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PROSPECTIVE MATHEMATICS TEACHERS' ARGUMENTATION WHILE INTERPRETING CLASSROOM INCIDENTS

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Abstract. *This paper reports research aiming to analyse the structure and quality of prospective mathematics teachers' (PTs)' argumentation when engaged in identifying and interpreting critical incidents from their initial field experiences. Toulmin's model and recent elaborations of it were used to identify the structure of the argumentation and characterize the emerging warrants and backings. Results indicate different argumentation structures and types of warrants, backings and rebuttals in the process of PTs' interpretations of students' mathematical activity.*

INTRODUCTION

Current approaches in research in mathematics teacher education acknowledge the importance of noticing as a construct to study what and how PTs attend to when observing, analyzing and interpreting teaching (Scherer & Steinbring, 2006). Thus noticing has been considered as a complex action that involves teachers in identifying what is significant in a classroom interaction, interpreting this noteworthy incident on the basis of their knowledge and experiences and linking these with broader principles of teaching and learning (van Es & Sherin, 2010). Moreover, at the level of teacher education and in collaborative contexts interpreting teaching phenomena is a joint action that involves the development of claims, conjectures and arguments. Studying teachers' argumentation is a means to understand the resources upon which teachers base their interpretations. Then a challenge is to get insight into the nature and structure of argumentation in relation to the PT's multiple experiences from school, teacher education courses and field experiences.

In this paper, we focus on PTs' argumentation in the process of selecting and interpreting critical classroom incidents as part of their fieldwork activities. In a recent paper (Potari & Psycharis, submitted) we used critical incidents as a structure to facilitate PTs' reflection and to study their conceptualizations of mathematics teaching and learning. The analysis showed PTs' shifts from observing teaching to questioning aspects of it and conceiving it in a more relational way. It also brought to the fore a richness of arguments developed by PTs as they supported their claims or challenged their peers' interpretations. In this paper, we use Toulmin's model of argumentation and recent elaborations of it to analyse the structure and quality of PTs' argumentation and its development while identifying and interpreting critical incidents.

THEORETICAL FRAMEWORK

Critical incidents and teacher noticing

Critical incidents in the form of signifant classroom events have been used as a structured framework promoting teachers' understanding of teaching and learning situations (Hole & McEntee, 1999). Critical incidents have been used in mathematics education for analytical and developmental purposes (Skott, 2001; Goodel, 2006). For example, Skott (2001) used the term "critical incidents of practice" to describe moments of a teacher's decision-making in which

multiple and possibly conflicting motives of his activity evolved that challenged the teacher's own school mathematics images and provided learning opportunities for students. Goodel (2006) used this structure to promote PTs noticing as well as to address her own development as mathematics teacher educator. The issues raised by PTs in her study, concerned: teaching and classroom management; student factors; relationships with colleagues, parents and students; and school organizational issues. In our study, we adopt van Es and Sherin's (2002) framework of teachers' noticing that addresses: (a) what teachers notice; (b) what are the sources upon which they base their interpretations; and (c) their suggested pedagogical actions. In this paper, our focus is on the second dimension of the framework and in particular on PTs' argumentation when analyzing and interpreting their selected critical incidents.

Teacher argumentation and the Toulmin's model

In mathematics education field, teacher argumentation has been studied in the context of classroom (Knipping & Reid, 2015), in teacher education programs (Metaxas, Potari & Zachariades, 2009) as well as in teachers' responses in hypothetical scenarios (Nardi, Biza & Zachariades, 2012). Toulmin's theory has been the basis of most studies in the analysis of teacher argumentation and in particular its model for the layout of arguments (Toulmin, 1958). His model (see fig. 1) consists of six basic elements: the claim (C) is the position or claim being argued for; the data (D) are the foundation or supporting evidence on which the argument is based.; the warrant (W) is a general rule of inference that authorises the step from the data to the claim; the backing (B) supports the legitimacy of the warrant; the modal qualifier (Q) represents the degree of force or strength that the data confer on a claim in virtue of the warrant; and the rebuttal (R) consists of exceptions to the applicability of the warrant.

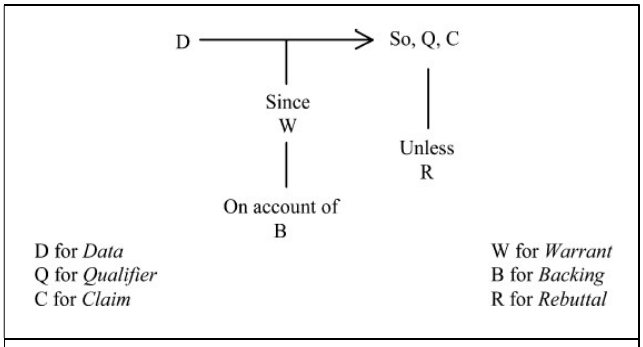

Fig.1 A representation of Toulmin's model

Often Toulmin's model has been combined with other frameworks so that to address not only the structure but also the quality of argumentation For example, Metaxas et al. (2009) used argumentation schemes to analyse the internal coherence of mathematics teachers' arguments in the context of a master's course, while Nardi et al. (2012) adopted Freeman's framework to identify the different types of warrants on the basis of which mathematics teachers support their claims when they interpret hypothetical classroom scenarios. Their approach seems to address the quality of teachers' argumentation by placing their arguments in relation to teacher considerations and priorities – pedagogical, curricular, professional and personal. In particular, they distinguish seven types of warrants in teachers' arguments: epistemological and pedagogical a priori; professional and personal empirical; epistemological and curricular institutional; and evaluative. This categorization is adopted in this study to analyse PTs' warrants backings and rebuttals in the process of their argumentation. Here, professional experiences are expected to be related to PTs' fieldwork and other practice-based activities in the context of teacher education.

Moreover, Knipping and Reid (2015) distinguished local from global arguments to study classroom proving processes. Local arguments represent a step of an argument that can be analysed by Toulmin's model. Global arguments lay out the structure of interconnected local arguments indicating the structure of an argumentation process as a whole. They identified different types of global argumentation structures (e.g., source-structure, spiral-structure) as constructs to address differences in the argumentative process. To explain these differences they consider the nature of local arguments that make up global structures.

In our study, we use Toulmin's model, the classification of the warrants proposed by Nardi et al. (2012) and the structures developed by Knipping and Reid (2015) to analyse PTs' interpetations of critical incidents indentified by them when reflecting on lessons observed and/or taught. Toulmin's model provides a structure to analyse PTs' local arguments, Nardi et al.'s approach helps us to characterize the sources of warrants, backings and rebuttals while Knipping and Reid's elaborations allows us to compare different argumentative processes to address potential shifts in PTs' interpretations of classroom phenomena. The combined use of these approaches offers us a tool to address the quality of PTs'argumentation.

METHODOLOGY

Context of the study and participants

The research took place in the context of a 14-week mathematics education undergraduate course with the philosophy to link theory-driven instruction on the teaching and learning of mathematics at the secondary level with mathematics teaching in classroom settings. Enrolling in the course, PTs had a background of undertaking at least four other mathematics education courses as a part of their teacher education program at the university. In parallel to their university studies, most PTs were helping school students on a private base with their mathematical homework. The aim of the course was to engage PTs in critically consideration of aspects of mathematics teaching as they emerge from the complexity of teaching practice in schools. Every second week (for the entire semester) 22 PTs (9 males, 13 females) working in pairs were asked to participate in a number of field activities such as to observe and analyse other teachers' lessons as well as to design, implement and analyse three lessons. In each week following the activities at school, a three-hour class session took place at the university. Instructional practice in the university meetings aimed to support PTs' reflective activities on their recent field experiences and to link emergent issues with existing mathematics education research.

Data collection and analysis

Critical incidents were considered as a methodological tool for triggering PTs' reflection on teaching practice. At the beginning of the course, PTs were introduced to the idea of critical incidents through examples provided by the teacher educator. In subsequent meetings, the groups of PTs were asked to select and present critical incidents that they had found in their fieldwork activities. All university meetings (eight in total) were video recorded. The data for this study was collected over the entire semester, and consisted of PTs' personal portfolios, video recordings of the university meetings and PTs interviews.

The present paper is based on the analysis of the transcripts of the university meetings. Initially adopting a grounded theory perspective (Strauss & Corbin, 1998), we identified four themes of

critical incidents discussed in the meetings (i.e. students' activity, epistemological issues, lesson planning and classroom management; wider contextual and social factors). Within each theme, we conducted a fine-grained analysis of the data in terms of the three dimensions of the van Es and Sherin (2002) framework (i.e. what they observed, sources of interpretations and potential teaching actions). Then, we analysed PTs' argumentation developed when interpreting critical incidents related to each theme across the meetings by using our combined theoretical approach mentioned above. In particular, for each theme we identified the claims that the PTs made while reflecting on classroom observations and their own teaching, the data on which they based their claims, the warrants and backings they used to support them and their rebuttals where the validity of the conclusion was questioned. Then we focused on the interrelationships among the arguments related to the specific theme throughout the university meetings. Our purpose in this part of the analysis was to describe argumentation structures and to trace their progressive development in order to identify shifts in PTs' interpretations. Finally, we analysed the warrants, backings and rebuttals to identify the sources on which PTs based their arguments following Nardi et al.'s (2012) classification. In this paper, our focus is on PTs' argumentation concerning the theme "students' activity" and in particular the construction of mathematical meaning.

RESULTS

Below, we analyze two extracts from the discussions at the university meetings taking place in different phases of the course to characterize the nature of PTs' argumentation as well as to illustrate its progression. The first one took place at the beginning of the course, after PTs' initial experiences from classroom observations, and the second towards the end of the course, after the completion of PTs' teaching.

Extract 1 (third university meeting)

The teacher educator encouraged PTs to report on critical incidents they had identified during their first classroom observation. Concerning students' activity, PTs' focus was on students' difficulties. The main claim (C) developed in the discussion was that students cannot move beyond a surface understanding to a deeper conceptualization of the underlying concepts and properties. PTs used a number of different data sources upon which they based their claim. These sources came from their classroom observations and concerned students' difficulty to transform a fraction to an equivalent one (D1), to use the algebraic properties to solve a first degree equation (D2) or to simplify an arithmetic or algebraic expression (D3).

In this phase of the discussion, the PTs reported critical incidents without using warrants in their interpretations. Later on, Orestis interpreted students' difficulty to conceptualize mathematical ideas offering as a warrant that in school textbooks mathematics loses its meaning: "we use terms or expressions that have nothing to do with mathematics. For instance the rule of three, central in school textbooks at primary level, is a technique rather than a mathematical method" (W1). Leonidas offered another warrant by referring to the different meaning of symbols in arithmetic and algebraic expressions in the school textbooks. He mentioned that "3 ½ is a mixed number while 3x where x is ½ is a product" (W2).

In a subsequent phase of the discussion, the argumentation was enriched by other data coming from classroom observations. PTs used warrants related to curriculum and to wider cultural factors

interfering to teachers' attempts to promote conceptual understanding. For example, Irene noticed that: "one student mentioned the term 'adjacent angles' from nowhere". The classroom teacher responded that "we have not said something about adjacent angles in the lesson" (D4). Other PTs brought similar examples from their classroom observations. For example, Marina mentioned a case where the teacher introduced the concept of angle in the 7th grade, but the students referred to its measure that they had encountered in primary school (D5). A variety of warrants and backings were given: "they have already met the same concepts in the primary school" (W3); "the students have taken private lessons"(W4); "they take private tuition because parents do not have the knowledge or the time to help their children with their homework" (B1); "the requirements of school mathematics are increasing so students need help in order to be successful" (B2); "the national examinations are rather demanding" (B3); "the students need more individualized teaching" (B4). In the realm of the discussion, Orestis expressed a rebuttal by questioning the tendency to support students to become good at mathematics through continuous guidance: "it is not necessary every child to be successful in mathematics. So, close guidance does not allow students to take decisions for their future according to their interests" (R1). Anta referred to her own experience with her parents who always helped her at home "although they worked all day" (R2).

We note that in this part of the discussion the PTs grounded their warrants, backings and rebuttals on their own learning experiences as students at school or on their broader views about teaching and learning. Towards the end of the session, the discussion was centered again on the barriers to students' conceptual understanding. At this phase, the PTs brought new data from their classroom observation: "the teacher gave the task to simplify the expression 2+4(2x+1) and one student wrote 4·3x. Although the teacher reminded him the distributive law, the student provided again a wrong answer" (D6). Here, the PTs started to identify elements of students' mathematical thinking by offering as a warrant that "the students conceive the distributive rule visually as a picture in their mind and use it without understanding its meaning" (W5).

Taking a more global view of the above analysis focusing on the interrelationships of arguments, we recognise a number of argumentation steps based on different data sources (D1, D2, D3) that appeared at the beginning of the discussion. These steps indicate the existence of parallel arguments supporting indermediate conclusions that are used as data later to support the main claim concerning students' difficulty to develop mathematical meaning in algebra. Later in the discussion, new data sources appear (D4, D5) that provide the basis of new argumentation steps where warrants and backings are used to support further the main claim. We could possibly argue that the argumentation structure emerging in this initial attempt of PTs to address students' construction of mathematical meaning follows characteristic features of what Knipping and Reid (2015) name as source-structure where the emphasis is more on collecting information (data and conclusions) rather than on the connections between the different argumentation steps.

As regards the nature of the PTs' warrants, backings and rebuttals, they can be characterized as a priori pedagogical (B3, R1), institutional-epistemological (W1), institutional-curricular (W2, W3, B2), empirical-personal (R2), empirical-professional (W4, B1), and evaluative (W5, B4).

Extract 2 (eighth university meeting)

We provide below an example where the focus of the discussion was on a critical incident reported by two PTs, Anna and Marina. The incident that constituted the data (D) for the subsequent

argumentative process concerned the difficulty that a student had to link the algebraic identity $(a+b)^2=a^2+2ab+b^2$ and its geometrical representation in a model provided by the PTs representing the design of the house with side a+b divided in rooms with areas, a^2, ab, ab and b^2. Both Anna and Marina described that although the student was encouraged to work in the geometrical model and then to recognize the algebraic identity, he only recalled the algebraic formula that already knew. The main claim (C) in the discussion was the students' difficulty to connect different representations of mathematical knowledge such as algebraic and geometrical.

Different PTs expressed their interpretations about this incident. Sofia reflected on her own experience as a school student to interpret the student's reaction: "Actually, the student offered a secure answer! I also used to do the same as a student at school. When the teacher asked me something that I did not know, I gave him one formula I could relate to the question. This is what the student did here". In her comment, Sofia refuted the initial claim (R1) and provided as a warrant that "the student offered a secure answer" (W1). She supported her warrant further by implicitely referring to existing norms in the classroom where a student feels obliged to give an anwer to any question (B1). She brought data from her own experience as a learner (D1). Later on in the discussion, Anna and Marina brought examples from their teaching to support their opinions related to the student's conceptualization of the algebraic identity. In the following excerpt, Anna supported the main claim by referring to the student's use of language: "The student used the word 'solution' to refer to the algebraic identity". The new warrant provided by Anna was that the student "cannot see the equivalence of the two parts of the identiy, but he considers it as a procedure that needs to be followed" (W2). Irene mentioned that the process of linking algebra and geometry was not a simple task even for PTs: "This connection is too difficult for the students" (W3) ... "It is difficult even for us to see how a geometrical situation can be expressed by algebraic symbols and operations" (B2). Here, the warrant is backed by PTs' similar difficulties as learners at the university. In this case, her experience at the university operated as a new source of data (D2). Later on in the discussion, she enriched her interpretation by bringing data from research and theory of mathematics education (D3) offering as a warrant that "the students are used to apply the mathematical content to exercises" (W4) and supported it further by offering a backing including the qualifier "I think" (Q1): "I think that this has to do with the didactic contract and the social norms of the classroom" (B3). Alexandros refuted W4 stating that "the children are more creative than adults" (R2) and offered a warrant for this: "for the kids to use models to form algebraic relations is like playing a game, so they are successful" (W5). He referred to his experience at the university (D4) and used this as data to back W5: "we [as university students] tend to follow complicated solutions. Our minds are not used to see the simple solution" (B4).

In terms of the argumentation structure, we see a number of backings and parallel arguments leading to the support of the main claim as well as emerging subclaims based on different sources of data. In this process, we also observe the presence of refutations in the argumentation structure, the formulation of new claims that are supported by warrants and backings as well as the use of qualifiers (Q1) in PTs attempts to consider the main claim from different viewpoints. This structure has features characteristic of the spiral argumentation structure of Knipping and Reid (2015).

As regards the nature of the PTs' warrants, backings and rebuttals, they can be characterized as empirical-professional (W2, W3, W4, B1, B3), empirical-personal (W1, B2, B4, R1), a priori

pedagogical (W5), and evaluative (R2) indicating different sources of data where PTs build their arguments. These sources include PTs' personal experiences as learners at school or at the university, their current classroom experiences, and research based findings encountered in the university courses. The fact that most of the warrants, backings and rebuttals are empirical-professional and personal indicates a shift in the way that PTs interpret students' activity as they have developed links between their prior and current experiences.

CONCLUSION

For interpreting students' activity, the PTs used different sources of data based on their prior school experiences, current university studies and fieldwork. In particular, they made links between students' conceptualizations and their own experiences as learners in school and university and they looked for evidence in their classroom observations and teaching. The analysis of the two extracts shows different argumentation structures. The structure emerged from the analysis of the first extract involves paraller arguments and the use of warrants and backings by the PTs to support the main claim without developing connections between them. In this case, the identified warrants and backings indicate a multiplicity of sources that the PTs used in their argumentation in a rather balanced distribution. In the second extract, the argumentation structure involves new claims, warrants and backings that offer different ways to support the main claim. The warrants and backings have been formulated mainly on an empirical basis related to the PTs' personal experiences as students as well as to their professional experiences from the course.

The analysis of the two extracts shows shifts in the process of interpreting students' activity. PTs initially focused on the factors that influence the construction of mathematical meaning (extract 1) while in the last meetings (extract 2) on the interplay between these factors. Although the PTs used their personal experiences as learners at school or at university to support their claims, towards the end of the course (extract 2) they brought also evidence from their classroom experiences blended with research findings in mathematics education to make interpretations. Thus, PTs' interpretations of critical incidents seemed to become progressively deeper as they were enriched by new elements emerging from the PTs' experiences in the course. Moreover, the practice of questioning mathematics teaching and learning established in the university meetings seem to have had an impact on the development of a more reflective stance on classroom phenomena.

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