CROSSING THE BOUNDARIES BETWEEN SCHOOL MATHEMATICS AND MARINE NAVIGATION THROUGH AUTHENTIC TASKS

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> Une carte marine est bien plus qu'un instrument indispensable pour aller d'un point à un autre; c'est une gravure, une page d'histoire, parfois un roman d'aventures.
>
> — Jacques Dupuet [1]

Despite the variety of studies that support the idea of using authentic workplace tasks in mathematics teaching, little attention has been paid to empirical investigations of the 'educationalizion' of such tasks in order to implement them in the classroom, as well as to their potential for students' learning. For instance, what are the requirements for relevant authentic workplace task design and how might students' mathematical work be promoted? In this article we explore to what extent authentic marine navigation practices and tools can offer a context for mathematics learning in upper secondary schools.

Our approach is informed by current views of the importance of integrating authentic workplace situations in mathematics teaching in general education (Wake, 2015). Many issues are raised in the literature when task design aims to educationalize authentic situations. Specifically, different definitions, perspectives and meanings of the adjective 'authentic' are described in the literature (see Niss, 1992; Palm, 2006; Vos 2011, 2015). Some researchers realize 'authentic' as something that is genuine (true, honest) while others accept as 'authentic' simulations or copies of reality. Some view authenticity as an internal characteristic of the task while others see task authenticity as an agreement reached through a social process. Furthermore, some researchers use a list of aspects that characterize task authenticity such as the event that the task describes, the mode of task conveyance, the language used and the experienced plausibility of students' solution strategies (Palm, 2006). Ways of educationalizing authentic situations that are proposed in the literature involve the design of the so-called realistic tasks and authentic tasks without a clear distinction between them. The two terms are often used interchangeably while each one's potentiality for students' work is not yet clear.

Our approach is based on the distinction between authentic and realistic tasks and their combined and sequential use while educationalizing authentic naval navigation practices. In resonance with Niss (1992), we consider an 'authentic' workplace situation an extra-mathematical situation "which is embedded in a true existing practice or subject area outside mathematics, and which deals with objects, phenomena, issues, or problems that are genuine to that area and are recognized as such by people working in it" (p. 353).

Following Palm's aspects to characterize authenticity, here we distinguish authentic tasks from realistic tasks in terms of: their purpose, the event described in the tasks and their mode of presentation. Specifically, the purpose of realistic tasks is to familiarize students with authentic tools, they are based on events similar to those encountered in mariners' training sessions and their mode of presentation constitutes a re-configuration of the authentic tools in a digital environment (*e.g.,* use of digital measurement tools).

The purpose of authentic tasks is students' exploration and open inquiry while engaging in genuine events *i.e.,* problem situations that professionals' meet in their everyday practice. Their mode of task conveyance is a traditional paper-pencil environment with the use of authentic physical tools such as the nautical map and marine measurement instruments. Common features in the two types of tasks in our case are: the school classroom as the context of implementation, so no consequences of students' errors exist (Vos, 2015); the language used, that is the marine navigation terminology (Palm, 2006); and the didactical goal served, *i.e.,* support students to identify relations between their school knowledge and the naval navigation practice. Our distinction between authentic and realistic tasks allows verification of their genuine characteristics and identification of their role as aid or hinderance in students' mathematical work. It also permits us to shed light upon the complex relationship between authentic situations and school mathematics, the multifaceted relationship between the process of problem solving, and underlying teaching/learning goals.

Given the complexity of introducing authentic tasks in the classroom and engaging students in unfamiliar practices, we study the move of students between navigation practice and school mathematics through the lens of boundary crossing (Bakker & Akkerman, 2014). The concept of boundary crossing has been used to describe how subjects deal with

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boundaries as they engage in unfamiliar practices and, to some significant extent, are unqualified to do so. This process allows familiar practices (*e.g.,* problem solving in typical school mathematical tasks) to be expanded to new ones (*e.g.,* problem solving in ship navigation contexts with the use of mariners' instruments). Specifically, boundary crossing is realized as a learning process that can emerge through the possible activation of four learning mechanisms: *identification* of the intertwined practices by defining one practice in light of another and thus, delineating the differences among them; *coordination* of practices to restore effective communication by developing tools/objects and making transitions between practices smoother; *reflection* while subjects become aware of their own perspectives in relation to the perspectives of others (*perspective making*) and through taking a new look to their own perspectives through the eyes of others (perspective taking); and *transformation* leading to changes in the existing practices, even the emergence of a new hybrid practice (Bakker & Akkerman, 2014).

Marine navigation and school mathematics: a boundary crossing perspective

Marine navigation consists of a set of professional practices applied to a nautical chart to ensure the efficient and safe travel of a ship. Despite the important role that technology plays in modern marine navigation, navigators need to know the established ways of vessel route tracking and position fixing with the professional authentic tools/artifacts (*i.e.,* nautical charts, parallel rulers), that are considered by us as the historically culturally developed tools. Navigation is referred to as an 'art' that has a scientific background formed historically, socially, and culturally within the naval tradition (Buchan, 2008). The most important tool/artifact historically used in navigation is the nautical chart. Thus, marine navigation requirements include knowledge about the position of points on the Earth's surface, the direction and distance between these points, and calculations and operations needed to avoid natural hazards. In addition, some marine navigation measurement tools look quite similar to tools used in school geometry classes. However, these tools have different usages in the navigation context. Figure 1 shows some examples of such tools/artifacts. These are: the nautical divider (its legs end in sharp points without a pencil), used by professionals to measure distances; parallel rulers (two connected rulers moving in parallel lines), used to measure the bearings of ships' courses; and the station pointer (a three-armed protractor, the middle arm is stable and the two others can open and close), used to measure two adjacent angles simultaneously.

Use of authentic marine navigation tools and measurements can enhance the authenticity of the tasks given to students. However, engaging students in using authentic measurement tools while facing a problematic naval navigation situation, for example, avoiding a dangerous area, is a new classroom practice for learners. This means that students need to be familiarized with the use of such instruments while their decision making needs to recognize the conditions posed by authentic practice, such as restrictions due to safety. As we consider the use of typical school knowledge in workplace situations as a boundary crossing activity, there is a need for coordination around both practices.

To facilitate boundary crossing between workplace and school mathematics, Bakker, Kent, Noss and Hoyles (2008) developed an approach to create mathematical learning opportunities along with what they called technology-enhanced boundary objects (TEBOs). These are reconfigurations of workplace artifacts, and thus real-world simulations of these artifacts, designed to facilitate access to the relevant mathematics tied up within these artifacts. According to our perspective, TEBOs occur in realistic tasks.

Here we explore students' boundary crossing as they work with a specially designed sequence of realistic tasks (including TEBOs) and authentic marine navigation tasks. We give special attention to if and how the realistic tasks mediate students' access to the mathematics involved in the authentic situation. We consider the authentic aspects of the navigation practice (*e.g.,* nautical chart, measurement tools), as well as TEBOs, as boundary objects and our aim is to see if and how these facilitate the sharing of information and restoring of communication between school mathematics and marine navigation.

Design of authentic and realistic tasks

The authors conducted a case study with four 10th grade students (S1, S2, S3, and S4) in a secondary school in Athens, Greece where Nikolaos (the first author) works. Our basic intention throughout the study was to engage the students in working on nautical charts while using authentic tools and familiarizing themselves with authentic measurements. Taking into account the complexity of engaging students in authentic tasks using school mathematics, our task design was informed by Wake's (2015) approach, which favors the coupling of authenticity and mathematics, to allow students to develop insight into, and understanding of, how mathematics occurs in workplace practice. According to this perspective, our task design was based on the use of events and authentic tools taken from the naval navigation profession as well as the use of digital artifacts that simulate naval navigation measurements.

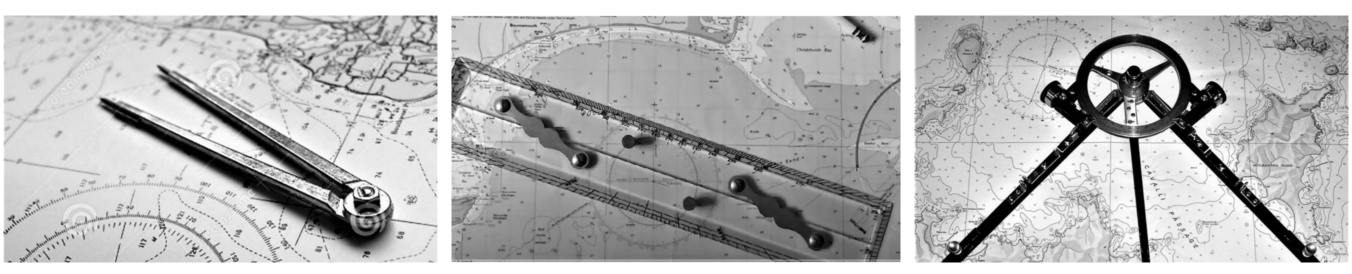


Figure 1. The professionals' tools: nautical divider, parallel ruler, station pointer.

To achieve those ends, Nikolaos decided initially to collaborate with a professional captain who familiarized him with the basic elements of the nautical chart, the use of the navigation tools and the way to carry out measurements in the authentic context.

Nikolaos also studied the navigation textbooks used at a marine academy (*e.g.,* Dimarakis & Dounis, 1986) before starting to design a series of authentic and realistic tasks. The authentic tasks were inspired by authentic problematic situations in which professionals have to make quick decisions on how to handle problems. They were designed in a close collaboration between Nikolaos and the captain. In this process, the captain validated the compatibility of the tasks with the authentic marine navigation practices. In addition, Nikolaos designed by himself realistic tasks that aimed to familiarize students with the authentic tools and measurements involved in professional practices. The other two authors supported Nikolaos to link the above decisions with the adopted approach and the existing literature.

Table 1 shows the sequence of the realistic and authentic tasks and their features. The realistic tasks aim to assist students to understand basic navigation measurements (range, bearing and horizontal angle [2]) while the authentic tasks aim to engage them in authentic marine navigation practices in which the professional uses these measurements to solve a problem (*e.g.,* safe voyage of the ship). In order for the students to be able to engage effectively with the authentic tasks, in Phase B realistic tasks preceded each of the authentic ones.

The captain was present during enactment of the tasks in the classroom and in one case (Task 8) he validated students' solutions and answers. The students were challenged by Nikolaos to reason about their ideas and solutions using mathematics and authentic aspects (language, constrains) (Figure 4).

Here we focus on the authentic practice of 'Passing safely through dangerous areas with the use of horizontal angle measurement' and the related Tasks 7 and 8.

Task 7 (Realistic – TEBO)

Testing possible ship positions was designed to familiarize students with the use of horizontal angle and support them in linking the horizontal angle (workplace measurement) to the inscribed angle (underlying geometrical concept). This task is considered as a TEBO (and realistic) as its design involves the reconfiguration of the horizontal angle and the use of the nautical chart as background in a digital environment (GeoGebra). The students had to conceptualize the inscribed angle as the locus of the vertex of a given angle subtending a given line segment. Initially the students were given the two landmarks A and B, one free point that represented the ship's position, and the corresponding horizontal angle. They were asked to test the alteration of the horizontal angle moving the point in three different ways (straight line, angle, and polygonal path, see Figure 2).

Table 1: The designed tasks and their features.

Authentic practices (routine/problematic)	Marine navigation tools	Mathematical tools	Student realistic and authentic tasks
Phase A. Introduction to nautical chart (2 hours)			
Identify/interpret landmarks on the chart and distances between them	Nautical chart, geographical coordinates	Calculation of distance	Task 1. (authentic) Exploring the nautical chart
		Polygonal line, geometric transformations	Task 2. (authentic) Plotting the ship's course
Phase B. Working with authentic measurements (6 hours)			
Avoid an obstacle with the use of range measurement	Range through the radar, nautical divider	Circle characteristics	Task 3. (realistic) Working with range
		Tangents of the circle	Task 4. (authentic) Avoiding an obstacle
Position fixing with the use of bearing measurement	Bearings, parallel ruler, compass rose	Straight lines, relative positions of lines in the plane	Task 5. (realistic) Working with bearings
		Parallel lines, parallelogram, and parallel shift	Task 6. (authentic) Finding the ship's position.
Passing safely through dangerous areas with the use of horizontal angle measurement	Horizontal angles, station pointer, nautical chart.	Inscribed angles, relative positions of circles in the plane	Task 7. (realistic – TEBO) Testing possible ship positions.
			Task 8. (authentic) Passing through a dangerous area

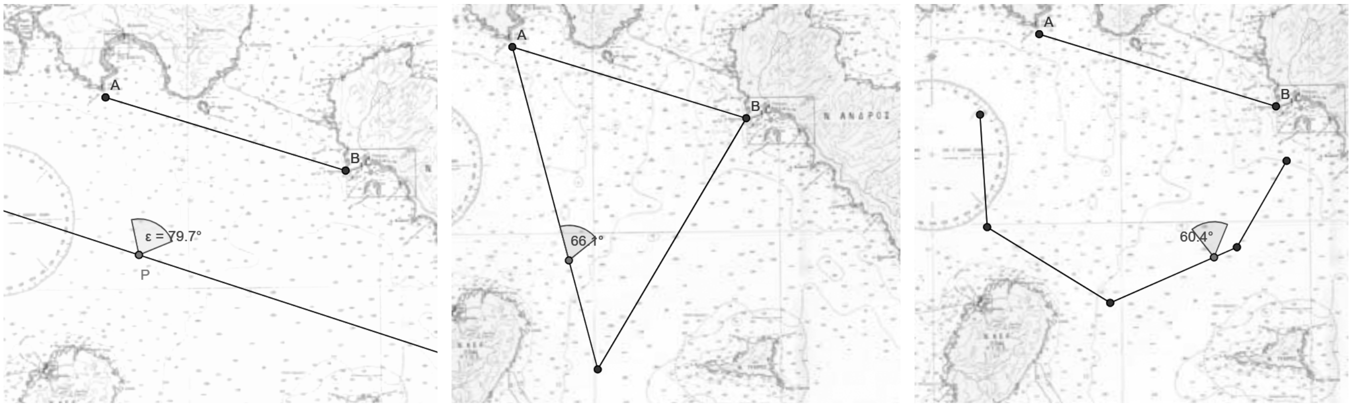


Figure 2. *Task 7: Testing the alteration of the horizontal angle.*

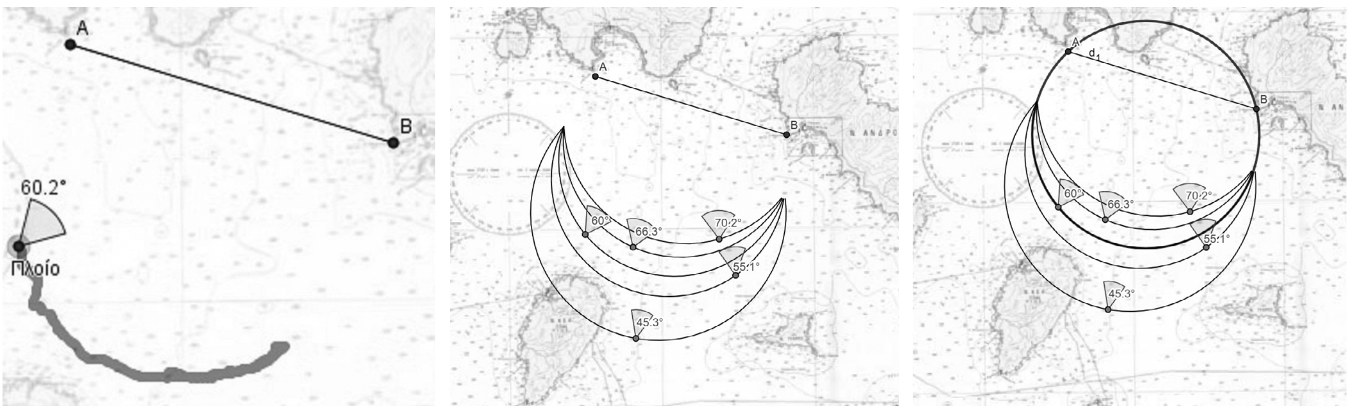


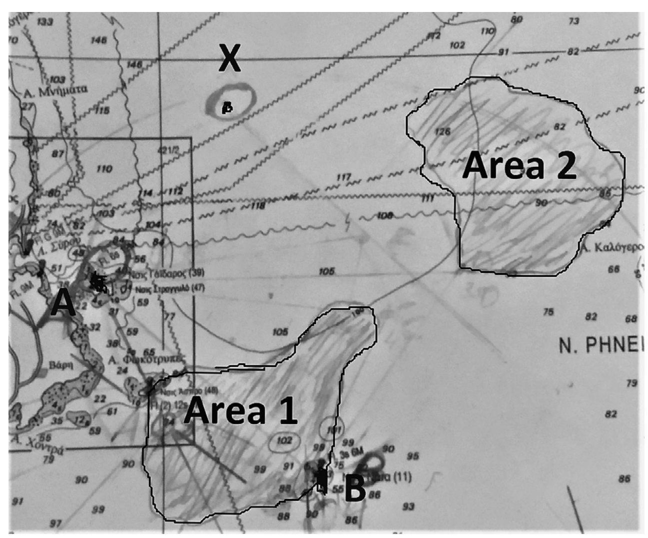
Figure 3. *Testing the ship's position line by keeping the horizontal angle constant.*

Subsequently, Nikolaos asked the students to move the free point with the trace activated so as to keep the horizontal angle constant. As the designed trace represents the ship's route, Nikolaos's aim was to support students in understanding that the ship's position line is an arc passing through the landmarks A and B (Figure 3).

In this task, the digital files given to the students had a nautical chart in the background so as to keep alive the feeling of the authentic context. The digital tool's affordances such as dragging, active trace, and hide/show objects were expected to cultivate the making of links between marine navigation and typical mathematical practices.

Task 8 (Authentic)

Passing through a dangerous area, as presented in Figure 4, was given to the students as a critical situation during the ship's voyage. We consider it as an authentic task since the students had available the same type of tools (*i.e.* printed nautical map, all types of marine navigation instruments) as professionals do and had to choose the appropriate tools in order to avoid specific dangerous areas (Area 1 and Area 2) presented on the nautical map.



Authentic Task 8: Starting from point X the ship must pass through a dangerous (hatched) area with non-visible, underwater obstacles. You have at your disposal landmarks A and B as reference points. Using the professional's measurements and tools, find a way to keep track of the ship's course to ensure its safe passage through the dangerous area.

Figure 4. *Passing safely through a dangerous area.*

Account of learning episodes

Here we provide an account of learning episodes based on the analysis the three of us carried out. In the task *Testing possible ship positions,* the students were expected to become familiar with the nautical chart and gain adequate experience of using professional's measurements (bearing, range) and authentic tools (compass rose, parallel rulers, and nautical divider). We had observed the activation of the *identification* mechanism of the boundary crossing while students familiarized themselves with the nautical chart, and identified commonalities and differences between this chart and the Cartesian coordinate system. In addition, the *coordination* mechanism also emerged as the students worked with Tasks 3–6 related to the measurements of range and bearing. At this point, the students appeared to be able to link the range measurement to the notion or circle with a given radius. Below, we provide an analysis of students' boundary crossing as they engaged in tasks *Testing possible ship positions* (realistic-TEBO) and *Passing through a dangerous area* (authentic).

In the realistic task *Testing possible ship positions* the students initially experimented with the possible curves satisfying the condition that the given point/ship faces the given line segment under an angle of 60 degrees. The possible paths are a straight one or a polygonal one. Then Nikolaos asked them "Which of the proposed lines satisfy the above condition?" The students observed that between possible paths, the smallest change was observed when the ship was moving on the polygonal one (S3: "In the last, the polygonal path"). Taking a cue from the idea of S2 ("It is a curve") student S1 refers directly to a circle relating the horizontal angle with the inscribed one (S1: "It is a circle! If we have designed a circle, it would be the horizontal angle of 60° in all positions of the ship that moves on the given circle").

S2 and S3 speak of a polygonal path and a curve while S1 refers directly to a circle relating the horizontal angle with the inscribed one. S1 uses a mathematical property of the inscribed angles and geometrical locus from school mathematics to refer to a horizontal angle that constitutes an authentic measurement in the navigation practice to describe the positions of the ship across a circle. Therefore, it provides an indication of a communicative connection between the two practices, that is, coordination.

The next challenge for the students was to realize that only one arc of the circle satisfies this condition. Testing a series of given arcs, they were surprised that only in one case (the circle should pass from landmarks A and B) was the horizontal angle unchanged.

This was something that opposed the property students knew from school mathematics for the equality of angles inscribed on the same arc (Figure 5). Since the possible positions of the ship were only on one of the two arcs created by the two landmarks, they were challenged by Nikolaos to justify their answer ("If I cross the point through the landmark, will the angle remain 60 degrees?").

Answering Nikolaos's challenge student S2 clearly used mathematical properties to validate their choice (S2: "180° minus 60°gives 120° They are complementary, this was also confirmed in the software. So, only the lower arc").

The realistic-TEBO task, acting as a boundary object, supported students to restore the communication between school mathematics and marine navigation, combining elements from both practices and making evident the activation of the coordination mechanism. We can observe that students worked mainly in an experimental way, empirically associating the underlying mathematical concepts to the workplace context through the provided simulation in the digital environment. The TEBO allowed the underlying mathematical concepts to come to the fore and helped students to establish links between the two practices.

Moving to the authentic task the captain marked point X as the ship's starting point, and pointed out A, B as visible landmarks on the map. In order to take measurements, the students could use only landmarks A and B (Figure 4). Then the captain asked the students to find a method to keep track of the ship's course through the hazardous area.

Initial solution: applying a parallel line model

Students started solving the problem by drawing the segment AB. After that, they used the parallel rulers to draw a

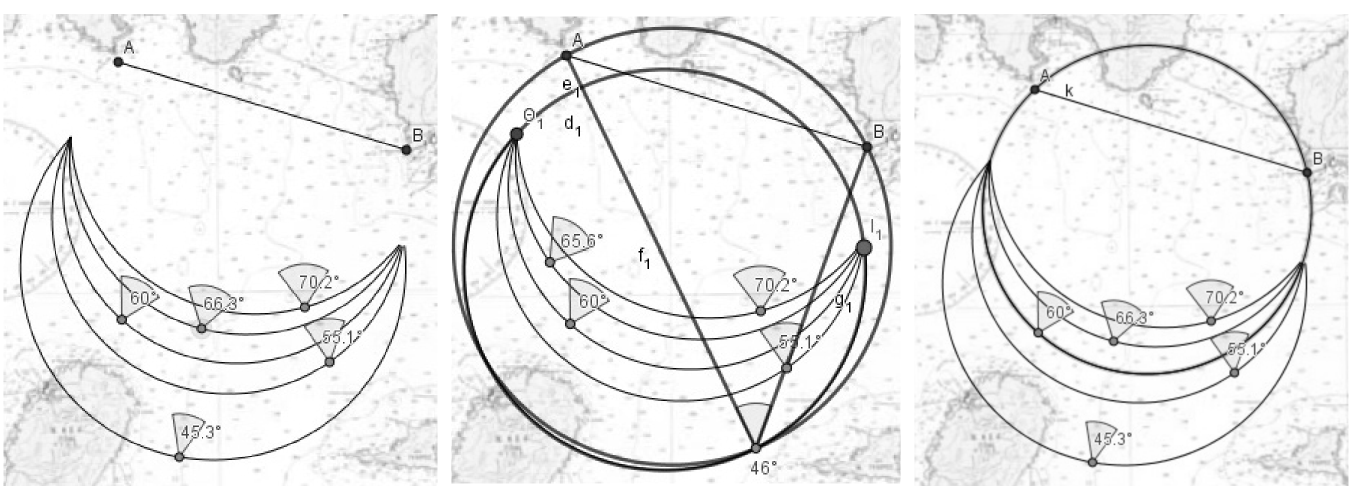


Figure 5. Finding the right circle.

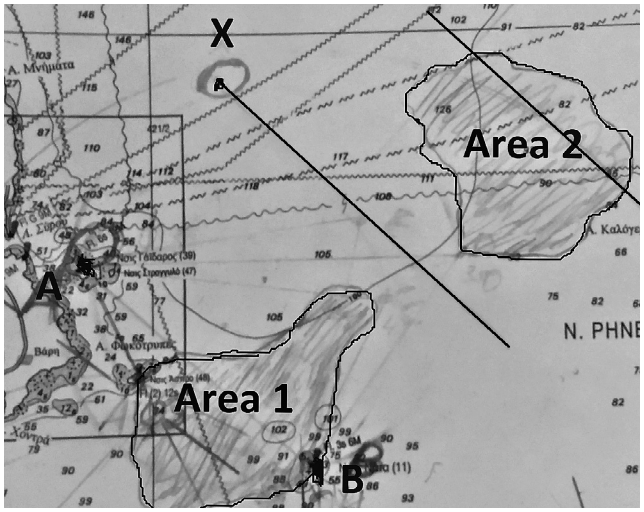


Figure 6. *The initial student solution.*

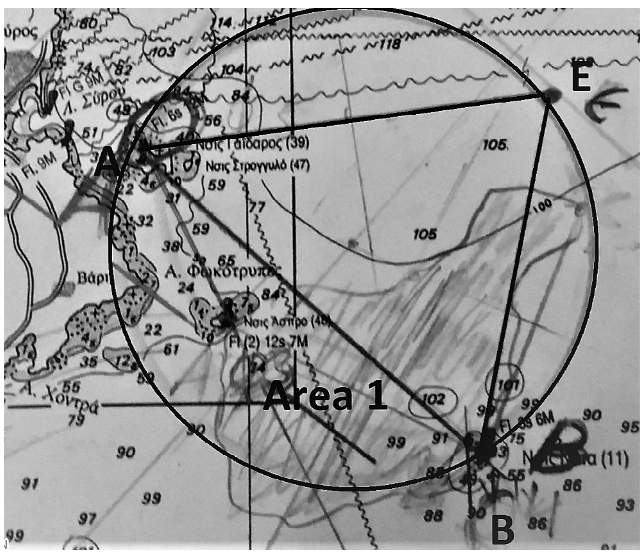


Figure 7. *The intermediate student solution.*

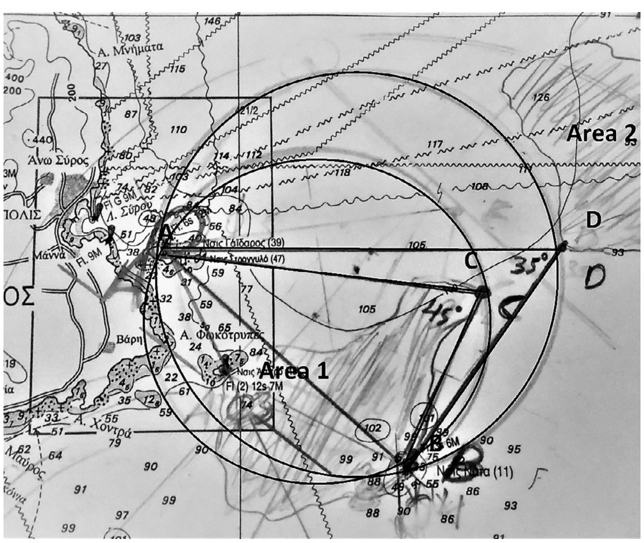


Figure 8. *Final student solution.*

line from point X parallel to segment AB as a safe route for the ship (S3: "Let's move like this, and measure the distance of the two lines". [*S3 draws a straight line from X parallel to the segment AB*]) (Figure 6). Just measuring the bearing of the aforementioned line from X did not provide an effective way to keep track of the ship's course, as the indication of the ship's compass that determined the course of the ship could correspond to many parallel routes. Student S3 proposed measuring the distance between the lines drawn from X and the segment AB to keep track of the ship's course. Although, this was a correct solution from a mathematical point of view, the students realized that it was not possible in the real-life situation to measure the distance between the ship and AB with the use of the available measurements/tools, since AB in reality is an imaginary straight line (S2: "How shall I know if I am far away from or near to the hazardous area? We cannot see line AB to measure the distance." [*S2 refers to the distance from the ship to the hazardous area in a real situation*]).

The constraints posed by the captain's measurements/tools and the navigational demands (ship safety) led the students to identify the inability of school mathematics to efficiently solve authentic problematic situations (S3: "The parallel line is not a good choice" [*All students agree*]), while S1 offered alternative idea (S1: "We must use horizontal angles […] Bearings or ranges will not work").

In this dialogue, we recognize the activation of boundary crossing (identification). Another fact that highlights boundary crossing is students' adoption of professional practice terminology. In addition, they devise a new strategy by choosing the most appropriate measurement among those that have been taught up to this point.

Intermediate solution: defining the dangerous area with a circle

Later on, Nikolaos's intervention was crucial for the students to overcome their difficulties in finding a strategy to exploit the concept of inscribed angle. Nikolaos urged students to mark a safe point on the chart and the students marked the safe point E, which was also a point on the line they had drawn in their initial solution. ("OK, why don't you mark a safe point for the ship on the chart?") In this case, the students used landmarks on A and B as reference points on the chart while they looked for a way to determine the safe area for the ship. Nikolaos indicated the need to specify the dangerous area on the map and S1 proposed drawing a circle that passed through the points A, E, and B to measure the corresponding inscribed/horizontal angle. (S1: "Wait. I think we need to design a circle passing through all three points" [S1 refers to points A, B, E]).

The first part of the dangerous area was somehow delineated by the circle passing through the points A, B, and E (Figure 7). As students answered Nikolaos's question, they had to come up with a way to use the horizontal angle to check if the ship was in a safe position. All the points on the arc AEB were safe, since all the corresponding horizontal angles are equal as inscribed angles to the same arc (S4: "As the area surrounded by the arc of a circle" [*S4 refers to arc AEB*]).

The communicative connection activated with the help of Task 7 is evident at this stage as students combined tools from both practices, leading to the coordination between them. Students experimented with other points, but they had not yet come up with a comprehensive strategy. They had created an intermediate model that could not yet give them the final solution, but it was gradually leading them to it.

Final solution: working with inscribed angles

Once the students discovered how to use the horizontal angle, they engaged in finding a measure to indicate that the ship was sailing in a safe area. For this, they had the idea of designing the point C (the ending point of the dangerous area near the shore, Area 1, Figure 8) with the circle passing through A, C, and B, and the inscribed/horizontal angle ACB. Their strategy was to use the value of the inscribed/horizontal angle (45°) to define the dangerous area oriented by the designed circle. With the intention of provoking students' mathematical reasoning, Nikolaos asked if the value of the inscribed/horizontal angle had to be bigger or smaller than the angle ACB to ensure the ship's safe passage through the dangerous area ("Fine, the horizontal angle is 45°. To be safe, do you need a wider or a narrower angle?"). Taking the perspective of the workplace, students suggested using point D (the beginning point of the dangerous area away from the shore, Area 2, Figure 8) as they had used point C before. Thus, through a reflection process the students designed a new circle that passed through points A, D, and B and the inscribed/horizontal angle ADB in order to have a visual representation of the 'safe' area for the ship's course. The students had to measure the inscribed/horizontal angles (from the points C and D) and observe the difference in their values, which were 45° and 35° respectively. (S2: "We must keep track of the horizontal angle. The limits are from 35° to 45° for a safe passage"). Hence, the students accepted that as the radius of the circle (passing through A and B) increases, the corresponding inscribed angle decreases (S4: "No, for a bigger radius, the angle becomes narrower") (Figure 8). In this way, they developed a strategy to check whether the ship is following a safe course or not, based on the value of the inscribed/horizontal angle having as its vertex the position of the ship and sides defined by the lines connecting the vertex with points A and B respectively. Finally, though, they generalized their answer (S1: "We must keep track of the horizontal angle"). In the end, the captain acknowledged that the students' final solution was identical to the one used in corresponding workplace situations.

Although coordination took place between the two practices in the previous realistic tasks, when it came to the authentic task, we identified a paucity of boundary crossing. Students did not immediately realize that horizontal angle measurement could help them in solving the task. They developed the parallel line model but the students themselves rejected this solution by evaluating the mathematical results through the constraints posed by the authentic situation. In this way, the students realized that their own mathematical knowledge and understanding of this situation (*i.e.* their choice to draw a line parallel to a line segment between two landmarks) could not help them to solve this problem because the available authentic measurement tool (the bearing) could not support this idea. Thus, the students became aware of constraints posed by the authentic context (identification). Then they expanded their own perspectives (perspective making) and were able to choose the relevant measurement (horizontal angle) so as to keep track of the ship's route (perspective taking). In accordance with other studies, these findings confirm that authentic tasks support students' metacognitive and reflective engagement (Nielsen, Nashon & Anderson, 2009). Moreover, it seems that boundary crossing follows a cyclic trajectory that is similar to that of the modeling cycle (Kaiser & Schwarz, 2006) in which the identity of authentic practice is at stake and needs to be clearly oriented (identification) before moving to more advanced types of learning such as coordination and reflection.

Crossing boundaries using realistic and authentic tasks

The complexity of engaging students in authentic situations in the mathematics classroom is an issue that has been addressed by many researchers. Such complexity raises a number of challenging questions: What is the appropriate task design for this purpose? What are the learning processes emerging during students' transition between mathematics and authentic practices? What are the pedagogical and epistemological aspects of this process?

In this article, the identification of subtle issues regarding students' engagement in working with authentic tasks—and the raising of the teacher's awareness of these issues—was made possible by the combination of the distinction between authentic and realistic tasks together with the boundary crossing perspective. Through illustrative episodes we highlighted how a didactic sequence of realistic and authentic tasks can facilitate boundary crossings between the practices of school mathematics and authentic marine navigation. We clarified how the realistic task (in the form of TEBO) enriched the 'didactical milieu', helped students access the mathematics black-boxed in marine navigation measurement tools and fueled their perceived need to find a meaningful solution for the authentic problem. This indicates the potential contribution of realistic tasks in students' engagement with authentic problems. Implementation of similar tasks based on authentic situations can be informed by the insights provided by the present work in many ways. For instance, the analysis may sensitize curriculum designers, teachers and researchers to orient students' activity during the task enactment in the classroom by identifying a priori areas of boundary crossing. These should take into account: (a) the critical role that realistic tasks can play in students' activity, especially those exploiting the dynamicity of digital tools; (b) the complexity of students' learning processes when engaged with authentic tasks; and (c) the requirements of task design at the nexus of authentic situations and school mathematics. The distinction between authentic and realistic tasks could also inform curriculum designers and researchers that support the idea of using authentic workplace tasks in mathematics classrooms. A similar point is also raised by Lagrange, Huincahue and Psycharis (2022), who argue for the need to design tasks that put at stake transitions or coordination between different working spaces that can be defined by a given authentic situation and the plurality of models related to it.

Finally, similar to Vos (2015), we conclude that students' experiences with authentic practices offer an epistemological insight into students' learning of mathematics. This insight refers to students' understanding about mathematics, or about the utility of mathematics, in these practices. Students' knowledge here was attained through a cyclic trajectory similar to that of the modeling cycle where the identity of authentic practice is at stake and needs to be clearly oriented before developing more advance but context-specific mathematical models.

Notes

[1] From his 1999 novel 'Marin': A nautical chart is much more than an indispensable instrument for getting from one point to another; it is an engraving, a page of history, sometimes an adventure novel. (our translation)

[2] The horizontal angle is the angle with its vertex at the eye of the observer and two obvious landmarks defining the legs.

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Sculptures on the San Vicente del Raspeig campus of Universitat d'Alacant