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Teacher Educators' Activity Aiming to Support Inquiry Through Mathematics and Science Teacher Collaboration

Chrissavgi Triantafillou[1] · Giorgos Psycharis[1] · Despina Potari[1] · Dionysia Bakogianni[1] · Vasiliki Spiliotopoulou[2]

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Abstract
This study explores teacher educators' (TEs') activity as they support mathematics and science teacher collaboration in co-designing and jointly implementing tasks. We view TEs' activity through the lens of Activity Theory and expansive learning and draw evidence from data generated within the mascil project that linked mathematics and science teaching with workplace situations through inquiry-based teaching. We focus on five TEs' actions and goals, use data from their professional development sessions with teachers and from the TEs' interactions during their own meetings, and highlight the illuminating case of one teacher educator. We trace evidence indicating paths of actions followed by each Teacher Educator and look for indications of their professional learning. Our analysis reveals generic and content-focused actions. All TEs faced different kinds of contradictions and had difficulties handling them. In terms of professional learning, all TEs adapted their prior teacher education practices and appreciated the critical role of epistemological differences between the two disciplines.

Keywords Epistemological contradictions · Mathematics and science teacher collaboration · Teacher educators' actions and goals · Teacher educators' professional learning

✉ Chrissavgi Triantafillou
chrtriantaf@math.uoa.gr

[1] Department of Mathematics, National and Kapodistrian University of Athens, Panepistimioupolis, Zographou, 15784 Athens, Greece

[2] School of Pedagogical and Technological Education, Patras, Greece

Introduction

High-quality teacher professional development (PD) is an acknowledged, global, and urgent need that over time, has attracted wide research attention (e.g. Borko, 2004). In contrast, the professional learning of teacher educators (TEs) has only recently become an area for research focus (e.g. Loughran, 2014). Existing research acknowledges the complexities of TEs' own professional learning (e.g. Even, 2008) and TEs' practices to promote inquiry into teaching and learning through collaboration between mathematics teachers (Llinares & Krainer, 2006; Zaslavsky & Leikin, 2004) or among science teachers (Berry & van Driel, 2012). Our study contributes to this research field by attempting to provide empirical indications of TEs' practices that aim to support mathematics and science teachers collaborating to adopt an inquiry stance of teaching and their professional learning in this context. We draw evidence from data generated within the project that linked mathematics and science teaching with workplace situations through inquiry-based PD approaches and collaborative experiences. Such approaches have particular importance today, as STEM has become a key focus in education (English, 2016).

Collaboration between mathematics and science teachers is acknowledged as crucial for teacher PD (Nelson & Slavit, 2007). Achieving it, though, likely raises additional challenges for the teachers and their educators (Potari et al., 2016). The differences in the two subjects' epistemologies may constrain their integration into secondary classrooms (Williams, 2011). We use the term epistemologies to describe different types of practices and ways of practitioners' thinking and understanding in each discipline (Reynante, Selbach-Allen, & Pimentel, 2020). Some ways to facilitate teachers' cross-disciplinary collaboration are fostering engagement in discerning disciplinary aspects and willingness to share classroom experiences (Frykholm & Glasson, 2005). However, TEs' role in such contexts remains an unexplored research topic. All the above bring to the fore the demanding character of TEs' work in cross-disciplinary PD activities and create a need to study their role in collaborative environments. Therefore, our study seeks to explore how TEs support collaboration among in-service mathematics and science teachers to link mathematics, science, and workplace situations with classroom teaching in a community of inquiry environment (Jaworski, 2006). Such an environment involves different epistemological assumptions and inquiry-based teaching approaches. It thus constitutes a fruitful terrain for gaining deep insight into content specific issues that may emerge in establishing a community of inquiry. Our research questions are: (RQ1) What is the TEs' activity to promote inquiry into teaching and learning through collaboration between mathematics and science teachers to integrate workplace situations into teaching? (RQ2) What indications of TEs' professional learning emerge in this activity?

Theoretical Perspective

Teacher Educators' Challenges in Collaborative Settings with Teachers

TEs' role in supporting teacher collaboration within PD settings has gained increasing research attention over the last decades due to the wide consensus that TEs are critical for developing teachers' collaborative activity (e. g. Borko, 2004). In mathematics teacher education, TEs and teachers often form communities that aim to develop mathematics teaching and learning. In our study we, as teacher educators, formed a community of inquiry working with teachers in the three layers of inquiry that Jaworski (2006) introduced and recently elaborated further (Jaworski, 2019). The first layer involves inquiry into mathematics where students and teachers are engaged in the classroom. In the second layer, teachers and TEs inquire into the practices of teaching and learning to learn more about creating mathematical opportunities for students. The third layer involves "research inquiry into the practices and processes in the two inner layers" (ibid. p. 277). In layer three, TEs have the central role in the inquiry activity that aims to promote teacher PD and learn more about teachers and the TEs' own professional learning. The joint work within a community of inquiry facilitates co-learning through reflective practice. Although this often entails challenges for teachers and TEs, there is limited empirical evidence in mathematics education to understand how TEs overcome them in forming and sustaining a community of inquiry (Robutti et al., 2016). The situation becomes more challenging for the TEs in communities that involve teachers from different disciplines (e.g. mathematics and science teachers). Many studies point out the importance of mathematics and science teacher collaboration in pursuing integrated teaching in the classroom. These studies generate ideas and present elements of collaborative activity that emphasize the need to communicate the different meanings of concepts and representations in science and mathematics (Frykholm & Glasson, 2005) and to inquiry-based teaching approaches (Nelson & Slavit, 2007).

Most existing studies on mathematics and science teacher collaboration have highlighted a number of important personal, social, or cultural factors that influence teachers' collaborative work. However, they have rarely focused on the content specific issues and common conceptual themes of mathematics and science teaching that our study addresses. Davis, Chandra, and Bellocchi (2019) did delve more deeply into the epistemological issues that TEs need to consider when they design and enact STEM teacher education programs. Their research points out that these programs should take synthetic epistemological orientations that "enable ways of knowing through integrated STEM" (p. 37), go beyond the boundaries of the disciplines, and include new pedagogical orientations where learning "as through and through context may be experienced" (p. 38). These examples concretize issues and challenges TEs are expected to face while supporting collaboration between teachers of different disciplines. Our recent work also suggests that the collaboration between secondary mathematics and science teachers and researchers involves tensions in the process of forming a community of inquiry (Bakogianni, 2021).

Teacher Educators' Practices and Professional Learning

In collaborative contexts, TEs' professional learning is closely related to teachers' PD. In most cases, PD occur simultaneously on the basis of teachers and TEs' histories, practices, and experiences (Zaslavsky & Leikin, 2004). This dual process poses questions in terms of how to investigate the development of these two communities through evidence-based research. Our special interest in this study is how TEs support groups of mathematics and science teachers to work together in the design and implementation of workplace situations into teaching. This entails the need to gain a deeper appreciation of the essence of the teachers' beliefs, actions, and participation in a team (Silva, 2000). It further involves TEs' continuous self-reflection and development towards research-informed teacher education activities that will offer teachers opportunities to consider bi-directional moves across disciplinary aspects of mathematics and science teaching as boundary-crossing events (Vale, Campbell, & Speldewinde, 2020).

Recently, in mathematics education research, the question has arisen of how TEs support teacher learning. It has primarily been answered by the description of TEs' moves to adopt an inquiry stance towards teaching and learning (Borko, Jacobs, Seago, & Mangram, in press). Sakonidis and Potari (2014) by adopting a sociocultural perspective, combined the Activity Theory and communities of inquiry frameworks to study mathematics TEs' practices and professional learning. The authors collaborated with mathematics teachers in two settings where inquiry into teaching and research was a basic element. Among the identified TEs' practices were co-analyzing lessons with teachers; exemplifying theoretical constructs and research tools through their own experiences as researchers and practitioners; and critiquing and supporting teachers' work. Further indications of TEs' professional learning included reducing guidance level, providing support relevant to teachers' needs, adopting a less critical attitude towards teachers' actions, and developing an understanding of teachings' complex nature.

As discussed above, TEs face challenges when working in collaborative settings with teachers and especially with teachers from different disciplines like in our study. In it, we use the Activity Theory to take into account the complexity of teacher education in such research contexts and scrutinize TEs' activity and professional learning. In the following subsection, we present the main elements of the theory and how we used it in this study.

Activity Theory as Our Theoretical and Analytical Tool

We use two main constructs to capture the interactions between mathematics and science teachers and TEs. Figure 1 shows the construct from Leont'ev's work on the three-tiered explanation of activity (Leontiev, 1981). Figure 2 presents Engeström's view of an activity system represented by a triangle linking relations between mediational means, the subjects, and the object of the activity with elements of the activity's relevant communities (Engeström, 1987).

According to Leontiev (1981), activity is a unit of life organized into three hierarchical layers. The top layer, the activity, is oriented towards a motive. Actions are conscious processes directed at goals which must be undertaken to fulfill the object. In this process, the object becomes the motive of the activity as it obtains its stimulating

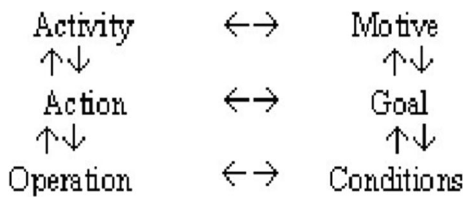

Fig. 1 The definitive hierarchy of Leontiev (1981)

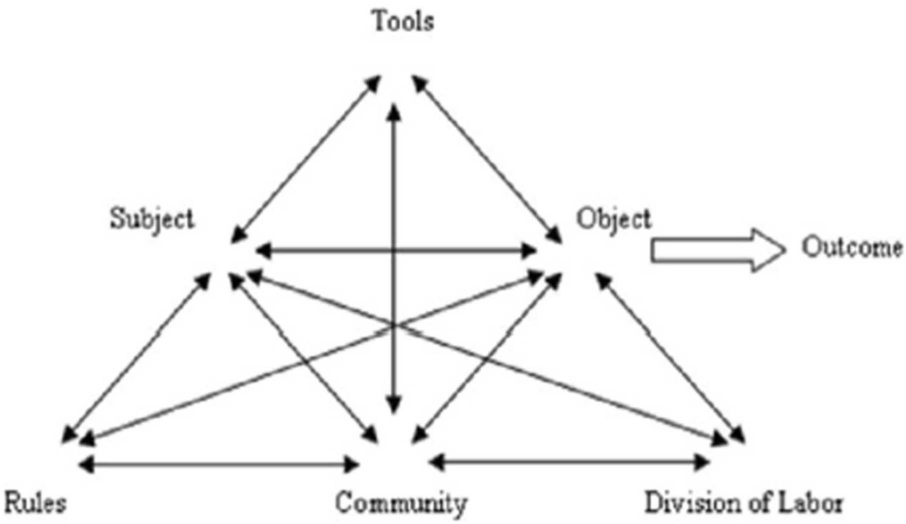

Fig. 2 Expanded activity triangle model (from Engeström, 1987, p. 78)

and activity-orienting function (Roth & Radford, 2011). The actions are implemented through the operations that are routine processes providing an adjustment of an action to the ongoing situation. Operations are oriented towards the conditions under which the subject is trying to achieve a goal. People are typically not aware of their operations that emerge as a result of step-by-step automatization of originally conscious actions. We use Leont'ev's three-tiered explanation of activity focusing on TEs' actions and goals with reference to the TEs' activity. The analytical framework of the activity system and its elements as described by Engeström are used in our study to capture the social influences involved in the TEs' activity. In particular, the framework allows us to map out relations between the subject and the object of the activity that are constructed through the mediation of tools, the community in which the subject participates, and its rules and the division of labor.

In this study, TEs' activity is directed towards the object, teachers' PD through inquiry into teaching and learning mathematics and science based on workplace situations. TEs undertake certain actions that are implemented with the use of tools and conditions (Leontiev, 1981). By focusing on the TEs' actions and goals and with successive references to the teacher education activity, we aim to gain insight into how the conditions and tools form TEs' actions and goals and transform the activity. We also link TEs' actions and their development to the elements of the activity system in order to conceptualize their role in achieving the outcome of the teacher education activity. In our study, the activity system is the teacher education activity in mixed groups of mathematics and science teachers. The subject is the TEs, while the object of the activity is teachers' PD in a community of inquiry. The main tools are inquiry approaches, workplace situations, research articles, classroom-based examples. The TEs participate in different communities (e.g. research) and follow specific rules and regulations that exist in them. Division of labor includes joint inquiry of teachers and TEs. The TEs fulfill project requirements and support teachers in jointly designing, enacting, and reflecting on classroom teaching activities and practices. Figure 3 shows an adaptation of Engeström's expanded activity triangle model our study uses. Contradictions constitute a key concept or principle in Activity Theory and are characteristic of activity systems.

They are not simply conflicts or problems, but are "historically accumulating structural tensions within and between activity systems" (Engeström, 2001, p. 137). Tensions often lead to transformations in activity systems when individuals begin to question and deviate from the established norms and re-conceptualize the object of the activity. This is related to what Engeström (ibid.) calls expansive learning that involves the creation of new knowledge and practices. In our study, TEs' professional learning is triggered by their actions to overcome contradictions and indicated by their questioning existing teacher education practices, developing awareness in the light of the new conditions, expanding the object of the activity, and responding to it in enriched ways (i.e. working together with mathematics and science teachers in the mascil project, developing new practices).

Methodology

Context. In mascil, 11 TEs worked for one academic year with 13 cross-disciplinary groups of about 10 practicing mathematics and science (physics, biology, chemistry) teachers. Two TEs worked with two PD groups. The PD program was mainly based on 78 PD sessions over a year (on average, 6 PD sessions per TE). In each PD session, teachers collaborated in designing inquiry-based tasks (design phase), shared their experiences from the implementations, and discussed emerging issues (reflection phase). The TEs also met alone before PD sessions to progressively develop a common agenda for PD activities within teachers' cross-disciplinary groups. Between the meetings, teachers often cooperated on co-designing tasks and lessons, and applied

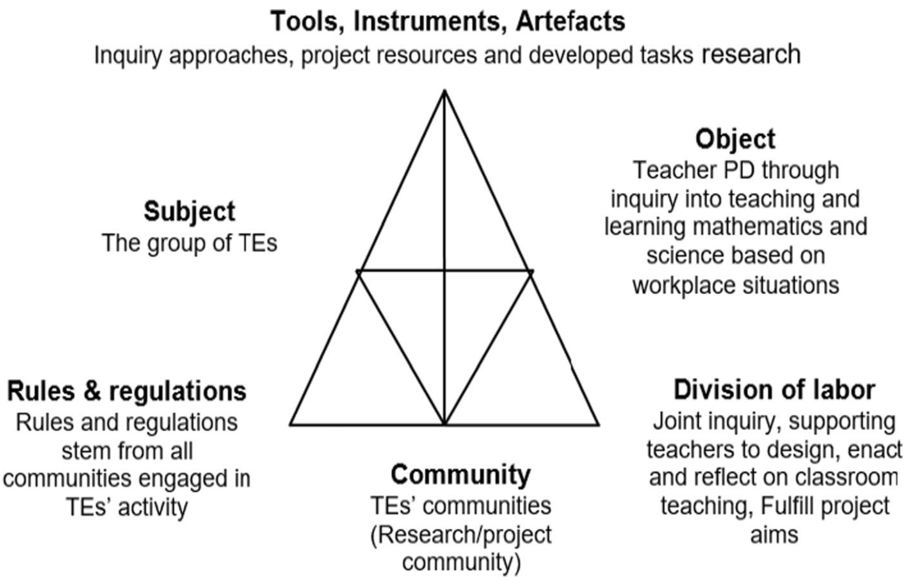

Fig. 3 Adapted model of the teacher education activity

their designs in their classrooms. TEs attended most implementations. We made audio or video recordings of all meetings and classroom interventions and transcribed the recordings.

Participants. Our study focuses on five TEs (the authors) from the 11 who participated in the project. These five cases were indicative due to their diverse academic backgrounds and experiences in mathematics and science teacher education. Table 1 presents the five TEs' profiles. We note that it was the first time of all TEs had worked together with cross-disciplinary groups of teachers. TE1 served as the program coordinator.

Data and Data Analysis. The data we used in this paper consists of the transcribed recordings of the PD sessions (30 in total, approx. 2 h each) and the TEs' meetings (6 in total, approx. 3 h each) to enrich our understanding of TEs' activity and its rationale.

The initial analysis of the discussions in the PD sessions focused on TEs' activity undertaken while trying to facilitate and support inquiry through cross-disciplinary teacher collaboration. Units of analysis are parts of the text (transcribed recordings) that provide evidence of how TEs in their PD groups supported teachers' development towards the object of the activity. To this end, we identified actions and goals throughout the whole set of data by adopting grounded theory approaches (Charmaz, 2014). This eventually led to the coding and categorization of the TEs' actions and goals, a process three of the researchers undertook and evaluated to ensure a sufficient fit between the data and the description of categories. Although these categories may not be always mutually exclusive, they allowed us to discern crucial aspects of TEs' activity. Moreover, we linked the TEs' actions and their development to the elements of the activity system in order to conceptualize how the activity system influenced the outcome of TEs' activity. We also addressed elements of the activity system such as contradictions, tools, and conditions that appeared throughout the PD experience.

Table 1 TEs' profiles

TEs	TEs' professional status	Research interests and teacher education experience
TE1	University researcher/mathematics teacher educator	Development of mathematics teaching and learning and teacher development with experience in the use of mathematics in different contexts and disciplines including workplace and science—rich experience in mathematics teacher education
TE2	University researcher/mathematics teacher educator	Design of learning environments for students/teachers with the use of digital tools and inquiry approaches in mathematics—rich experience in mathematics teacher education
TE3	Post-doctoral researcher in mathematics education	Use of mathematics in workplace situations and its transfer into mathematics and science teaching—limited experience in mathematics teacher education
TE4	PhD student in mathematics education	Development of teaching and learning of statistics—limited experience in mathematics teacher education
TE5	University researcher/science teacher educator	Development of science teaching and learning and teacher development—rich experience in science teacher education

In the next step of the analysis, we traced evidence indicating paths of actions each TE followed so as to look for indications of professional learning. Here we have chosen to detail the path of actions TE1 followed while handling a specific contradiction. So, we have used specific instances from TE1's meetings with her group of teachers to reveal not only how the PD occurred and TE1 supported the teachers, but also how her own learning evolved. We selected TE1's case as she reached a positive outcome in terms of engaging mathematics and science teachers in co-designing and jointly enacting inquiry-based tasks connected to workplace situations and she dealt with the emerging contradiction concerning the epistemological distance between mathematics and science teaching and learning.

Results

We present study results in two main subsections. First, we illustrate the main TEs' activity and second, we present TEs' professional learning.

TEs' Activity

TEs' activity is directed towards the object, teachers' PD through inquiry into teaching and learning mathematics and science based on workplace situations. The analysis indicated that the TEs developed two categories of actions in order to fulfill the object of the activity. *Generic* actions consist of TEs' pedagogical actions and appear in TEs' efforts to highlight the importance of collaboration and value inquiry in co-deploying mathematics and science in workplace situations. The second category, *content-focused* actions, refers to TEs' actions that are related to the content of the two school domains (mathematics and science). Table 2 presents the categories and subcategories of actions and specific goals of these actions and is illustrated by a brief description of the related TEs' actions.

Generic Actions. Two subcategories of these actions emerged: supporting teachers to value inquiry and encourage them to collaborate. The actions of s*upporting teachers to inquire into their own teaching and students' learning in mathematics and science* emerged in order to fulfill the project aim to support teachers' PD through inquiry. For example, when a chemistry teacher in a group presented a task he used with his 12th grade students, TE3 asked "What do you think about this task? Do you think that the students had the opportunity to engage in a fruitful inquiry?" (TE3, 4th PD session). Sometimes the TEs asked teachers to reflect on their own teaching: "How did you pose the question to students?" "What changes do you suggest in a future task implementation?" (TE2, 4th PD session). The second subcategory of *encouraging teachers to co-design tasks and jointly reflect on their classroom enactment*s was supplementary to the first one. The special interest in collaborating to design common tasks emerged and was shared by the TEs in the 2nd TEs' meeting as a way to develop a common ground for collaboration between mathematics and science teachers. (i.e. TE1: "We should encourage them to jointly develop tasks.") Thus, almost all TEs, especially in the introductory PD sessions, referred to the importance of collaboration between science and mathematics teachers. For example, TE4 developed her argument based on the PD

Table 2 Categories and subcategories of TEs' actions

TEs' actions	Subcategories	Clarifying descriptions	Goals
Generic actions	Supporting teachers' inquiry into their teaching and students' learning in mathematics and science	Moving the lens on students' activity and on specific teaching events	To support inquiry
	Encouraging teachers to co-design tasks and jointly reflect on their classroom enactments.	Highlighting the value of collaboration; Encouraging teachers' in co-peered discussions; Handling emerging tensions	To create a common ground for discussion between mathematics and science teachers
Content-focused actions	Exemplifying and/or extending tasks across disciplines	Providing examples from various sources (e.g., project materials; practice-based examples)	To support the design of interdisciplinary tasks
	Focusing on epistemological issues	Analyzing scientific and mathematical dimensions of tasks; Questioning the interrelations between scientific phenomena and mathematical ideas; Dealing with epistemological contradictions	To help teachers understand how common concepts and tools are used in school mathematics and science

group diversity: "It helps that you are nine teachers from five different subject areas, and you have the opportunity to collaborate and co-design" (TE4, 1st PD session). Also, this type of TEs' actions was mediated by research-informed tools (e.g. critical incidents) and targeted both collaboration and adoption of an inquiry stance. For instance, in an attempt to highlight the role of critical incidents in promoting co-learning, TE2 mentioned: "By discussing these incidents, we will reach some conclusions. In this community, we are also learners who learn from each other" (TE2, 4th PD session). However, in various cases, TEs faced emerging tensions such as teachers' unwillingness to collaborate. For example, in TE4's and TE5s' PD groups, the teachers of the two subjects expressed from the start their reluctance to work on common tasks with their peers from the other discipline.

Content-Focused Actions. We identified two main subcategories of content-focused actions. The action *of exemplifying and/or extending tasks across disciplines* stems from TEs' goal to support the design of tasks based on workplace situations through the co-deployment of mathematics and science. The tools used in this action included mainly ready-made examples from various sources (e.g. research and/or project materials). For example, TE3 proposed using authentic workplace tasks from her research work where "mathematics and physics are interrelated" (TE3, 2nd TE meeting). However, in some cases TEs exploited teachers' emerged ideas. For instance, after one science teacher's idea of a task based on seismology, TE2 suggested that: "Locating the epicenter of an earthquake involves mathematical notions such as circle intersection, thus it could be a mathematical task as well" (TE2, 3rd PD session). This type of action also targeted teachers' awareness of the knowledge elements that each discipline fosters and common areas of mathematics and science curricula. The sub-category of *focusing on epistemological issues* aimed at making teachers aware of the different epistemologies of the two school subjects by using common notions and representations as well as to deal with epistemological contradictions. TEs' agenda included questions to the teachers to discern the mathematical and the scientific ideas in their proposed tasks and in students' activity. For example, TE3 prompted teachers to comment on common concepts and tools across disciplines and workplace situations: "What differences do you realize when using the same representation in mathematics and science?" (TE3, 1st PD session). Sometimes, TEs made explicit their views about epistemological differences between the two subjects, e.g. "The inquiry process is possibly closer to the experiment in science than in mathematics" (TE1, 4th PD session) or "The same representations of a concept are used differently in science and in mathematics" (TE2, 4th PD session). Nevertheless, focusing on epistemological issues seemed rare or loose in TEs' activity, as it required handling emerging contradictions between mathematics and science teaching and learning. One main contradiction concerned the epistemological distance between mathematics and science in school. Particularly, TEs' actions to promote teachers' awareness of epistemological differences between the two subjects resulted in reinforcing the boundaries between them and this weakened the possibilities of collaboration. TE1 is the only teacher educator from the analyzed cases who seemed to handle effectively epistemological contradictions and succeeded in managing mathematics and science teachers' collaboration.

We consider that the two main categories of actions (generic and content-focused) have a progressive character: The more content-specific they become, the more the TEs obtain meaning in terms of teachers' awareness of mathematics and science concepts and processes and their co-deployment in workplace situations. The appearance and intensity of the various actions varied among the five cases of TEs. To some extent, not all the actions appeared in all TEs' PD activities. For instance, TE4 and TE5 developed content-focused actions mainly related to their own discipline and their group did not develop cross-disciplinary designs.

Development of TEs' Activity While Dealing with an Epistemological Contradiction: the Case of TE1

We trace TE1's activity in managing this contradiction.

Initiation of the Contradiction by TE1. From her previous research collaboration with a colleague from science education, TE1 was aware of the epistemological distance between science and mathematics. In particular, she knew about the fragmented ways of teaching common concepts in mathematics and science (e.g. volume, periodicity). She also had experience in working in communities of inquiry with mathematics teachers. However, in the project, she was motivated for the first time to take the challenge of promoting inquiry and collaboration between teachers of the two disciplines. In this direction, TE1 introduced the epistemological distance in the group discussions from the second PD session when the teachers presented their initial ideas for planning their lessons. In this session, TE1 undertook content-focused actions and made explicit her view about differences between science and mathematics in modeling tasks and in their relation to realistic context: "While mathematicians tend to eliminate contextual factors in modeling tasks, scientists usually take them into account since context is a coherent part of the scientific phenomenon" (TE1, 2nd PD session). The discussion about experimenting in science and inquiring in mathematics also appeared in the other PD sessions. Below, we describe and analyze two extracts: One concerns a design-phase-related discussion between teachers and TE1 (3rd PD session); the other is in the reflection phase (4th PD session).

Extract 1: Dealing with the Contradiction in the Design Phase. Two science teachers (ST1 and ST2) have prepared a task about a rope breaking during a bungee jump. In this context during the third PD session, they discuss the elastic behavior of a metal string and the breaking limit. They plan to set up an experiment where the students will explore the law of Hook by hanging weights on a spring. For safety reasons, the teachers will demonstrate another experiment to show how a string's elasticity changes before it breaks. The first reaction of the mathematics teacher (MT1) is that mathematics is trivial. TE1 initiates an exchange of ideas that the science teachers follow. Then, the discussion focus is on the concept of ratio as a possible connection of the proposed task to mathematics. Next, TE1 asks the science teachers to explain the phenomenon. They clarify and detail the two experiments. As MT1 does not participate in the discussion, TE1 calls attention to what is expected of the students and references common tools used in science and mathematics, such as graphs and measurement instruments. MT1's immediate response is: "It is not an open-ended task that could support inquiry." A period of lengthy interactions follows where TE1 primarily takes content-focused actions to provide specific ideas on how the task can be extended beyond students or teachers merely performing the experiments by suggesting specific questions that promote the passage to mathematics: "You can use different materials leading to different relations and graphs and pose questions such as 'How can you interpret the diversity of the graphs?' She then adds, "It seems to me that discussion about the breaking limit for different materials can be connected to important mathematical concepts" (turn 324). For some time, ST1 and ST2 continue focusing on the experiments and the variety of materials given to students such as strings, springs and ropes. This way all teachers consider that the exploration can become more open. In the following dialog, a ground for co-designing seems to emerge. TE1 refers directly to the mathematical models and concepts related to the phenomenon and asks the science teachers to confirm their relevance. She challenges ST1 and ST2 to consider this dimension in their planning.

TE1: This is an interesting topic. Maybe this behavior of the wires can be modelled through mathematics. I do not know how.

ST1: As far as I know it cannot be modelled.

TE1: Only its linear part.

ST1: Yes. I do not know about the non-linear part of the graph …

TE1: We can also observe how the graph behaves. Is it increasing, decreasing?

ST1: Maybe MT1 can see what you suggest. He can choose different graphs.

ST2: And possibly ask students to interpret them.

TE1: Yes, what the graph means.

MT1: The students can compare the graphs and choose the ropes according to their elasticity and endurance.

Through this interaction MT1 starts to pose questions to the science teachers about the phenomenon and how the underlying relations are linked to graphs.

Then TE1 shifts to more generic actions addressing the school context conditions that could facilitate or hinder implementing a common task with the same students: "Do you have the same students? It would be interesting to engage them in both mathematical and scientific dimensions of the task" (turn 383). MT1 wonders if 10th grade students have the prior knowledge about function that would allow them to engage with the task. TE1 again uses content-focused actions to challenge the teachers to think beyond the content and consider the added value of the approach in terms of the learning outcomes for students: "It deserves taking a global view of graphs and building on the modeling process to engage students in doing mathematics in the scientific context" (turn 390). ST1 questions the potential of the task to challenge students' conceptions of linearity. TE1 highlights that this conception also exists among students in mathematics (turn 392). This point appears again later when TE1 highlights the role of modeling in mathematics and science: "It is very important to realize that a law in physics models a phenomenon under certain conditions. I think that this is important in mathematics as well" (turn 418). After this meeting, the group of ST1, ST2 and MT1 co-designed and implemented the task *Ropes and strings* together in a 3-h classroom session.

Extract 2: Dealing with the Contradiction in the Reflection Phase. In the fourth PD session, the three teachers present their joint implementation. Initially, their comments are rather general such as, "The students enjoyed the lesson and they understood the main ideas." Adopting both generic and content-focused actions, TE1 challenges teachers to inquire into students' thinking and provide evidence of their claims. The distance between mathematical and scientific activity becomes evident through specific references to how tools, conventions, and processes operate in the two disciplines. For example, the teachers observed that: the conventions of graphs are different in science and mathematics; the concept of rate has units of measurement in science, while in mathematics, it is an absolute number; and evidence is conceived differently in mathematics and science (proof versus experiment). TE1 builds on teachers' observations and supports their reflection by challenging them to broaden their views about the differences and interrelations of mathematical and scientific concepts and methods. This is carried out by a series of actions focusing on epistemological issues in science and mathematics and supporting teachers to inquire into teaching. For example, TE1 emphasizes the epistemological differences and similarities in mathematics and science as regards: diagrams and graphs (turn 71, "The discussion among the teachers is very interesting as they realize how differently diagrams are handled in mathematics and in science"); argumentation and proof (turns 77, 124–127, "How you verify a conjecture is different in science than in mathematics"); trial and error (turn 109,"Trial and error is not a process that is developed in mathematics. Can you think where we see it in mathematics?"); inductive process (turn 121, "The inductive process is common in mathematics and science"); intuition (168, "Intuition is not only in science but in mathematics"); and operational versus structural understanding (151–153, "The most important thing in both science and mathematics is to emphasize the processes and not only the procedures"). She also brings examples from research to interpret teachers' observations (161, 183, "It is true that the transfer of knowledge from one context to the other is not straightforward. Researchers claim that knowledge is situated"). In this extract, the interaction between TE1 and the teachers is more dialogic than in extract 1. The teachers bring examples from the lesson and formulate claims while TE1 challenges some of their claims or extends them by posing questions to consider. Thus, the classroom observations provide a common ground for the teachers and TE1 to develop more integrated conceptualizations of mathematics and scientific activity in teaching. In the last two PD sessions, the teachers continue presenting their designs and reflect on their classroom implementations. In these sessions, TE1 emphasizes the need to take a research-informed stance towards studying teaching and learning through selecting and analyzing critical events. Through this, TE1 attempts to bring the discussion about the epistemological distance closer to teaching and learning from a research-oriented perspective.

TEs' Professional Learning

All TEs encountered different kinds of contradictions and faced difficulties and dilemmas on how to handle them as we have illustrated in the previous section. Due to their limited experience in handling similar issues, TEs sometimes found it difficult to achieve cross-disciplinary collaboration or address emerging conflicts between school mathematics and science (e.g. diverse curriculums). Thus, in some cases (e.g. TE4 and TE5) allowed teachers to design tasks individually or jointly with teachers from the same discipline. Moreover, TEs' emphasis on diverse methods and ways of knowing in the two school subjects often discouraged mathematics and science teachers from engaging in cross-disciplinary activities.

In terms of TEs' professional learning, all TEs recognized the need to adapt their prior teacher education practices and started to question their existing approaches. They also appreciated the critical role of epistemological differences between the two disciplines in their mostly ineffective attempts to manage mathematics and science teacher collaboration. An exceptional case is TE1 who performed all types of actions and managed a cross-disciplinary collaboration by providing a synthetic view of the different epistemologies of the two disciplines. When analyzing TE1's actions in handling the epistemological distance between science and mathematics teaching and learning, we see that TE1 makes explicit this distance to the teachers from the first meetings. She also uses content-focused actions that mainly illustrate similarities and differences between science and mathematics aiming to encourage co-designing and deeper reflection on the implementation. To support teachers in co-designing, TE1 provides examples of epistemological similarities and differences and directs teachers to ways that they could extend a science task to integrate a mathematical dimension. In the reflection phase, TE1 builds on specific examples that the teachers bring from the classroom and promotes connections between mathematics and science teaching at a more theoretical level. Generic actions such as focusing on students' activity, supporting teachers to identify specific critical episodes as evidence from their observations, seem to interrelate with content-focused actions. Encouraging teachers to select critical episodes indicates TE1's professional learning as she moves from more guided ways to handle the epistemological contradiction to more open ways to engage teachers in becoming aware of it through reflecting on specific events of learning and teaching. In reference to the activity system of teacher education, TE1 brings tools from the community of mathematics teacher educators and researchers, like the use of critical events, the research evidence, the focus on students' activity and the inquiry stance of teaching and learning mathematics. Her own participation in research that links mathematics and science learning and teaching has made her aware of the need to support such a goal, of the different rules that exist in the communities of mathematics and science teachers, and of common tools like concepts, processes, and methods involved in mathematics and science teaching and learning. The institutional context and its rules (curriculum, textbooks, and school hours/schedule) are also elements that TE1 had to take into account to make the co-designing possible. TE1's participation in the abovementioned communities provided her with tools to handle the teachers' reluctance to collaborate. Additionally, the context of the PD sessions and of the classroom implementations provided specific examples that acted as boundary objects between the communities of mathematics and science teachers. Integrating these examples with research-informed tools seems to have had more relevance to teachers towards the end of the PD sessions.

In terms of expansive learning, we see new elements in TE1'a practices. These are characterized, for instance, by her conscious attempts to expand inquiry practices that were already familiar to her in the mathematics teacher education context into the new conditions by actions and goals. This allows her to promote the synthesis of the different epistemologies in mathematics and science teacher collaboration. We identified similar expansions in all TEs' activity. Questioning their teacher education practices was apparent in all TEs' meetings. Even the non-experienced TEs promoted cross-disciplinary inquiry. For example, TE3 brought workplace ideas and tools to engage both mathematics and science teachers in the modeling process. Others brought tools based on their research experiences that met teachers' common interests and facilitated their collaboration. For example, TE2 encouraged the use of digital tools to promote workplace simulations and joint explorations. In TE5's group, where science and mathematics teachers opted not to co-design, the TE drew teachers' attention to the epistemological distance between school mathematics and workplace mathematics, or school science and workplace science and also created opportunities for all teachers to discuss the distanced critically. TE4, who also did not achieve cross-disciplinary collaboration in her PD groups, became more conscious about the particularities of teachers' professional development in mathematics and science: "I learned a lot through collaborating with science teachers, I came to realize many more conceptual connections to mathematics but also in terms of inquiry practices between mathematics and science." (TE4, 6th PD session).

Discussion and Conclusions

This study aimed to obtain a deep insight into five TEs' activity to facilitate mathematics and science teachers' collaborative inquiry while tackling tasks situated in the workplace. The actions that emerged from the data analysis brought to the fore two main categories (generic and content-focused) that seem to have a prominent role in achieving the object of the activity. The generic actions are more related to the formation of conditions that seem crucial in allowing inquiry and collaboration among teachers. For example, supporting teachers to collaborate and participate in co-peered discussions are actions that promote both the establishment of a community of inquiry (Jaworski, 2006) and the dialogic inquiry in teachers' work (Nelson & Slavit, 2007). In contrast, the content-focused actions seem to directly address issues of teachers' engagement in co-designing, and respond to particular contradictions that stem from existing epistemological differences between mathematics and science in terms of content or teaching practices. The use of examples that aim to emphasize epistemological aspects, modeling situations within the context of science or providing support to integrate content areas from mathematics and science curriculum, are indicative actions aiming to exemplify how to make connections. Such actions are also acknowledged and highlighted in the existing literature (e.g. Frykholm & Glasson, 2005; Furner & Kumar, 2007). The theoretical background of this study, in the principles of Activity Theory, let us further connect these actions with particular elements of the teacher education activity system. These include engaged communities, their rules, and tools, and how these elements form the outcome of TEs' activity. A noticeable finding in our study is that although epistemological differences between mathematics and science learning and teaching appeared in PD sessions, TEs rarely handled the differences in a synthetic way. As TEs participated either in a mathematics or science education research community, it was highly demanding for them to deal with the diversity of epistemological orientations of mathematics and science that are considered important in achieving cross-disciplinary teaching (Davis et al., 2019). The special focus on the case of TE1, who managed rather effectively the aforementioned epistemological distance, helped us to recognize how the various categories interrelate and are affected by the rules and conditions of the communities in which a TE participates. Throughout the PD sessions, she moved from more guided ways of handling the epistemological contradiction to more open ones that engaged teachers themselves to cope with this contradiction. Moreover, this development reflected TE1's interactions with various elements of the activity system (research community, teachers' community, TEs' community, project tools, school-based tools etc.). Although not all TEs always achieved cross-disciplinary co-design and implementation, our analysis brings to the fore indications of shifts in teacher education activity. Working with mathematics and science teachers allowed all TEs to scrutinize their actions through evidence based on interactions between TEs and teachers and develop their understanding about the complexity of forming and sustaining a community of inquiry.

To sum up, we can argue that TEs' professional learning in the context of mathematics and science teacher collaboration involves addressing numerous challenges and complexities (diversity of disciplines, curriculum-based issues, TEs' research backgrounds, teachers' priorities, etc.). In order for the TEs to address these challenges and deal with the emergent complexities, they developed meanings of cross-disciplinary collaboration leading to deeper understanding of the underlying activity. Our analysis highlights factors that seem to be catalytic in supporting STEM teacher education: the context and the specific aims of the PD program that facilitate inquiry and collaboration among TEs and the STEM teachers (e.g. the workplace situations suggested by the project); TEs' teacher education and research experiences; and contextual elements like school conditions that allow or prevent cross-disciplinary task implementations (e.g. curriculum constraints). Beyond identifying factors that impacted TEs' learning, our study adds to the discussion on the content focused character of professional development (e.g. Prediger, Roesken-Winter, & Leuders, 2019) and highlights the need to reconsider its meaning in cross-disciplinary settings. For teacher educators, it is vital to become conscious of epistemological differences between STEM disciplines and develop competencies to address these in collaborative PD settings.

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