop interdisciplinary units connecting both topics. In this direction, Potari et al. (2016) argue that
workplace seems to provide a context for collaborative work of mathematics and science practicing teachers that helps them to connect meaningfully the two subjects.
The study reported in this paper took place in the context of a European project, mascil (see: www.mascil-project.eu), aiming to integrate workplace in the teaching and learning of mathematics and science through implementation of inquiry-based tasks in classrooms. Thirteen partner countries participated in the project and developed a body of exemplary classroom and teacher education materials as a basis for the organisation of PD activities and classroom implementations. Our aim is to examine how practicing mathematics teachers integrate workplace tools and practices when designing and implementing problem-solving classroom activities and what factors facilitate or constrain this integration. We adopt an AT perspective to focus on how workplace situations - considered as activity systems - interact with the mathematics teaching activity in the context of mascil.

## THEORETICAL FRAMEWORK

Thinking beyond dichotomies such as school versus work, Bakker (2014) argues on the importance of developing research-informed understanding of what happens at the boundaries of schools and workplaces settings. However, the task of building connections between the two is rather demanding from an epistemological and didactical point of view. From an epistemological point of view, a number of studies emphasize the extent and depth of mathematical concepts and sophisticated mathematical skills encountered in the workplace. However, the conventional epistemological view of mathematics fails to capture this richness (Hoyles \& Noss, 2001; Triantafillou \& Potari, 2010). At the level of teaching, viewing the workplace context as non-mathematical might eliminate teachers' opportunities to explore its pedagogical potential. Wake (2015) argues that modelling the structure of a contextual situation could provide teachers an opportunity to create a nexus of mathematics and reality.
We adopt Engeström's (2001) approach to investigate mathematics teachers' activity when they are challenged to integrate workplace into their teaching. We consider two activity systems: the system of workplace and the system of mathematics teaching in which the teachers have been engaged to study the interaction between the two. The "activity system" is a basic concept of AT that is collective, tool-mediated and needs a motive and an object. Individual and group actions are studied and interpreted against the background of entire activity systems. Activity systems are transformed over lengthy periods of time when the object and the motive of the activity are reconceptualized to embrace a radically wider horizon of possibilities than in the previous mode of the activity. Central to the process of transformation are contradictions within and between activity systems emerging when a new element comes from the outside. Figure 1 shows a representation of two interacting activity systems under Engeström's (2001) perspective. The two triangles represent the two activity systems considered in the present study.


Each system involves the basic dimensions of AT with elements the subject and the object of the activity (object 1) that is constructed through the mediation of tools and it is framed by the community in which the subject participates, its rules and the division of labor. In the interaction of the two systems object 1 moves from constructed by the activity system (object 2 ) and to a potentially shared or jointly constructed object (object 3 ).
In this study, we analyze teachers' goals and actions when acting as subjects into the activity systems of workplace and mathematics teaching. Our aim is to explore the role of tools (workplace artefacts, teaching resources) and the specificities of the workplace and classroom contexts (rules and division of labour) in the formulation of a new object incorporating elements of both activity systems.

## METHODOLOGY

## The context of the study

In mascil implementation in Greece, thirteen groups of practicing secondary teachers (about 10 in each group) from mathematics, science and technology have been established to work in the spirit of lesson study (Hart, Alston \& Murata, 2011). In each group, teachers collaborated with the support of a teacher educator for a school year to design and implement inquiry-based tasks related to workplace non-routine situations and reflect on their teaching. Before and after each implementation of the designed lessons PD meetings took place. In the initial PD meetings, the teacher educator informed teachers about the rationale of the project and introduced them to the main principles of inquiry-based tasks and to the nature of workplace mathematics. In the subsequent meetings, teachers were asked to collaborate in transforming the exemplary mascil tasks or designing new ones in the same spirit, share their experiences from the implementations and discuss emerging issues. In the highly centralized Greek educational system, mathematics teaching in secondary school is rather traditional with a strong emphasis on mathematical content without connections to real life contexts. Moreover, PD activities are often limited to lectures and short term courses of a top down philosophy. Thus mascil was a rather innovative project both in terms of its teaching objectives and its PD approach.

## Participants

In this paper, we focus on five groups of practicing teachers (22 mathematics teachers, 14 science and 9 technology). Teachers in these groups worked in upper or lower secondary schools and they had long teaching experience (more than ten
years). We analyze the work of the mathematics teachers in these groups who collaborated together and/or with science and technology teachers. Participation in mascil was on a voluntary basis and most of the teachers had qualifications beyond those required by their profession (e.g., master or PhD degrees in mathematics, science or technology education).

## Data collection and analysis

The data collected from the five teacher groups included: audio and/or video recordings of the PD meetings ( 7 two- hour meetings per group - 35 in total) and classroom implementations ( 71 in total, 2 teaching hours each); teachers' portfolios (tasks, worksheets, written accounts/journals, power -point presentations, digital materials, students' work, students' evaluation reports) and selected interviews with teachers and teacher educators.
In this paper, under a grounded theory approach (Charmaz, 2006) we analyse the discussions in the PD meetings and teachers' portfolios. Initially, we identified parts of the data concerning the activity of mathematics teaching and the activity of workplace. Then we analysed teachers' goals and actions looking for possible intersections between the objects of the two activity systems identifying emerging contradictions and convergences in relation to: (a) the origins of their ideas for tasks (e.g., personal experiences); (b) the tasks and resources by which they targeted students' familiarisation with workplace (role playing, workplace tools, representations used); (c) the links they made between workplace and mathematics; (d) the supportive factors and/or constraints in the process of integration; and (e) the teachers' reflections on the contribution of workplace in improving their teaching. The rationale of the goals and actions was analyzed by taking into account the bottom elements of the extended mediational triangles of the activity systems (community, rules, division of labor).

## RESULTS

In this section, we present the case studies of three teachers from different groups indicating three emerging ways of interaction among the elements of the activity systems. In case 1 , the workplace context is mostly used for motivating students to see the applications of mathematics while the teaching goals are not linked to the workplace activity. In case 2 , the workplace context is smoothly integrated into the classroom teaching through a modelling process linking the workplace activity with problem solving in the classroom. In case 3 , a simulation of the workplace activity in the classroom facilitated a strong integration of workplace into mathematics teaching.

## Case 1

The teacher studied in this case, James, is a mathematics teacher with more than 20 years of teaching experience who participated in one PD group consisted of four mathematics, one technology and four science teachers.

The initial idea of his design was based on a contextual textbook task: "Two villages are situated on the opposite sides of a river and their distances from the sides are unequal. In which place do we have to construct a bridge perpendicular to the sides of the river so that the two villages to have the same distance from the bridge". His proposal was negotiated in the group and he was challenged to make more explicit the workplace connection. The mathematics teachers invited a landscape engineer to inform them about the design of a bridge and the main issues involved in it. The engineer pointed out that at his workplace context the main goal was to reduce the cost of the bridge construction. The cost was related to the width of the river and that the distance from the villages did not matter. The science teachers started to propose non mathematical parameters from the realistic situation to take into account in the task design such as "rivers with varying width" or "rocky landscape". After this exchange of ideas, James did not feel happy with this workplace complexity: "it would be better not to have all these factors interfering".
James reformulated the problem of the design of the bridge by referring to a specific very old bridge that it had been awarded a prize for its original construction. The students were asked to find the parabolic curve given the length and the height of the bridge that a technician could use to build it. This task was an extension of a similar textbook problem. In the group discussion conflicts emerged as the other teachers and the teacher educator could not see any connection with the workplace. James presented to the group a technical method that he had found in the internet about the construction of this bridge.
In the classroom implementation ( $11^{\text {th }}$ grade students, 17 -year -olds), James took the following teaching actions: (a) familiarized the students by asking them to read information about the history of the bridge; (b) engaged them is solving a textbook task (drawing the graph of $y=-x^{2}+6 x$ and find its maximum value); (c) asked them to find the formula of a parabola when they knew that it passed through three points; and (d) explained on the board the technical process of joining together different parts of a bridge. He closed the lesson by asking the students "What would you recommend to the constructor of the bridge?"
James based his task design on a familiar to him tool, the school textbook. In the collective process of transforming this task the inputs from the science teachers in the PD group brought realistic factors that he could possibly include into his design. However, for him it was not easy to take these factors into the account in his implementation. Although, he tried to be familiarized with the specific workplace context and tools (talking with the professional, finding relevant information about different techniques of bridge construction) the gap between his teaching goals and the workplace goals still remained. This can also be explained by the fact that mathematics teaching practice in upper secondary education in Greece is characterized by norms and rules targeting students' conceptualization of abstract mathematical ideas while connections with contextual situations are rather limited.

Nevertheless, James made an attempt to introduce a contextual task into his teaching but did not succeed in overcoming norms and rules established in his professional community.

## Case 2

The mathematics teacher, Elena, had 15 years of teaching experience. In mascil she participated in a group of five mathematics, three science and two technology teachers. She chose to use a task (the Solar Cells) that was included in the exemplary mascil materials (www. mascil-project.eu). The task concerned the installation of solar panels on a house rooftop. In this task, the students had to decide whether a specific installation of solar panels on a house rooftop was a profitable choice for a family in relation to the cost of electrical supply provided by the National Electricity Company. In this process, students had to explore how to place the panels on the roof in order to maximize their number by studying their projections. In terms of mathematics, the problem required students to visualize relations between the three-dimensional context of the task and its two-dimensional representation.
Elena collaborated with the science teachers during and between the PD meetings in order to be familiarized with the scientific context of the task. Also she discussed specificities of panel installation with a professional working in a solar panel company. In her design, she used resources provided in the initial version of the task (e.g., actual panel dimensions, panel inclinations, video from the workplace). Furthermore, she adapted the problem to be closer to reality on the basis of the information that the professional provided to her (e.g., the distance between horizontal rows of panels).
During classroom implementation ( $8^{\text {th }}$ grade students, 14- year-olds), Elena supported students' familiarization with the scientific aspects of the problem by asking them to interpret authentic representations. For example, she provided the representation (Fig. 2) of sun's positions during the spring and the winter equinox and asked students "what case we could consider as important in order to decide about the shadow effect on the panels' installation?" Furthermore, she challenged them to consider the advantages of using solar energy as power supply for houses: "why making your house energy sustainable is a profitable investment?"
The main part of students' activity concerned the modelling of the problem through the development of different strategies such as: defining the rooftop area dimensions to be covered; translating the problem in the three-dimensional space by utilizing the projections of the panels on the rooftop through the use of trigonometric ratios; and examining alternative ways to place the panels and comparing the expenses in each case.


In her reflection, Elena realized that the modelling process revealed unexpected students' weaknesses and strengths that she had not noticed in her day-to-day mathematics teaching.
Elena's willingness to integrate the workplace of solar cells in her teaching was followed by a number of actions such as her own familiarization with the workplace context (i.e. discussion with science teachers in the group and one professional) and students' familiarization with this context by emphasizing situational aspects of it (e.g., technicians' installation practices, how panels' energy capacity is related to sun's position). The emerging rich interaction between the two activity systems was unfolded as a multifaceted modelling process involving the use of workplace tools, scientific representations, mathematical concepts, strategies and inquiry processes.
Case 3
This case refers to a mathematics teacher, Katerina, who had about 10 years of teaching experience. She participated in a mascil group with thirteen members (eight mathematics, one technology and four science teachers). Katerina developed a task entitled Seismologists for One Day where the students had the role of a seismologist responsible to study main features of a specific earthquake (e.g., the epicentre).
The initial idea of the task was provided by a group member whose specialization was geology. The teacher educator had suggested collaboration between mathematics and science teachers as a way to help them integrate workplace context into their classroom teaching. The geology teacher designed and implemented a similar task in his classroom and shared his materials (e.g., description of the main features of earthquakes and how they are studied by specialists) with the PD group. Katerina was teaching mathematics and geography in the $7^{\text {th }}$ grade ( 13 -year-old students) in her school, so she found as a challenge to develop a task for integrating the context of seismologists into her teaching by combining mathematics and geography. Her familiarization with the context of earthquakes in the PD meetings allowed her to use it as a context for designing a task for her students.
In classroom implementation, Katerina presented and discussed scientific aspects of the earthquakes based on her knowledge from physics and geography and provided students with authentic data from the National Institute of Geodynamics. The data included: (a) the velocity of $\mathrm{p}\left(\mathrm{V}_{\mathrm{P}}\right)$ and $\mathrm{s}\left(\mathrm{V}_{\mathrm{S}}\right)$ waves and the exact time these waves were recorded in specific seismic stations; (b) the mathematical formula $\mathrm{D}=\mathrm{t} \cdot\left(\mathrm{V}_{\mathrm{P}}\right.$ $\cdot \mathrm{Vs}) /(\mathrm{VP}-\mathrm{V} \mathrm{s})(1)$ where D is the distance (in Km$)$ of the epicentre from the seismic station and t the difference of the time arrivals of the waves; (c) a geographical map indicating all the seismic stations in the country with the corresponding codes (e.g.,

LKD2 for the seismic station in Lefkada island); and (d) the specific measures recorded in the seismographs of six stations in western Greece (see Fig. 3).


In terms of mathematics, the students had to identify that the epicentre of the earthquake was the common point of three intersecting circles whose centers were situated on three seismic stations (Fig. 4). In particular, they
had to: substitute given quantities into the formula (1) to calculate the distance of the epicentre from the different stations; model the situation through the use of map scales; conceptualize the calculated distances as radii of different circles; and design them with the use of ruler and compass.
Katerina's attempt to integrate the workplace of seismology into her teaching was followed by a number of actions such as: her own familiarization with the workplace context through discussions with the geology teacher in the PD group and her involvement in teaching mathematics and geography in the same classes; her decision to connect the topic of earthquakes included in geography curriculum with aspects of the mathematics curriculum (e.g., scales, properties of geometrical figures); the use of authentic workplace worksheets and tools; and the assignment of the role of seismologist to the students simulating the actual workplace practice.
In the case of Katerina, we see that the two activity systems are strongly connected and a new object started to be formulated in the intersection of the two systems. In this case, sharing of artefacts, goals and actions between mathematics teaching and workplace emerged through the simulation of the workplace activity in the classroom.

## DISCUSSION

Our study builds upon existing research indicating that integrating workplace situations in mathematics teaching is pedagogically sound in two ways: mathematics can be helpful to broaden students' understanding of a situation and conversely, the out-ofschool situations provide students the opportunity to deepen their mathematical knowledge. Through the above case studies we explore how the three teachers attempted to integrate workplace in their teaching and what factors facilitated or constrained this integration.
Our results indicate different forms of interaction between the activity system of workplace and the one of mathematics teaching. We address here these forms of interaction by focusing on two dimensions: the process by which the teachers
attempted to integrate workplace into their teaching and the factors that supported or hindered this integration.
As regards the process of integration, teachers' goals and actions included their familiarization with the workplace context, students' engagement in modelling activities, students' familiarization professional contexts and their attribution of a professional role. Modelling was a process that triggered all teachers' interest and, as Wake (2015) argues, it operated as a means of building connections between mathematics teaching and workplace. This process was adopted by Elena and Katerina who engaged students in mathematizing workplace situations such as panels' installation and identifying geographical maps' scaling. James, on the other hand, considered the modelling of the bridge construction as an application of mathematics by engaging his students in working in a ready- made model. Modelling in Elena's case was primarily based on a problem solving activity. In Katerina's case it was embedded in a process of simulating authentic workplace practice in the classroom while in James' case modelling remained bounded in the context of school mathematics. Teachers' attempts to familiarize themselves with workplace practices were carried out through either their personal communication with professionals or through their cooperation with science teachers in their PD groups. Familiarization of students with workplace was carried out in the following ways: engaging them in working with authentic contextual or scientific representations such as photos of bridges or diagrams of sun's route or geographical maps; and including in the task aspects of the broader scientific context (e.g. the solar energy). Finally, only one teacher (Elena) assigned her students a professional role (i.e. seismologist) and a task (i.e. to find the epicentre of an earthquake) strongly related to the workplace practice.
The analysis brings to the fore the following categories of supportive factors and constraints that facilitated and/or hindered the interaction between workplace and mathematics teaching: the collaboration between teachers from different disciplines in the PD groups in co -designing a task; the use of exemplary resources and materials provided by mascil; teachers' experiences in teaching subjects related to the workplace broader scientific context; and the rules underlying mathematics teaching. The collaboration between mathematics and science teachers supported the integration in the cases of Elena and Katerina since they co- designed the task with science teachers from their PD groups. This supports recent research findings that acknowledge workplace as a fertile ground for science and mathematics teachers' collaboration (Potari et al., 2016). In case 1, however, James and physics teachers in PD group did not find a ground for co-designing a task. The use of exemplary mascil materials favoured Elena's attempt to integrate workplace in her teaching. Katerina's teaching experiences of geography supported the smooth integration of the workplace practice of a seismologist in her classroom teaching of mathematics. Finally, the established rules of mathematics teaching in upper secondary level, as in the case of James, provided barriers to the integration of workplace in his teaching.

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## REFERENCES

Bakker, A. (2014). Characterising and developing vocational mathematical knowledge. Educational Studies in Mathematics, 86, 151-156.
Charmaz, K. (2006). Constructing grounded theory. A practical guide through qualitative analysis. London: Sage.
Engeström, Y. (2001). Expansive learning at work: Toward an activity theoretical reconceptualization. Journal of Education and Work, 14, 133-156.
Frykholm, J., \& Glasson, G. (2005) . Connecting science and mathematics instruction: Pedagogical content knowledge for teachers. School Science and Mathematics, 105(3), 127-141.
Hart, L. C., Alston, A. S., \& Murata, A. (2011). Lesson study research and practice in mathematics education. Dordrecht: Springer.
Hoyles, C., \& Noss, R. (2001). Proportional reasoning in nursing practice. Journal for Research in Mathematics Education, 32, 4-27.
Nicol, C. (2002). Where's the math? Prospective teachers visit the workplace. Educational Studies in Mathematics, 50, 289-309.
Potari, D., Psycharis, G., Spiliotopoulou, V., Triantafillou, C., Zachariades, T., \& Zoupa, A. (2016). Mathematics and science teachers' collaboration: searching for common grounds. Proceedings of PME 40 (Vol. 4, pp. 91-98). Szeged, Hungary: PME.
Triantafillou, C. \& Potari, D. (2010). Mathematical practices in a technological workplace: the role of tools. Educational Studies in Mathematics, 74(3), 275-294.
Wake, G. (2014). Making sense of and with mathematics: the interface between academic mathematics and mathematics in practice. Educational Studies in Mathematics, 86, 271-290.
Wake, G. (2015). Preparing for workplace numeracy: A modelling perspective. $Z D M$, 47(4), 675-689.

