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Digital Technologies and Mathematics Teaching and Learning: Rethinking the Terrain

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Developing a joint methodology for comparing the influence of different theoretical frameworks in technology enhanced learning in mathematics: the TELMA approach

TELMA (KALEIDOSCOPE) European Research Team[2]: **<** *Artigue M., Bottino R., Cerulli M., Mariotti M., Morgan C.> & <Alexopolou E., Cazes C., Chaachoua H., Georget J.P., Haspekian M., Kynigos C., Lagrange J.B., Latsi M., Maffei L., Maracci M., Papadopulou A., Pedemonte B., Psycharis G., Robotti E., Souchard L., Trgalova J., Vandebrouk F.>*

This contribution deals with the work of the European Research Team TELMA (Technology Enhanced Learning in Mathematics) of the Kaleidoscope network towards understanding the role played by theoretical frames in design and research in that area, and building tools to improve communication between researchers from different cultures. We present two facets of TELMA work: a 'cross-experimentation' project in which each TELMA team experimented with an Interactive Learning Environment (ILE) for mathematics designed by another team; and the design of a methodological tool for systematic exploration of the role played by theoretical frames in the design and analysis of uses of ILEs. We focus on the methodological dimension of this work, showing how we employ the construct of didactical functionalities as a means of comparing and integrating the research conducted by the teams. We provide some preliminary results of the joint experiment and use of the methodological tool.

Introduction

This contribution originates from TELMA, a European Research Team (ERT) established as one of the activities of Kaleidoscope, a Network of Excellence (IST–507838) supported by the European Community (www.noe-kaleidoscope.org). The contribution reports on the work developed within TELMA for analysing the influence of different theoretical frameworks in the design and/or use of digital technologies for shaping mathematics teaching and learning activities and discusses some of its outcomes that we think of interest for this ICMI Study. It is especially related to theme 4 of the ICMI Study: "Design of learning environments and curricula". Indeed it addresses some of the research questions raised in this theme: "How can theoretical frameworks be helpful for understanding how design issues impact upon the teaching and learning of mathematics?", "What kind of mathematical activities might different technologies and different theoretical

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backgrounds shape and how can learning experiences (including the tools, tasks and settings) be designed to take advantage of these affordances?", and, last but not least, "Which methodologies can be developed and applied to understand and compare different approaches, theoretical frameworks, and backgrounds?".

Background: Kaleidoscope and TELMA

Kaleidoscope's central aim is to address the lack of harmonised research in the field of Technology Enhanced Learning (TEL) in Europe by integrating various existing initiatives and research groups. The aim is to develop, on the one hand, a rich and coherent theoretical and practical research foundation, and, on the other hand, new tools and methodologies for an interdisciplinary approach to research on learning with digital technologies at a European level. The network is doing this by supporting a range of integrating actions, including *European Research Teams* (ERT) such as TELMA. ERT are integrating activities, which aim to network European excellence through specific research challenges. The key idea of creating an ERT is to stimulate the mutualisation of knowledge and know-how of a number of recognized research teams on the identified issues, and to favour the construction of shared scientific policy, building complementarities and common priorities.

TELMA is specifically focused on Technology Enhanced Learning in MAthematics. It involves six European teams with the aim of building a shared view of key research topics in the area of digital technologies and mathematics education, proposing related research activity, and developing common research methodologies. Each team has brought with it particular focuses and theoretical frameworks, adopted and developed over a period of time. Most of the teams have also contributed learning environments integrating digital technologies for use in mathematics learning, designed, developed and tested in accordance with their own theoretical perspectives[3]. We will refer to these as Interactive Learning Environments (ILEs).

The starting phase of TELMA was very challenging, requiring six teams with different backgrounds, work methodologies, and ILEs, to begin to share knowledge, developing a common language and common topics of interest. This demanding task was addressed by working on a number of topics considered important for mutual knowledge and comparison (including research areas and goals, theoretical frames, ILEs implemented or used, contexts, work methodologies). Each team had responsibility for one topic and, on the basis of materials sent by the other teams, produced a report analysing the different contributions and developing them into an integrated presentation (the result is available the TELMA web site (www.itd.cnr.it/telma).

This first effort, based on the descriptions provided by the teams and analysis of papers they had published, helped to identify some common sensitivities to, for example: the contextual, social and cultural dimensions of learning processes; the

[3] For instance Ari-Lab2 (CNR-ITD), Pepite and Casyopée (DIDIREM), Aplusix (MeTAH), E-slate (ETL), L'Algebrista (CNR-ITD and UNISI).

role played by semiotic mediation; instrumental issues. It also made it evident that the diversity of the theoretical frames we employed[4] affected the ways we dealt with these common sensitivities in the design or use of ILEs. But reading and exchanging descriptions and research papers left us unsatisfied as we felt that our understanding of the underlying processes and their possible effects on practice remained too superficial. For that reason, we decided to develop a strategy allowing us to gain more intimate insight into our respective research and design practices. This strategy consisted of a 'cross-experimentation' project and simultaneous development of a methodological tool for systematic exploration of the role played by theoretical frames in teaching and learning in mathematics using digital technologies.

In this contribution, we focus on this second phase of our collaborative work, introducing first the idea of "didactical functionality", which we used as a tool for approaching the relationships between theory and practice.

The notion of "didactical functionality" of a tool

The notion of didactical functionality of an ILE (see Cerulli, Pedemonte, Robotti, 2005) was developed as a way to link theoretical reflections to the real tasks that one has to face when designing or analysing effective uses of digital technologies in given contexts. It is structured around three inter-related components:

- a set of features / characteristics of the ILE;
- an educational goal;
- modalities of employing the ILE in a teaching/learning process related to the chosen educational goal.

These three dimensions are inter-related: although characteristics and features of the ILE itself can be identified through *a priori* inspection, these features only become functionally meaningful when understood in relation to the educational goal for which the ILE is being used and the modalities of its use. We would also point out that, when designing an ILE, designers necessarily have in mind some specific didactical functionalities, but these are not necessarily those which emerge when the tool is used, especially when it is used outside the control of its designers or in contexts different from those initially envisaged. In the second phase of TELMA work, this notion of didactical functionality thus took a central and unifying role:

- on the one hand, the cross-experimentation aimed to explore the didactical functionalities the different teams involved would associate with ILEs they had not designed, and how their particular educational contexts and the theoretical frames they used would influence their constructions;
- on the other hand, this notion was also used to structure the methodological tool for exploration of the role played by theoretical frames.

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[4] These were mainly: activity theory, socio constructivism, Vygotskian theories of semiotic mediation, social semiotics, theory of didactic situations (TDS), anthropological theory, Rabardel's theory of instrumentation, situated abstraction, AI theories.

In what follows we present these two facets of TELMA work, focusing on the methodological dimension.

The cross-experimentation

The idea of cross-experimentation is a new approach to collaboration, seeking to facilitate common understanding across teams with diverse practices and cultures and to progress towards integrated views.

Some important methodological choices:

There are three principal characteristics of this cross-experimentation:

- the design and implementation by each research team of a teaching experiment making use of an ILE developed by another team;
- the joint construction of a common set of guidelines expressing questions to be answered by each designing and experimenting team in order to frame the process of cross-team communication;
- the specific role given to PhD students and young researchers.

Each team was asked to select an ILE among those developed by the other teams. This decision was expected to induce deeper exchanges between the teams, and to make more visible the influence of theoretical frames through comparison of the vision of didactical functionalities developed by the designers of the ILEs and by the teams using these in the cross-experimentation.

The cross-experimentation involved a rich diversity of ILEs, educational contexts and theoretical frames, but important attention was paid to the control and productive exploitation of this diversity, especially through the joint construction of guidelines, developed through an on-line collaborative activity. On-line collaboration allowed the teams to communicate the results of their within-team discussions and resulted in an agreed joint set of guidelines (http://www.itd.cnr.it/telma/documents.php), negotiated between the teams to be as relevant as possible to their interests and theoretical frameworks, while remaining feasible in light of the constraints of time and empirical settings. These guidelines structure and support a priori and a posteriori reflective analysis of the cross-experimentation.

In order to allow as much comparability as possible between the research settings, it was also agreed to address common mathematical knowledge domains (fractions and algebra), with students between years 7 and 11 of schooling in experiments lasting approximately one month. Table 1 summarises the ILEs chosen, the teams who developed the ILEs and the teams conducting the experimentation.

An important role has been given to young researchers and PhD students. Starting from three draft sets of questions addressing the issues of contexts, representations and theoretical frames, they developed the guidelines through the Kaleidoscope Virtual Doctoral School platform, and have taken charge of the experimentation. This role is coherent with the general philosophy of TELMA and Kaleidoscope. It also has the benefit of allowing “fresh” eyes to look at the teams’ approaches, theoretical frameworks, and consolidated practice in order to make explicit those factors that often remain implicit in the choices made by more experienced researchers.

ILE	Developer's team	Experimenting team(s)
Aplusix	MeTAH-Grenoble	CNR-ITD, UNISI
E-Slate	ETL-NKUA	UNILON
ARI-LAB 2	CNR-ITD	MeTAH, DIDIREM, ETL-NKUA

Table 1: The tools employed by TELMA teams in the cross experiment

The selection of research questions, experimental settings and modes of use of the ILE, methods of data collection and analysis were all determined by each experimenting team after a period of familiarisation with the ILE itself, following the common guidelines developed through the on-line activity. Each team thus conducted an independent study of the use of an ILE. At the same time, however, the framework of common questions provided a methodological tool for comparing the theoretical basis of the individual studies, their methodologies and outcomes.

The current state of the project

The experimentations took place during the first term of this academic year. A priori analysis of the experiment, according to the guidelines, has been produced by each team. The a posteriori analysis, following the guidelines, is in progress. Comparison and discussion of similarities and differences between the reflective analysis carried out by the PhD students and young researchers and the analyses, expectations, and results obtained by the ILE designers is planned in the following months. However, preliminary comparison of each team's results has highlighted interesting issues and indicates directions for future investigation, as described in the following section.

Some preliminary results from the cross-experimentation

In order to point out and compare the preliminary results of the cross experiments a meeting was held. Each team reported on its own experiment focusing on the defined/employed didactical functionalities of the ILE used, trying to make explicit the relationship between such didactical functionalities and the team's theoretical assumptions. In order to structure this preliminary analysis each team was asked to complete a form before the meeting, focusing on the three dimensions of didactical functionalities. The form followed the principle of "necessary conditions" in the sense that not all the details of the experiments needed to be given, but only those that the team believed to be necessary conditions for the experiment to be successful according to the team's theoretical assumptions (http://www.itd.cnr.it/telma/documents.php).

Comparison of the forms completed by each team, and of the oral reports of the experiments, highlighted a set of issues that seem promising in terms of future investigations and in relation to the key question: *How can the use a given ILE within a specific context be characterized by specific theoretical frameworks and by cultural and/or institutional constrains?*

During the cross experiment some difficulties arose when teams attempted to use a given ILE in a context (both in the sense of school and of research context) different from that in which it had been developed. For example, the software Aplusix has been designed (by the French team MeTAH) to facilitate the teacher's work, and to offer him/her a good level of autonomy with respect to standard algebra curricular activities. The software allows students to build and transform algebraic expressions freely and to solve algebra exercises by producing their own steps as on paper; for each step the system gives an indication of correctness as feedback. Aplusix was designed to support the standard activity of algebraic manipulation, based on the solution of calculation tasks like *expand*, *factorize*, *solve the equation*, etc. However, the CNR-ITD team, adopting a socio-constructivist approach, faced the problem of planning open-ended tasks. According to this theoretical framework, open-ended tasks favour pupils' construction of meanings through exploratory activities. This was achieved by interpreting the feedback concerning the correctness of steps as feedback concerning the equivalence of expressions and/or statements. This change of perspective implied also that Aplusix was no longer used autonomously by students, but required the teacher to orchestrate the activity by asking the students to make their strategies explicit, to justify them and to discuss them with their classmates.

Adapting the way in which an ILE is used to a changed context, even if possible, may also be complicated by the role played by different curricular constraints and school praxis. As an example, we consider ARI-LAB2 (developed in Italy by the ITD team). ARI-LAB2 is composed of several microworlds designed to support activities in arithmetic problem solving and in the introduction to algebra. One of these is the "fraction" microworld, which provides a graphical representation of fractions on the real line, allowing the user to build fractions by means of commands based on Thales theorem. Some teams encountered difficulties using this microworld in their school context due to the fact that Thales theorem is usually introduced in the curriculum later than fractions. The MeTAH team tried to use it as a "black box" but found this caused problems when pupils needed to make sense of feedback. Similarly, the DIDIREM team decided to switch to other microworlds of ARI-LAB2 because they judged it was not realistic to ask a teacher to change the mathematics organisation of the school year.

During the cross-experimentations another aspect has been highlighted related to the influence of theoretical frameworks on the use of ILEs. This is related to the role assigned to feedback by different teams. For example, the DIDIREM team, drawing on the theory of Didactic Situations, found the feedback provided by ARI-LAB2 too limited with respect to what is generally expected from a "milieu" offering an a-didactic potential for learning. On the contrary the ITD team, who had developed the ILE, drawing on a more general constructivist framework, considers the feedback sufficient because the teacher's role and feedback are considered as fundamental as those of the ILE.

These examples show how, in order to employ an ILE in a context different from that taken as reference by its designers, one has to face a set of problematic issues. To sum up, the highlighted issues include:

- educational goals
- typologies of tasks proposed to the students
- computer's feedback and autonomy of the ICT tool and/or of the pupils
- settings and role of teachers

These issues point to significant investigative directions to be addressed in the next year of the work of TELMA. In fact, in order to refine the comparison between the experiments, our preliminary analysis raises the need to refine the lenses through which the experiments are analysed and compared. Starting from the idea of didactical functionality, we need to address its three dimensions in more detail, and in order to do so, a first methodological tool has been developed in parallel with the preliminary analysis and will be refined and employed in the next year. Below we present the tool and indicate the kinds of analysis it can help to bring forward, showing how it was employed by one of the TELMA teams.

A methodological tool for systematic exploration of the role played by theoretical frames

Some preliminary remarks

In the design of this methodological tool, we were inspired by the work already developed in TELMA about theoretical frames, the common sensitivities and the evidence of differences in the ways these were dealt with, and also by the meta-study previously developed by DIDIREM researchers involved in TELMA (Lagrange & al., 2003). This led us to consider this methodological tool as a multidimensional tool structured around the notion of didactical functionality, and to associate to each component of the notion of didactical functionality a set of 'concerns', expressed in the most neutral way. Analysis using the tool will try to determine for each of these concerns whether and how it is addressed, and to elucidate the role played both explicitly and implicitly by theoretical frameworks. It is assumed that such a tool will help to establish productive links between different frames, and will support partial integrative views when these appear accessible and possibly productive, keeping in mind that a global integration is certainly out of reach, and even not desirable, the strength of any approach being attached also to the specific lens it chooses for approaching the complexity of the reality we study.

The methodological tool

a) Analysis and identification of specific ILE characteristics :

The analysis of an ILE using the definition of didactical functionalities generally involves two different dimensions, questioning on the one hand how the mathematical knowledge of the domain is implemented in the ILE, and on the other hand the forms of didactic interaction provided by the ILE. Both the implementation of the knowledge of the domain and the didactic interaction can be approached through different perspectives, which are neither independent nor mutually exclusive. We have thus selected according to this dimension the following concerns:

- concerns regarding tool ergonomy
- concerns regarding the characteristics of the implementation of mathematical objects and of the relationships between these objects
- concerns regarding the possible actions on these objects
- concerns regarding semiotic representations
- concerns regarding the characteristics of the possible interaction between student and mathematical knowledge
- concerns regarding the characteristics of the possible interaction with other agents[5]
- concerns regarding the support provided for the professional work of the teacher
- concerns regarding institutional and/or cultural distances

b) Educational goals and associated potential of the ILE

It is more the relationship between potentialities and goals rather than each of these considered separately which can contribute to illuminate the role played by theoretical frames in relation to this dimension, complementing what is offered by the information provided by the analysis of the ILE. It seems interesting to investigate the relative importance given in the definition of educative goals to considerations of an epistemological nature - referring to mathematics as a domain of knowledge or as a field of practice, considerations of a cognitive nature - focusing on the student in her relationship with mathematical knowledge, considerations focusing on the social dimension of learning processes, and finally institutional considerations. Thus the concerns we selected for this dimension are:

- Epistemological concerns focusing on specific mathematical content or practices
- Cognitive concerns focusing on specific cognitive processes or difficulties
- Social concerns focusing on the social construction of knowledge and on collaborative work
- Institutional concerns focusing on institutional expectations and on compatibility with the forms and contents valued by the educational institution

c) Modalities of use

The design of modalities of use and the a priori analysis of their implementation supposes a multiplicity of choices of diverse nature. It is reasonable to hypothesize that only a small part of these are under the control of theoretical frames, explicitly or even implicitly, many other being dictated consciously or unconsciously by the educational culture and the particular context within the realization takes place. The categories are:

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[5] Other agents can be other students, the teacher, tutors as well as virtual agents such as the companions implemented in some ILEs.

- Concerns regarding contextual characteristics
- Concerns regarding tasks proposed to students, including their temporal organisation and progression
- Concerns regarding the functions given to the tool including their possible evolution
- Concerns regarding instrumental issues and instrumental genesis
- Concerns regarding social organisation, especially the interactions between the different actors, their respective roles and responsibilities
- Concerns regarding interaction with paper and pencil work
- Concerns regarding institutional issues, especially the relationships with curricular expectations, values and norms, the distance from the usual environments

Using the methodological tool

The methodological tool has been used first in order to explore the role played by methodological frames in the preparation of one experiment: that carried out by DIDIREM. There is no space here for a detailed presentation of this analysis which is accessible on the TELMA website (Artigue, 2005). It shows that, in this experimentation involving two different microworlds offered by ARI-LAB2, nearly all the concerns mentioned above were addressed, but with evident differences in emphasis (four different levels of emphasis were distinguished). The analysis also shows how the three main theoretical frames used: instrumental approach, theory of didactic situations, anthropological approach (together with didactic knowledge about the mathematical domain at stake) influenced the choice of the selected microworlds and the design of the experiment. However, an interview with the young researchers of the team involved in the project also shows that many of their choices were not under the explicit control of these theoretical frames. Some of the choices can be explained a posteriori by referring to theoretical frames but had been used as naturalized and implicit conceptual tools. Others were dictated by institutional and cultural habits. We can hypothesize that, from one team to another one, differences in the implicit theoretical tools and cultural habits will be made visible by the exchanges organized around the cross-experimentation, offering us insights into the real influence of theory on research and design practices that the reading of papers hardly offers.

Conclusion

In this short text, we have tried to make clear the kind of contribution we can offer to this ICMI Study. This work tries to overcome the difficulties generated by the existing diversity of theoretical frames and the lack of communication between these, through a better understanding of the role played by theories, the development of methodological tools and cross-experiments. This is on-going work and, by the time of the Conference associated to this study, all the analysis being completed, we hope to be able to offer a sound contribution on these difficult but crucial issues.

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