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Activity Theoretical Approaches to Mathematics Classroom Practices with the Use of Technology.

Part 1

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Activity Theoretical Approaches to Mathematics Classroom Practices with the Use of Technology – Part 1

Guest Editorial

Activity Theoretical Approaches to Mathematics Classroom Practices with the Use of Technology.

By Fabrice Vandebrouck, Giampaolo Chiappini, Barbara Jaworski, Jean-Baptiste Lagrange, John Monaghan and Giorgos Psycharis

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1 INTRODUCTION

Activity Theory (AT) is a philosophical and cross disciplinary theory adopted for studying various forms of human practices, such as teaching/learning, that are seen as developmental processes mediated by artefacts, where individual and social levels are simultaneously interlinked (Kuuti, 1996). It gives us a framework, which includes a terminology and notions associated with those terms to analyse and describe human activity and its transformation. This framework can be very useful for understanding and informing the design of technology-enhanced learning orchestrations in the domain of mathematics as well as conceptualising various mediations occurring in these situations and their associated outcomes.

In November 2011, a symposium, ATATEMLO (Activity Theory approaches to technology-enhanced mathematics learning orchestration), took place in Paris with the goal to share and integrate competences, methodologies and ideas of researchers in the field of AT and mathematical teaching and learning, in order better to address issues related to orchestrations of mathematics learning with technology. This special issue has emerged from that symposium and subsequent work by its participants as expressed further below. In this discussion document we introduce Activity Theory and its roots together with brief accounts relating a number of particular perspectives. We address also our focuses in mathematics learning and teaching with technology, introducing and explaining some key terms. This is followed by details of the work that has taken place in the symposium and beyond. We introduce a set of papers which were first presented at the symposium and which, after further review and revision appear here in the special issue.

2 ACTIVITY THEORY (AT) AND ITS ROOTS

Activity Theory has its roots in the classical German philosophy of Kant and Hegel, the dialectical materialism of Marx and the socio cultural and socio historical tradition of Russian psychologists such as Vygotsky, Leont'ev and Luria. AT is concerned with the historical development of activity and the mediating role of artefacts within it. The essence of Activity Theory is the dialectical transformations of individuals and their community as result of the involvement in an activity. This derives from the fact that human beings do not merely react to their life conditions but they are able to change the conditions that mediate their activities. Engaging in activity collectively not only increases action potential but also opens up a zone of proximal development for individual learning and transformation (Engeström, 1987). Hence, the study of the human activity and its changes is central to understanding how individuals learn.

In Activity Theory, an *activity* (or sometimes an *activity system*) is motivated by an object (the object of the activity), and it is the object that distinguishes one activity from another. In some texts, this object is referred to as the 'motive' of activity. According to Leonte'ev, *all* activity is motivated, even though the motive may not be explicit; when it is explicit it is referred to as a *motive-goal* (Leont'ev, 1979). Transforming the object, or motive-goal, into an outcome is essential to the existence of an activity. Subject and object form a dialectic unit, which is at the basis of change and of cultural evolution: the object transforms the activity of the subject and at the same time it is itself transformed by the psychological reflective activity of the subject.

More recently, Cole and Engeström (1993) conceived a systemic model expressing the complex relationships between elements mediating activity in an activity system, useful for studying the relationships that take place in teaching/learning activity with technological tools. Engeström (1999) acknowledged the "hidden curriculum" in which he recognised mediation through *rules*, *community* and *division of labour* (as well as through tools) as central to an activity system and contributory to tensions and contradictions in activity systems. Moreover, Engeström elaborated the notion of a *cycle of expansive learning* (Engeström and Sannino, 2010) to describe activity transformation processes which may determine a re-definition of objects, tools, and structures of the activity by participants able to promote conceptualisation among them according to a different and innovative perspective.

Wertsch (1991) refers to "goal-directed action" and writes, "human action typically employs 'mediational means' such as tools and language". He goes on to emphasise that "the relationship between action and mediational means is so fundamental that it is more appropriate, when referring to the agent involved, to speak of 'individual(s)-acting-with-mediational-means' than to speak simply of 'individual(s)'" (1991, p. 12). Leont'ev makes the following point, "in a society, humans do not simply find external conditions to which they must adapt their activity. Rather these social conditions bear with them the motives and goals of their activity, its means and modes. In a word, society produces the activity of the individuals it forms" (Leont'ev, 1979, pp. 47-48). So, according to Wertsch (1991, p. 27), rather than "the idea that mental functioning in the individual derives from participation in social life", "the specific structures and processes of intramental processing can be traced to their genetic precursors on the intermental plane". (Extract from Jaworski and Goodchild, 2006. See also Lerman, 1996).

Within this work, three generations of activity theory are recognized: the first deriving from Vygotsky, the second from Leont'ev and other immediate followers of Vygotsky in the Russian school, and the third a more recent set of conceptualizations from scholars more broadly, including Cole, Wertsch and Engeström.

3 ACTIVITY THEORY AND THE TEACHING AND LEARNING OF MATHEMATICS

AT gives us a framework, namely terms and notions associated with those terms, useful for describing the interactions emerging in the learning environment in relation with both the object and outcomes of activity. Using Activity Theory as a framework, we can state that the learning environment is constituted by the enactment of a teaching/learning activity oriented towards an object involving students, teacher, and artefacts; for example, the solution of a task, the reading of a document, the class discussion on a specific issue, etc. motivated by an object which might be students' mathematical knowledge development, or mathematical learning. The object of a teaching activity from the point of view of the teacher is a didactical objective, namely the students' acquisition of a specific knowledge or of an ability (Bellamy, 1996). Obviously, the student's involvement in an activity can be motivated by different objectives (to get good marks, to please her parents or the teacher, to receive a reward, and so forth). These 'objectives' contribute to the overall object or motive-goal of the activity. Studying the learning environment means studying how the elements and the relationships that characterise the teaching/learning activity oriented to a didactical objective can determine the expected outcomes. Studying the changes that learning environments undergo when technology-based artefacts are introduced means analysing how activity changes as a consequence of tool introduction and how this change is meaningful for the students and the teachers (Bottino and Chiappini, 2008).

Some of the terms we use in this volume perhaps need explanation. The first concerns the word *activity* itself. In some contexts there is confusion between Activity as in *Activity Theory* (often written with a capital A) and activity as in 'classroom activities' which is a general way of referring to things that teachers and students do in classrooms (often written with a small 'a'). An *artefact* is a material or psychological object (Wartofsky, 1979), often something that is made by humans for a specific purpose, or in other words some cultural and historical human construct. An artefact becomes a *tool* when it is used by an agent, usually a person, to do something. The use of the artefact depends on the object of its user. For someone who has no object in mind, the artefact remains an artefact (but in that case there is no activity). So, seeing the artefact as a tool requires both the artefact itself and some instructions for its use, i.e. the user's way of using the artefact, sometimes referred to as the user's *schemes of utilization*. Another term we shall use in connection with artefacts and tools is the *instrument* which can be seen as the artefact together with its schemes of utilisation. An *instrument* is a combination of the characteristics of an artefact and of utilization schemes relating to tasks involving the characteristics (the instrumental genesis, see below). Rabardel (2003) writes as follows:

> … artefacts are not given to users in the sense that they can pick them up and use them. Rather, through a series of adaptations and customisations users instrumentalise the system – i.e., they transform it into a functional unit which consists partly of the properties of the artefact and partly of the attributes of the user.

In this special issue, we talk about our use of *technology*, and by this we refer to some discourse or reflection on techniques in the use of artefacts as tools. The term *digital technologies* is used to refer to artefacts such as computers, calculators, etc. as well as visions, reflections, etc. on their use. Thus, when we use the encompassing word "technology" we include use of artefacts which go beyond such digital technologies. An example might be the grouping of students to achieve a discursive environment. We talk about *technology enhanced learning orchestrations*. By *orchestration* we mean the process of productively coordinating supportive interventions across multiple learning activities occurring at multiple social levels (Fischer and Dillenbourg 2006). These include classroom orchestration of technology involving students and teachers as well as other settings where human beings can learn mathematics from various sources and collaborations.

4 DEVELOPMENTS OF AT BY THE FRENCH SCHOOL[1]

The ATATEMLO symposium originated through the interests of French researchers who have used AT extensively for some years. The AT proposed by Leont'ev (1978) was particularly expanded during the two past decades in the French community of research in work psychology (cognitive ergonomics) and professional didactics.

In this context, AT starts with the theoretical distinction between task and activity (small 'a') and focuses on the individual as a subject and an actor in her[2] activity (individual activity is considered as being embedded in a

[1] This paragraph has been written mainly by Janine Rogalski

[2] "She", "her" and "her" were chosen for avoiding a more "neutral" plural "they", "them" and "their" which could have been confusing with regards to Engeström's theory of activity (system).

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collective system). In work as well as in training situations, both task demands (in the context of a given situation) and existing competence[3] co-determine the subject's activity. The dynamics of activity produces feedback effects in a twofold regulation loop (Leplat, 1997; Rogalski, 2004). On the one hand, the object of the task[4] is modified, giving rise to a new condition for the activity (a new task or the pursuit of new actions for attaining the initial task goal). On the other hand, the subject's own knowledge is modified: the discrepancy between expectations and results of action exerts a pressure for adapting her activity in the short term, and the resulting experience may modify her knowledge in the long term.

Instruments – concepts, language, symbolic representations – elaborated in previous socio-historical human development – ensure a crucial mediation between the subject and the object of her activity, while human mediation introduces another social dimension in development, particularly important in training and learning situations. It may be a direct human mediation: for example, in Bruner's theory of instruction (Wood, Bruner & Ross, 1976), through a process of "scaffolding", the student's activity in performing a task in his "zone of proximal development" (ZPD; Vygotsky, 1962/1986). It can also be indirect, organized through technological environments (digital technologies), as in Rabardel and Vérillon's theory of instrumental genesis (Vérillon and Rabardel, 1995). The word 'instrument' refers to a distinction between a tool (a material object) and an instrument (a psychological construct). A tool is just an artefact until someone attempts to integrate it into activity. But tool and person are interrelated: the tool shapes the actions of the person (instrumentation) and the person shapes/uses the tool in specific ways (instrumentalisation). The process of turning an artefact into an instrument is called "instrumental genesis".

In the 1990s predominantly French researchers, under the intellectual leadership of Michèle Artigue, developed an approach to understand the complexities of attempts to integrate technology into 'normal' classrooms. This approach included the notion of instrumental genesis, developed by Lagrange (1999) and Guin and Trouche (1999).During the 2000s it became apparent that this approach included two distinct theoretical frameworks: the 'ergonomic approach', see Vérillon and Rabardel (1995); and the anthropological approach (AA), see Chevallard (1999). The ergonomic approach is concerned with the agent-artefact dialectic of turning an artefact into an instrument to execute a task. AA focuses on "praxeologies" (idiosyncratic practices) via four elements in two pairs: tasks and techniques; technologies and theories. Monaghan (2007) details these approaches and discusses their differences.

[3] Competence is a complex organization involving operational knowledge, schemes of action, perceptive, motor, and emotional characteristics, which is "locally stabilized". The subject's state is a more contextual determinant.

[4] It may be a material object, a symbolic one, or may concern a human being (e.g., as a trainee).

An AA lens focuses researchers' minds on the importance of techniques in mathematics (with and without the use of technology), the role of 'institutions' and values associated with these techniques and embedded in institutions, as Lagrange (this issue) discusses. Instrumental genesis was initially used to study the consequences of technology-rich environments for the students' learning. It has been developed in several directions more recently in which artefacts are considered not as *isolated*, but as inscribed into *systems* of artefacts. This leads to notions of *instrumental orchestration* (Trouche, 2004) and *documentational genesis* (Gueudet and Trouche, 2009) for describing teachers' activity and use of resources supporting students' instrumental geneses. These developments lead us to take into account teachers' own geneses; considering these geneses requires a holistic view on the teachers' activity (Abboud-Blanchard and Vandebrouck, this volume). Robert and Rogalski (2005) also developed a didactical and ergonomic approach to study teaching practices and learning situations initially not devoted to technological situations but now adapted to such situations (Robert, this volume) .

In further developments of professional didactics, AT was expanded through the introduction of a distinction between "productive" activity and "constructive" activity (Samurçay and Rabardel, 2004). In fact, in its *productive dimension*, activity is object-oriented – toward the work process. In its *constructive dimension*, it is subject-oriented: the subject's object is to develop or at least preserve herself (her competence, health...). The relationships between these two dimensions differ depending on the type of situations. In work situations, the productive dimension is crucial. The possibility for the constructive dimension to be deployed is linked to the developmental opportunities open in the situation as well as to the subject's intentions. In education and training situations, the constructive dimension in the student's activity is a key point. However, if the educator/trainer is conscious of the fact that tasks are proposed to the student in order to develop her competence, and particularly her conceptualisation, it quite often happens that this focus is absent in the student's intentions, centred on the task as the only object of her activity, or aiming at scholarly success and not at understanding. In fact the teacher's mediation not only concerns the productive dimension of student's activity but also its constructive dimension. Pastré (1997) stressed this point when developing the role of debriefing in professional training for focusing on the trainee's activity and her conceptualisation and not only on the success or failure of her task performance.

5 OTHER AREAS OF THEORY RELATING TO ACTIVITY THEORY AS REPRESENTED IN THE PAPERS WHICH FOLLOW

Several researchers in mathematics education, focusing on teaching and learning with use of technology, have used some of the above AT concepts in order to address specific issues linked to technology-enhanced orchestrations in mathematics classrooms, especially the role of the teacher in organizing and conducting technology-based lessons. These works are relevant, but remain largely unconnected. In parallel, the functioning of a mathematics classroom using technology, and more generally the complex orchestration of learning with technology, remains very problematic (and this problem is one reason why researchers initiated the ATATEMLO symposium). We believe that AT approaches offer tools to address learning orchestrations in terms of dynamical features of the activities to be carried out by learners and teachers. These features are related with new elements involved in classroom orchestration, such as the nature of the tasks for the students: new forms of interactions occurring in computer-based pedagogical settings offer more opportunities for experimentation with dynamic and interconnected mathematical representations and with the challenge of integrating these aspects into teachers' activity.

Recent reflections amongst researchers have expressed concern around a fragmented and unconnected emergence of constructs and frameworks, each inevitably bound to the context within which it was produced and used. When seen from the outside, the knowledge emerging from the mathematics education research community can seem fuzzy and its growth slow. Researchers recently have highlighted the need to find ways to generate connections amongst frameworks and constructs, to enhance their explanatory power and the efficiency of communication of research results (Sierpinska et al., 2002; Artigue, 2009). This has been one of the goals of the ATATEMLO symposium.

A number of ATATEMLO papers have attempted to network/combine/use/integrate different theoretical approaches, or tools/constructs of different theoretical approaches (which can be categorised as activity theoretic approaches or simply have some similarities to activity theoretic approaches). Thus, articles submitted in the symposium ATATEMLO can be seen as 'position papers' where the 'position' is, quite often, comparing two related approaches. Sometimes such attempts lead to the construction of new models/tools/constructs/frames for studying particular aspects of technology integration and/or use in mathematics classrooms.

For instance, under the lens of the Theory of Semiotic Mediation (TSM), Mirko Maracci and Maria Alessandra Mariotti provide an explicit model – consistent with the activity-actions-operations frame – of the actions which are expected from the teacher in order to make mathematics learning occur through the mediation of an artefact. The teacher's actions are identified by their goals and are related to the motive of the teaching-learning activity. Jean-Baptiste Lagrange evaluates the contribution of the anthropological approach (AA) concurrently to AT in view of overarching questions about classroom use of technology for teaching and learning mathematics. He uses particular tools/notions of the above theories to analyse a particular research study. Maha Abboud-Blanchard and Fabrice Vandebrouck use a number of theoretical developments of AT in the French didactic and research context (i.e. AT in cognitive ergonomics and double didactic and ergonomic approach) to build a frame for better understanding the evolutions of teachers' practices and to interpret them in terms of geneses of technology uses. Chronis Kynigos and Giorgos Psycharis attempt to discuss possible connections between constructionism and instrumental theory (a direction of AT in the French school of cognitive ergonomics). John Monoghan uses theory from Valsiner, a development psychologist who developed his 'zone theory' in the 1980s. The starting points for this theory were children's actions and Vygotsky's ZPD. Valsiner's 1987 book (Valsiner, 1987), referenced in the paper by Monaghan in this volume, does not explicitly refer to activity theory but his focus on actions and the ZPD certainly places him in a related (if not the same) field as activity theorists.

It seems important to connect these examples with the international discussion concerning the networking between theoretical perspectives, and the necessity of investigating the exact role played by theoretical frameworks in design and use of technological tools in mathematics classrooms. One point for reflection can be: What characteristics/ideas/concepts of AT – considered as "multi-voiced" theory (Engeström, 1999) – give AT a high level of connectivity with a number of different approaches/frameworks and thus contribute to the development of a core of shared "ideas" or "concepts" suitable to study the teaching and learning of mathematics with the use of technological tools? Or, in other words: how are the current developments of AT related to its use in the field of technological tools in mathematics classrooms? Why does AT seem to play the role of an umbrella under which so many different approaches are situated?

Such questions are addressed more concretely in the context of particular papers. For instance, we can compare/contrast the different ways by which the proposed ATATEMLO approaches consider the teaching and learning of mathematics based on the use of technological tools. In the case of Maracci and Mariotti who elaborated the notion of mediation in relation to teachers' practices in mathematics classrooms based on the use of technology, the actions which are expected from the teacher are connected to the signs produced by the students' interaction with the artefact and to the mathematical signs related to the official mathematical knowledge. Other approaches, however (e.g., Kynigos and Psycharis, to appear in the next volume of this special issue), consider mathematical meanings as contingent on the available computational tools in their specific context of use and seem to concentrate on the mathematical meanings emerging directly through the students' interaction with the available tools.

Saxe (1991) developed a cultural approach with three components which Lagrange (this issue) describes. The first component centres on emergent goals that arise in practices (any practice) and posits four parameters (social interactions, prior understandings, tools/conventions and activity structures) which impinge on practice-linked emergent goals. Saxe's approach has been used in studies examining teaching with technology, for example, Monaghan (2004). Saxe's approach is not explicitly activity theoretic though Monaghan (2004) and Lagrange (this issue) both view it as having strong links to activity theory.

These considerations take us into the set of papers which comprise this special issue, since each paper addresses these issues in its own ways and to differing degrees.

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6 THE SYMPOSIUM AND THE BASIS OF THIS SPECIAL ISSUE OF IJTME

In advance of the symposium, it was stated that outputs of the symposium were expected to contribute to existing research at two levels:

1) the level of theoretical work in technology enhanced mathematics learning (e.g. integrating research approaches following an AT perspective, highlighting the role of AT concepts in designing, implementing and analysing research studies in the field of mathematics education);
2) the level of operational approaches for orchestrating and instrumentalising mathematics learning with technology and for taking critical aspects of context into account.

Each contribution to the symposium was expected to address one or more of the four questions:

1) What key aspects in the complexity of technology enhanced learning (TEL) situations does your AT framework help to address? What tools does it provide to support teachers and designers?
2) How does your AT framework help to conceptualise the changing roles and relationship to mathematics knowledge by learners and teachers?
3) How does your AT framework help to conceptualise constraints specific to the orchestration of learning, like the necessity of assessing and certifying knowledge and skills in the context of digital technologies?
4) How does your AT framework help to take into account the ways in which TEL orchestration are mediated by the context.

Participants at the symposium were asked to read all papers in advance. At the symposium participants offered a brief introduction to their paper which was followed by contributions from two respondents and discussion of concepts and issues. After the symposium authors were invited to modify their papers drawing on respondents' comments and to submit a 4000 words version for consideration for this special issue. These papers were sent to three reviewers, including one from the scientific committee who fed back responses to authors. Authors were invited to revise papers according to the recommendations and to submit a final version for publication in the special issue. All but four of the original papers were thus re-presented in this way.

Members of the scientific committee categorised these papers into themes and organised the themes into sections as you will find below.

7 THE CONTENTS OF THIS SPECIAL ISSUE

Here we introduce briefly each of the set of papers comprising the special issue. This discussion document and the first five papers comprise the first half of the special issue. The second part comprises six papers and a brief conclusion.

The first paper by Giampaolo Chiappini, examines the construct 'cultural affordance' with regard to the software Alnuset, designed for teaching and learning algebra. The construct 'affordance' refers to an animal-environment relationship, what the environment offers (affords) the animal in terms of actions; a broom closet affords a small dog to lie down but it may not provide this affordance to a large dog (see Gibson, 1979, for an authoritative introduction). Affordances are commonly referred to in mathematics education research, largely due to the work of James Greeno (see Greeno, 1994). Affordances have also been taken up by activity-theoretic researchers in the field of human-computer-interaction (HCI) and it is the work of two HCI researchers (Turner and Turner, 2002) that is the springboard for Chiappini's paper. The Turners introduce the construct 'cultural affordance'. This resonates with Vygotskian cultural psychology whereby learning mathematics involves learners appropriating "historically constituted cultural significations and forms of reasoning and action" (Roth and Radford, 2011, p.48). Chiappini considers the potential of Alnuset for learner transformation, in the sense of Engeström's 'expansive learning' cycle, of affordances for embodied actions to affordances for "historically constituted cultural significations".

In their article, Maha Abboud-Blanchard and Claire Cazes use activity theory to analyse interactions between teachers and their students using a computer environment, *Electronic-Exercise-Bases* (EEB). Elements of Engestrom's expanded meditational triangle are used to address the questions: why and how do teachers use EEB? What effect does this use have on their teaching activity? Data were gathered and analysed to show teachers' preparation for a lesson, teachers' activity with students during a lesson, and teachers' reflection after a lesson. Tensions are revealed between the tool used (the EEB) and the subject or the object in the AT system; in teaching terms these refer to teachers' wish to control the students' activity contrary to specificities of the EEB and teachers' focus on mathematical process in contrast with the EEB's focus on answers only. Elements of rules, community and division of labour are useful in highlighting the wider influences on the teachers encouraging them to use the tool despite their reservations on its use.

Barbara Jaworski, Carol Robinson, Janette Matthews and Tony Croft use the expanded triangle of human activity elaborated by Engeström and three levels of human activity elaborated by Leont'ev to analyse the teaching of a first year mathematics module for engineering students based on an innovative approach that comprises the use of inquiry-based tasks and a computer environment (GeoGebra). The two models rooted in the AT enable the authors to make sense of the conflicts which emerged, in the teaching activity experienced, between the intentions of this approach to teaching mathematics and students' responses, engagement and performance. In their article the authors use these two models to provide insight into the contradictions emerging through the observation of teaching situations, to juxtapose the key elements of the areas of conflict and to emphasise the different ways in which the nature of the teaching activity is perceived by teachers and students.

The paper by Aline Robert draws an Activity Theoretical frame specific to mathematics at school with reference to both Vygotskian and Piagetian approaches[5]. At a local point of view, the frame is oriented toward analysis of students' mathematical activities in the classroom. This local point of view is extended to a global point of view, to gain access to what happen in the classroom, linking students and teachers' activities and the context of the whole activity system. This Activity Theoretical frame is named "Double Approach", referring both to didactical tools coming from the field of didactics of mathematic and also to an ergonomic approach, coming from cognitive ergonomics. This double perspective has been developed during the last twenty years (Robert and Rogalski, 2005).

The paper by Maha Abboud-Blanchard and Fabrice Vandebrouck deals with teachers' practices in technology-based lessons. The authors used a number of theoretical developments of Activity Theory in the French didactics and research context (i.e. AT in cognitive ergonomics and a twofold didactic and ergonomic approach) to build a frame for understanding better the evolutions of teachers' practices and to interpret them in terms of geneses of technology uses. The authors consider these geneses as movements which articulate three levels of organization of practice: a micro level, a local level and a macro level. The analysis highlights also internal and external aspects of teachers' activity in terms of teachers' instrumental genesis of specific artefacts. The main contribution of the paper is that it provides an activity theoretic framework which can be used to grapple with the complexity of the emergence and evolution of teachers' practices in technology-based mathematics lessons. Thus, it contributes to our understanding of teaching development.

These five papers (above) conclude the first part of the special issue. The six papers summarised below can be found in the second part.

The paper by Silke Ladel and Ulrich Kortenkamp addresses an Activity Theoretical framework for capturing the complexity of learning processes in environments exploiting multi touch affordances increasingly offered by digital artefacts. This frame, called ACAT (Artefact Centric Activity Theory), is adapted from Engeström's frame and it guides the design activities with the artefact. It can been seen as an attempt to enrich the model, first of all by discussing the way it addresses the role of the artefact and secondly by mixing Activity from a systemic point of view and Activity from a more psychological point of view. It addresses the ways a student can internalise and externalise concepts regarding an object (in this case, numbers) via an artefact (the multi touch environment) and relies on the Instrumental Approach and Instrumental Geneses developed by Rabardel in the French community.

In the article "Mediation of a teacher's development of spreadsheet as an instrument to support pupils' inquiry in mathematics" by Anne Berit Fuglestad, the author uses instrumentation theory and the Engeström expanded triangle of activity to analyse a case of collaboration among teachers aiming to utilise a spreadsheet to orchestrate for pupils' investigations. The focus of the analysis is on one teacher in the team and his development of the spreadsheet as a tool. The model of Engeström is used by the author to describe the main aspects of the collaborative activity where the case study takes place with the aim to achieve a better comprehension of the instrumental genesis processes involved in the teacher work on task design for students. In particular the AT framework is used to conceptualise constraints specific to the orchestration of learning and the ways in which orchestrations of technology are mediated by the context.

The paper from Chronis Kynigos and Giorgos Psycharis connects the idea of "reciprocal shaping" between the learner and the tool on the one hand, and the instrumental genesis of mathematical instruments on the other hand. Hoyles, Kent and Noss (2004) pointed out that situated abstraction has the potential to complement the idea of a process of instrumentation, "shaped by the tool", as means to precisely state how mathematical knowledge is constructed in this process. In a similar way, the authors consider the parallel between instrumentalisation and the idea of "shaping of the tool by the learner". Design of tools and of classroom situations is an important dimension in the instrumental approach. The authors take the issue of design in a constructionist perspective, different from the "Theory of Didactical Situations" or "Didactical engineering" perspective generally adopted by researchers referring to instrumental genesis. They claim that their perspective helps to "design for instrumentalisation", that is to say it provides a priori tools so that intrumentalisation by the learner will happen in particular ways conducive to the generation of meanings. In support for this claim, they report on an experiment where the tool and the task were particularly tuned in order that productive intrumentalisation occurs. The analysis shows how the design actually promoted intrumentalisation processes interacting with instrumentation in a fruitful instrumental genesis.

Under the lens of the Theory of Semiotic Mediation (TSM), the article by Mirko Maracci and Maria-Alessandra Mariotti provides an elaboration of the notion of mediation in relation to teachers' practices in mathematics classrooms based on the use of technology. In particular, the idea of mediation has been employed to address the potential of a specific artefact to foster mathematics learning processes. Based on assumptions of TSM for the development of mathematical knowledge through the use of artefacts, the authors investigate the teachers' intentional engagement in semiotic mediation processes with the aim to bridge the gap between the students' use of the artefact for accomplishing a particular task and the mathematical knowledge at stake. The paper provides a framework that can be used to raise teachers' awareness as to the actions they need to put into practice (i.e. in classroom discussions) in order to enable students to link their experience of tool use to the targeted

[5] In the debate "divergence vs convergence" concerning Vygotsky's and Piaget's theories about development, convergence and complementary were emphasized, for instance, by Cole and Wersch (2001), as well as in the domains of professional didactics (Rogalski, 2004).

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mathematical knowledge. It acts as a 'position paper' in which the 'position' is coordinating two related approaches: TSM and an activity theoretic approach based on the activity-actions-operations frame.

In his article, John Monaghan uses a sociocultural framework based on Valsiner's zone theory to consider the potential of an analysis of a teaching development resulting from a non-ICT innovation, onto research using ICT in classrooms. Valsiner's theory, introducing zones of "free movement" and "promoted action", is not explicitly linked to activity theory, although it develops from the same origins in the work of Vygotsky, particularly the concept of zone of proximal development. Zone theory offers the potential to capture aspects of both teachers and students' learning over time and ways in which these may be related to each other.

The paper by Jean-Baptiste Lagrange compares the usefulness of the anthropological approach (AA) with that of activity theory (AT) for highlighting issues in the classroom use of (digital) technology. Classroom extracts from two teachers/classes using spreadsheets illustrate issues arising from the point of view of each approach. In the language of the CERME working group focused on theoretic approaches (Kidron, Bikner-Ahsbahs, Monaghan, Radford and Sensevy, 2012), this paper provides a concurrent comparative 'networking' of these two approaches ('networking theories' refers to connecting theoretical approaches while respecting their underlying conceptual and methodological assumptions). AT frameworks considered are those of Leont'ev, of Engeström and of Saxe. Lagrange argues that AA and AT "share a common view of knowledge as a product of a human activity", that AT can help us trace the evolution of knowledge and that AA helps researchers see the institutionally-based values associated with the transformation of knowledge.

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