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Studying mathematics teachers' documentational and identity trajectories over time

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We study the evolution of long-term interactions between one mathematics teacher and resources, especially digital ones, with a dual focus on his documentational and identity trajectories. We combine a theoretical framework on teachers' work with resources with a framework based on social practice theory. The analysis allowed us to describe what digital resources the teacher interacted with and when, as well as to explain how and why he interacted with these resources the way he did over time.

Keywords: Digital resources, DAD, documentational trajectory, Patterns-of-Participation, identity trajectory.

INTRODUCTION

The aim of this paper is to study the evolution of long-term interactions between mathematics teachers and resources – especially digital resources (DRs) – with a dual focus on professional and identity development. A multiplicity of studies focuses on mathematics teachers' professional development as an outcome of their participation in specific, often short-term initiatives, while only fewer studies have examined teacher development over a long period of time as a result of their participation in daily activities (e.g., Little, 2002). Recently, calls have been made for extended studies, for instance by Trouche (2019), who argues that teachers' deep instructional changes often need long time. Taking into account the transformative potential of DRs in terms of stimulating learning experiences to students, motivating teacher collaboration and design capacity (Pepin, Gueudet, & Trouche, 2017), we consider the study of teachers' long-term interaction with DRs as a promising research area.

Our literature review revealed only few studies focusing on teachers' long-term development with DRs, predominantly within the framework of *Documentational Approach to Didactics (DAD)* (Trouche, Gueudet & Pepin, 2018) that concentrates on teachers' work with resources. For instance, Rocha (in Loisy, 2019) analyses when, where, why, and how teachers interact with which resources at critical points during their professional life. This analysis provides mainly overall descriptions of teachers' interactions at the macro level by focusing on special events outside classrooms or schools and the corresponding resources. Though contributing valuable insights, this analysis focuses on when and where teachers interact with resources and what promote their actions, and to a lesser extent on why and how teachers interact in the described ways by taking broader social perspectives into account.

In this paper, we adopt a social practice framework, *Patterns-of-Participation (PoP)* (J. Skott, 2019), that favours understandings of how broader social constellations and

modes of reasoning inform teacher action and meaning making. PoP allows a social perspective on a teacher's professional identity that is defined as his/her experiences of being, becoming and belonging as a teacher. We aim to combine (Prediger, Bikner-ahsbahs, & Arzarello, 2008) DAD with PoP to get a multi-faceted insight into a Greek mathematics teacher's (Victor's) long-term development and especially into the hows and whys of Victor's interactions with DRs over time. For this, we analyse his documentational trajectory (i.e. his interactions with resources over time) in conjunction with his identity trajectory (i.e. his identity formation over time).

THEORETICAL APPROACHES

DAD (Trouche et al., 2018) acknowledges the crucial role of resources for teachers' work and professional growth retaining a broad meaning for *resources* as material and non-material elements (e.g. textbooks, discussions with colleagues) and considers that teachers' work with and on resources constitutes a dialectic process where design and enactment are intertwined. Teachers use different kinds of resources that shape not only the mathematical content and the ways it is (re)presented, but also students' mathematical learning. Teachers adapt their appropriation and use of resources to their needs and customs. This dynamic process of (re)-design and interpretation continues during enactment of the resources and orients teachers' *documentation work* leading to their creation of *documents*. The set formed by all the resources used by a teacher defines his/her *resource system* (RS). Rocha (in Loisy, 2019) introduces the concepts of *documentational experience* (i.e. a teacher's appropriation of professional events acquired during his/her interactions with resources, that were remarkable to his/her documentation work from his/her perspective) and *documentational trajectory* (i.e. the set of professional events of both individuals and collectives that ground teachers' documentational experiences) to study the long-term evolution of interactions between teachers and resources. She also proposed two new methodological tools: a *Reflective Mapping of a teacher's Documentational Trajectory* (RMDT) made by the teacher and an *Inferred Mapping of a teacher's Documentational Trajectory* (IMDT) made by the researcher. These tools map a teacher's experience throughout his/her professional life by indicating on a timeline crucial events/collaborations to his/her documentation work (Fig. 1).

Trouche (2019) indicates a need to consider the broader contexts in which teachers operate as they make decisions about what resources to use and how to use them. To take into account how social constellations and modes of reasoning are reflexively related to and co-determiners of teachers' acts and meaning-making, we use PoP (J. Skott, 2019). In line with other studies of teacher identity, PoP draws on the notions of practice and figured worlds in social practice theory, where practice "connotes doing [but] doing in a social and historical context that gives structure and meaning to what we do" (Wenger, 1998, p. 47). Figured worlds are "socially and culturally constructed realm[s] of interpretation, in which particular characters and actors are recognised, significance is assigned to certain acts, and particular outcomes are valued over others" (Holland, Lachicotte Jr., Skinner, & Cain, 1998, p.52).

PoP differs from other identity studies using social practice theory by focusing on shifts in a teacher's ways of participating in social interactions over time (e.g., classroom interactions) and not focusing on how a teacher move towards a more comprehensive participation in a specific, pre-established community or figured world. In PoP, a teacher's participation in social interactions is understood as influenced by his/her interpretation of the immediate situation and his/her simultaneous meaning-making where he/she continuously interprets "others' actions symbolically, including their actual or possible reactions to one's own behaviour" (J. Skott, 2019, p. 472). In terms of symbolic interactionism, the teacher takes the attitude to him/herself of others. The others can be a colleague or a parent, but it can also be a social group or community in which case the teacher takes the attitude to him/herself of generalized others. In PoP, practices and figured worlds are interpreted as possible generalized others. Using PoP, we define Victor's professional identity as his experiences of being, becoming and belonging as a teacher at his school and beyond (e.g. research projects), and Victor's identity trajectory as his identity formation over time. Our research questions are:

- (1) What are the major professional events and corresponding resources in Victor's long-term documentational trajectory? How do these events/resources contribute to his professional development at the macro level?
- (2) How do Victor's experiences of being, becoming and belonging influence his interactions with DRs? How do these experiences contribute to forming his identity trajectory at the macro level?

METHODOLOGICAL APPROACH

We study Victor's professional life focusing on his interactions with DRs over 18 years from when he starts working as a new mathematics teacher in his own auxiliary school to his current position as a teacher in a multicultural public school in northern Greece. Victor's case is illustrative as he explores the different opportunities for interacting with DRs through his participation in multiple research projects and development initiatives. We aim to understand how Victor's participation in such major events at the macro level influence his interactions with resources, especially digital ones, but also, why Victor engages in these events and what motivates him in the long run. In line with the principles of reflective investigation (Trouche et al., 2018, p. 8) we (a) analyze Victor's long-term documentation work ("long-term follow-up" principle) at different time moments in- and out-of-class ("in- and out-of-class follow-up"), (b) address his reflections on his choices and experiences ("reflective follow-up"), (c) take into account a "broad collection of the material resources" produced and used throughout the follow-up, and (d) "permanent confronting Victor's views on his documentation work and the materiality of this work". For this paper our data is: 8 individual interviews, e-mails on his documentation work in different time periods, resources for his teaching of a specific unit in grade 8 (e.g., worksheets, lesson plans), and an activity template where he described aspects of his design and experiences from the implementations. Based on the interviews and teaching resources, we made first

drafts of IMDT including major events and resources. We then asked Victor to provide written comments (wr) on the IMDT to (a) examine this version and let him modify it by adding missing events, (b) highlight the most critical events for him and explain why, and (c) provide examples of resources related to these events. Based also on a follow-up discussion (fd), we constructed a combined IMDT and RMDT (we name it I/RMDT).

For the analysis of Victor's documentational trajectory, we cross-referenced I/RMDT with other data and identified key points operating as 'entrances' into his trajectory and examined if and how they influenced his professional activity. For the analysis of his identity trajectory we used mainly his written reflections on I/RMDT and the follow-up discussion. This analysis was carried out in two steps. First, under a broadly grounded approach we analyzed line-by-line Victor's utterances and practices in relation to his orientations towards important actors, significant acts and values, and gathered these into distinct practices, characters and figured worlds. Second, we analyzed how the roles of these practices, characters and worlds shifted in dominance to Victor's experiences of being, becoming and belonging when interacting with DRs, drawing corresponding landscapes of their constellations. By identifying shifts among these landscapes, we constructed Victor's identity trajectory.

VICTOR'S DOCUMENTATIONAL TRAJECTORY

Our analysis of Victor's I/RMDT shows four categories of events and corresponding representative resources (Fig. 1) that constitute his professional life: (a) taking part in professional exams (E2, E4); (b) teaching secondary students and pre-service teachers (E1, E3, E6, E9); (c) engaging in research and developmental work in mathematics education especially with the use of DRs (E5, E8, E10, E11, E12, E13, E14, E15, E16, E17); (d) participating in scientific and professional associations (E7). By combining the I/RMDT with our data, we identify three overall/key events and their corresponding time periods: participation in the national exams (E2) (2000-2001), collaboration with teachers and researchers (2004-2010), and participation in teams designing curriculum/curriculum resources such as in the projects *New School* and *Digital School* (2010-2015). These events provide the basis for three major documentational experiences underlying his trajectory: collaborating with colleagues and researchers; collaborating within specific frames/contexts and norms regarding resource design especially with DRs; approaching mathematics education research.

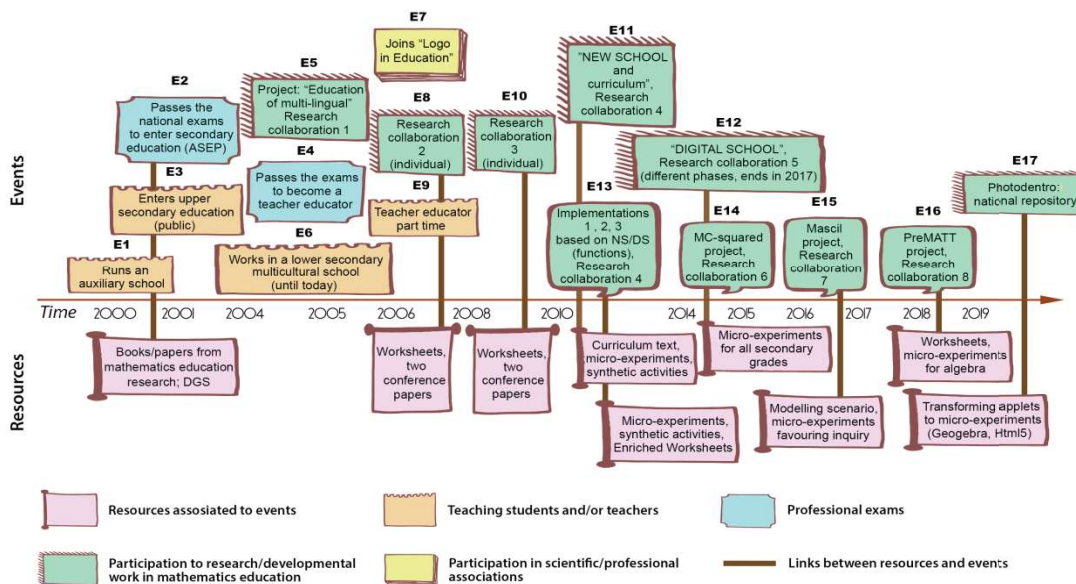


Figure 1: Victor's I/RMDDT

The reform aspects of New School and Digital School concerned multiplicity of curriculum resources and conceptualization of teachers as active partners in designing teaching with DRs. Furthermore, the new curriculum was structured in learning trajectories and used two novel DRs: 1) *micro-experiments*: interactive applets developed through different categories of proliferating digital media (CAS, programmable software), and 2) *synthetic activities*: generic scenarios promoting project-work based on modelling, inquiry and integrated representations. A set of principles guided the work of those two projects like combining formal math representations (e.g., graphs), text and/or simulations (e.g., math behaviour), promoting students' dynamic manipulation of math objects in order to explore and discuss their relationships/properties, and promoting inquiry. Victor's collaboration with researchers in the projects affected deeply his interactions with DRs and his RS. We highlight three points related to a new constitution of this system. (1) The new curriculum design and the enrichment of textbooks with DRs mark Victor's entrance to design DRs and to integrate them systematically in his RS. As a result, this set of new resources gives birth to a new RS that includes novel resources such as *Enriched Worksheets* consisting of theory (e.g., definitions), different tasks (e.g., problems, exercises), multiple representations and links to DRs (e.g., micro-experiments). (2) Although worksheet was an innovative resource in Victor's initial system, micro-experiments occupy dominant positions in all subsequent forms of his RS. (3) The two projects mark Victor's passage from looking for new resources (e.g., by reading research literature for ASEP) to aligning his RS with resources designed in the projects and later on to being able to design such resources himself. In terms of structure, we consider micro-experiments as pivotal resources (Trouche et al., 2018) used by Victor in various activities (e.g., lesson planning, tasks). Also, they seem to play a central role in helping Victor to restructure and reorganize his RS. Finally, Victor's participation in New School and Digital School seems to influence his professional development as

a consequence of the evolution of his RS. Resources designed in the projects are different from the resources (tasks, worksheets) normally used in classrooms. The new resources aim to introduce teachers to new design approaches as regards functionalities and use of available DRs as well as to open discussions and to share ideas for future use. Further, developing such resources is a demanding process requiring associated documentation work that in Victor's case led to deep reorganization/restructuring of his RS at the level of both technical (e.g., design capacity in Geogebra) and pedagogical demands (e.g., synthetic activity).

VICTOR'S IDENTITY TRAJECTORY

Preparing for the national exams to enter secondary education (1997-1998).

As a new teacher Victor works in his own auxiliary school (1997-2001) and belongs to a world of *Para-education* (e.g. a parallel private educational system behind the formal school system in Greece that supports students in passing the higher education exams). This world is characterised by “stereotyped perceptions of math” (wr, p.6), and it does not encourage use of DRs as this is not required for obtaining its purpose: helping students to pass exams. However, when preparing for his own exam to teach at

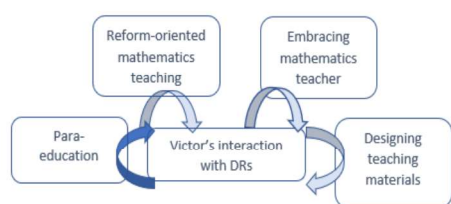


Fig. 2: Victor's first landscape

secondary level, Victor realises that “something was wrong [in his teaching] as it did not help all students” (wr, p.1). His realisation marks a critical shift in his identity formation. He begins to orientate himself towards a world of *Reform mathematics teaching* and a new character of an *Embracing mathematics teacher* emerges that

combines the inquiry part of the Reform-world with his political ideas of education for all. Thus, to teach student-centred, Victor starts designing worksheets with mathematical graphics which is new in Para-education. These worksheets also represent Victor's first steps in using DRs in his teaching, which is rather rare in Greece by this time. Figure 2 shows his first landscape of practices, characters and figured worlds (a dark blue arrow shows strong prominence).

Collaborating with teachers and educational researchers (2004-2010)

The equity issue gains importance for Victor, and hence his devotion to become an embracing character, when he begins to teach at a multicultural school with heavy language problems. Two types of experience prove important in this period: (1) Victor gains experience with designing resources that include DRs that students can use (e.g. dynamic geometry systems); (2) he starts collaborating with colleagues and researchers: “Our [collaboration] was based on my classroom teaching. She [researcher] supported me theoretically providing me a framework for functions” (wr, p.2). He experiences being recognised by colleagues and researchers as competent collaborator and teacher due to his affiliation with the Reform-world. Fig. 3 shows Victor's second landscape of practices, characters and worlds.

Participating in New School and Digital School (2010-2015)

This period is the most important for Victor's identity formation in relation to DRs, "It was a very big step ... one of the most important things I have done so far" (fd, p.5). While Victor transforms his ways of collaborating and designing, his landscape of practices, characters and worlds is similar with the previous period. We therefore focus

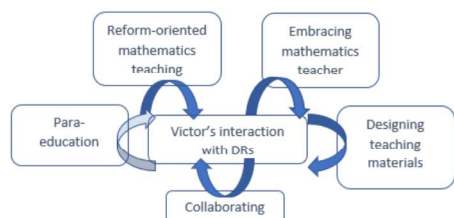


Fig. 3: Victor's second landscape

on the two practices. Regarding collaborating, Victor experiences being part of a complex and diverse collaboration both in terms of more participants, multiple actors (i.e. researchers, teachers, ministry advisors), larger scope (e.g. grade 1-9) and comprehensive purposes. By engaging in new ways of collaborating especially with researchers, Victor gains experience that helps him become an even more competent teacher "I knew a lot, but it was first when I consider the curriculum [in terms of learning trajectories] that I understood it deeper" (fd, p.5). Regarding the second practice, designing, Victor gains experiences with creating novel DRs in new ways, especially micro-experiment, that is "an autonomous application" with text, questions and digital constructs integrated and "starting with a problem. You need to design the problem to create space for the next step, the mathematics" (fd, p.8). By being required to engage deeply with the affordances of DRs relative to the targeted reform-intensions Victor experiences how to use DRs to facilitate these, "I understood deeper what using technology means ... the role of representations, the idea of exploration, the dynamism of dragging a slider and observing what happens ... Before, all these ideas circulated around and teachers always asked "Who can do all this?" (fd, p.6). At the same time, Victor tries out the micro-experiments in his classroom. By engaging in these new ways of collaborating and designing DRs and experimenting in classrooms, Victor gains experiences with implementing the Reform-world in classrooms. He thus becomes more like the embracing character.

In summary, Victor's identity formation undergoes a big transformation during these 18 years. From belonging to the traditional Para-education world and not using DRs, to becoming an embracing teacher who uses DRs as a tool to implement the Reform-world in his classrooms. The motivation for Victor's identity is his insistence on becoming the embracing character. By engaging in the two practices, collaborating and designing of DRs in New School and Digital School, Victor gains experiences with and recognition for his ways of implementing the Reform-world in his classrooms. These experiences contribute further to explain how and why Victor becomes like the embracing character.

CONCLUSION

We used two theoretical constructs, documentational and identity trajectories, to study a Greek teacher, Victor's, interactions with DRs over time. The DAD analysis shed

light on major events and corresponding resources that promoted Victor's interaction with DRs. It stressed the evolution of his RS and how it was influenced by his collaboration with specific agents or collectives. The PoP analysis highlighted Victor's trajectory focusing on big shifts in his identity formation related to his long-term experience of being, becoming and belonging as a teacher interacting with DRs. The analysis shows that his dominant motivation was to become an embracing math teacher and how this influenced his ways of participating in social interactions, and furthermore that the roles shifted between the practices, characters and figured worlds that were crucial to his professional experience. Our combined theoretical focus thus allowed to describe which DRs Victor interacted with and when, and to explain how and why he developed his interactions with these DRs.

REFERENCES

- Holland, D., Lachicotte Jr., W., Skinner, D., & Cain, C. (1998). *Identity and Agency in Cultural Worlds*. Cambridge, MA: Harvard University Press.
- Little, J. W. (2002). Locating learning in teachers' communities of practice: opening up problems of analysis in records of everyday work. *Teaching and Teacher Education*, 18, 917–946.
- Loisy, C., et al. (2019). Analyzing Teachers' Work with Resources: Methodological Issues. In L. Trouche, G. Gueudet, & B. Pepin (Eds.), *The 'Resource' Approach to Mathematics Education* (pp. 257-321). Cham: Springer.
- Pepin, B., Gueudet, G., & Trouche, L. (2017). Refining teacher design capacity: Mathematics teachers' interactions with digital curriculum resources. *ZDM - Mathematics Education*, 49(5), 799–812.
- Prediger, S., Bikner-ahsbahs, A., & Arzarello, F. (2008). Networking strategies and methods for connecting theoretical approaches: first steps towards a conceptual framework. *ZDM Mathematics Education*, 40, 165–178.
- Skott, J. (2019). Changing experiences of being, becoming, and belonging : Teachers' professional identity revisited. *ZDM Mathematics Education*, 51(3), 469-480.
- Trouche, L., Gueudet, G., & Pepin, B. (2018). *Documentational approach to didactics*. <https://arxiv.org/abs/2003.01392>.
- Trouche, L. (2019). Evidencing Missing Resources of the Documentational Approach to Didactics. In L. Trouche, G. Gueudet, & B. Pepin (Eds.), *The "Resource" Approach to Mathematics Education* (pp. 447–489). Cham: Springer.
- Wenger, E. (1998). *Communities of Practice: Learning, Meaning, and Identity*. Cambridge, UK: Cambridge University Press.