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Activity Theoretical Approaches to Mathematics Classroom Practices with the Use of Technology – Part 2

Designing For Instrumentalisation: Constructionist Perspectives On Instrumental Theory

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In this paper we aim to contribute to the process of networking between theoretical frames in mathematics education by means of forging connections between Constructionism and Instrumental Theory to discuss a design for instrumentalisation. We specifically focus on instrumentalisation, i.e. the ways in which students make changes to digital artifacts and generate meanings in reference to these, as something which will not inevitably happen during activity with digital media. We discuss the issue of designing artifacts and corresponding activities in order to facilitate an instrumentalisation process which will be rich in the generation of mathematical meanings. We report findings from research aimed at shedding light on the meanings of angle in 3D space generated by 13 year olds pupils while using a specially designed Turtle Geometry microworld. The analysis indicates that connections between the two theories on the issue of designing for instrumentalisation enhances our efficiency to explore the links between tool design and students' instrumental genesis in technology-rich environments.

1 INTRODUCTION

Many researchers have recently begun to contribute to the networking of different theoretical frameworks in the field of technology enhanced learning of mathematics (for example, Artigue, 2009). Our aim in this study is to forge some further connections between Constructionism and Instrumental Theory by taking the issue of design in a constructionist perspective in distinction to the Theory of Didactical Situations (TDS) or "Didactical engineering" generally adopted by researchers referring to Instrumental Genesis (IG). By adopting this approach – which we refer to as "design for instrumentalisation" – we aim to further promote connectivity between the two theories by addressing the design of digital artefacts so that learners' uses of them may happen in particular ways conducive to the generation of meanings.

2 THEORETICAL FRAMEWORK

Constructionism and Instrumental Theory emerged through the study of ways in which technology can be designed and used to enhance mathematical learning. However, they have different historical roots and were developed in different research cultures. Constructionism gave special emphasis to the epistemology of mathematics and of learning mathematics and was developed as a vision of using digital media to generate leaning-by-doing cultures. It was originally associated with expression of mathematical ideas through programming with Logo-based Turtle Geometry and embodied the vision of students tinkering with digital models (Papert, 1980). The French ergonomic approach (i.e. IG, Lagrange, 1999, Artigue, 2002) on the other hand was developed in the direction of Activity Theory (AT) aiming to address activity that employs and is shaped by the use of instruments over a series of research studies concerning the didactic exploitation of Computer Algebra Systems (CAS). Reflections upon the constructionist vision (Healy and Kynigos, 2010) show how constructionist approaches have progressed over the years from a primary interest in the individual learner to a growing focus on both individual and social aspects of learners' appropriation of a tool in different communities of practice. Despite this disparity in the two theories, they both suggest that there is a mutual transformation of learners and artefacts in the process of constructing knowledge with technology. This connection was pointed out some years ago (Lagrange, 1999, Hoyles, Noss and Kent, 2004), i.e. that Constructionism and IG share a common consideration concerning the two sided relationship between tool and learner. In particular, IG elaborated a language to describe student-tool "reciprocal shaping" as a process leading to the integration of the tool in students' practices for the accomplishment of particular tasks. In a similar way, constructionist approaches focus primarily on student's constructions of mathematical meanings (i.e. *situated abstractions*, Noss and Hoyles, 1996) and the ways in which these structure and are structured by the use of available tools.

In this study we focus more closely on the "reciprocal shaping" process by looking at instrumentalisation as our first priority. How could the process of making changes to a digital artefact enhance meaning making? Furthermore, how can we design artefacts which invite meaning making through the process of changing them? IG was generally developed in a culture of design especially through the lens of TDS and/or "Didactical engineering". As far as tool design is concerned, although in the early steps of IG the design of artefacts was given by CAS designers (Monaghan, 2007), subsequent French CAS work has moved to developing tools integrating a CAS kernel in new systems that enhance students' engagement with mathematics targeting productive instrumental genesis by the students (Lagrange, 2005). Constructionism has also given an emphasis to the design particularly in the sense of design as a learning process where learners engage in individual and collective 'bricolage' with ownership and production at the core of the constructionist agenda (Harel and Papert, 1991).

In our research we thus found it useful to draw from both theories in order to think about issues related to designing artefacts for instrumentalisation, i.e. the design of digital media *so that* their use will enhance the likelihood of meaningful and rich instrumentalisation by the students. This approach involves students' interaction with artifacts explicitly developed to invite changes by the students and engage them in exploration and construction activities. Some years ago, Kynigos (2007) coined the term 'half-baked microworlds' to discuss artefacts designed so that students would change them in order, e.g. to 'fix' something which they see as a bug or to extend the behaviour of a model. Designing for instrumentalisation addresses both the design of tools for learning (e.g. by researchers) and the process of learning itself as a design process where students tinker with digital models in the role of engineers. In this paper we elaborate on the "design for instrumentalisation" idea through empirical results of students' instrumental genesis in the classroom as they worked with a set of activities we designed under a constructionist theoretical perspective. In our analysis we avoided considering instrumentalisation merely as a process which inevitably happens during instrumentation by asking how it may depend on the design and the nature of the activity and on the nature of the artefact. Our aim was to challenge students' intuitions and ideas concerning angle in space. We used a 3D Logo-Based Turtle Geometry software called MaLT (Machine Lab Turtleworlds) which contains two additional affordances firstly a variation tool allowing for dragging for continuous change to variable values in order to provide dynamic change to the figure generated by the corresponding procedure and secondly the dynamic change of the viewpoint in space allowing for a perusal around the generated model including zooming in-out. Traditionally, the teaching of angles at school revolves around static approaches of angle while the notion of angle as turn is usually underrepresented or done only through static 2D representations which may delay the development of dynamic aspects of the concept (Clements, Battista, Sarama and Swaminathan, 1996). In the reported experiment our purpose was to better understand how students might think of angular relationships between objects and angles as turtle turns in space. In our analysis, we considered both levels of instrumentalisation, our own designs as researchers (for a discussion see for instance Pratt and Noss, 2010) and the students' changes to the buggy procedure we gave them to investigate and fix.

3 THE COMPUTER ENVIRONMENT

As mentioned above, MaLT (Kynigos, et al., 2009) connects Logo-based Turtle Geometry with dynamic manipulation of variable procedure values and dynamic change of the users' viewpoint. The software provides three components available to the user at all times, the Turtle Scene (TS), the Logo Editor (LE) and the Variation Tools (Fig. 1). The Logo language (a version of MSW Berkley Logo built with Java) contains two additional kinds of turtle turn which make it a 3D environment, i.e. 'UPPITCH/DOWNPITCH n degrees' ('up/dp n') which pitches the turtle's nose up and down on a plane perpendicular to the one defined by right-left turns and 'LEFTROLL/RIGHTROLL n degrees' ('lr/rr n') which moves the turtle around its own axis.

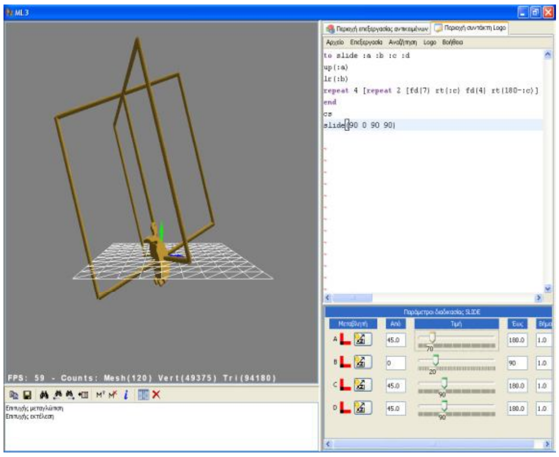

Figure 1 A revolving door simulation in MaLT.

MaLT affords dynamical manipulation of geometrical objects in two ways:

(a) Through the use of the Uni-dimensional Variation Tool (1dVT) whose main part consists of 'number-line'-like sliders, each corresponding to one of the variables used in a Logo procedure. Dragging a slider has the effect of the figure dynamically changing as the value of the variable changes sequentially.

(b) Through changing users' viewpoint of TS either by a toggle fashion (i.e. by using buttons to pick among 3 default views: front, side, top-down) (Fig. 3) or by dragging a specially designed vector tool, which we called 'the active vector', where the user can define the camera's direction or position.

4 THE EXPERIMENT

The reported design experiment (Cobb, Confrey, DiSessa, Lehrer and Schauble, 2003) took place in a secondary school with one class of twenty grade 7 students (13 years-old) and one experimenting teacher who also acted as a researcher. The class had in total 18 teaching sessions with the experimenting teacher over two months. The activity sequence was divided into two phases. In the first two tasks students were asked to freely navigate the turtle inside the TS and then to bring it back at its initial position or to navigate the turtle in such a way so as to simulate the take-off and the landing of an aircraft. In the next three tasks the students were given a dynamic procedure which contained a bug and were asked to fix it so that it created the representation of a dynamic object in 3D. In particular, in task 3, the students were asked to construct rectangles using the buggy procedures in at least two different planes of the TS simulating the windows of a virtual room. In task 4, the students were asked to develop a parametric procedure so as to simulate the opening and closing of a door.


```
to slide :a :b :c :d
up(:a)
lr(:b)
repeat 4 [repeat 2 [fd(7) rt(:c) fd(4) rt(180-:c)]
lr(:d)]
end
```

Figure 2 The Logo code of the 'slide' half-baked microworld.

In this paper, we analyse episodes of students working with task 5 when they were given a half-baked microworld in the form of a procedure called 'slide' (fig. 2). The 'slide' procedure was presented to the students as a means to create a revolving door with four orthogonally positioned rectangles simulating the door panels. They were told that it contained bugs since dragging the sliders did not result in such a model. They were asked to investigate its behaviour, find the bug(s) and fix it(them). The procedure was designed to have more than the variables needed. We wanted to see if the students would identify the role of each variable and make changes in the code so as to develop a simulation of a revolving door with the least possible number of variables. In this analysis we use two episodes selected (a) to have a particular and characteristic bearing on the students' interaction with the available tools and (b) to represent clearly aspects of the students' construction of meanings for particular aspects of angle in 3D space (i.e. geometric shape, dynamic amount and measure) emerging from this use.

5 ANALYSIS

5.1 Perspective taking in 3D space

In this section we look at the way the students started to change the viewers' perspective functionality in their efforts to understand the behaviour of the model as they changed the values of variable *d*. Early in their work during tasks 1 and 2 this group of students (group A) encountered difficulties in coordinating turtle turns and trace with the notion of angle as a slope. This episode finds the group of students engaging in investigating the role of variable *d*, which determines the measure of the angle between the panels of the revolving door. At that time the students began to make use of the tool provided to facilitate taking different views of the TS and thus of the current geometrical construction. The model is correct only when *d* takes the value of 90 plus a multiple of 180. The panels look as if they fold into one with multiples of 360. Fixing the bug would involve understanding that *d* should in fact not be a variable at all but instead just a constant value of 90. The students conjectured about the number of the visible rectangles (doors) if the value given to *d* was 720. However they did not find the front default view convenient and after testing all the available default views they chose to continue working with the top-down view, where the rectangles created by the turtle were more clearly visible.

S1: Lets see how many doors there are if the value is only one? This perspective is not convenient, I will change it (*He activates successively all the default views and opts for the top –down one*).

S2 Yes, exactly like in the case of 360. It turns two rounds.

S1: Yes, it collects all of them in one. When we move it, the doors are changing position. They are sticking together or they are unsticking.

S2: We can't say that. With :d we determine their place. Look, if it is 90° they are turning and they are forming a cross, they form right angles, yes right angles, whereas with 360° or 720° they are placed together in the same line.

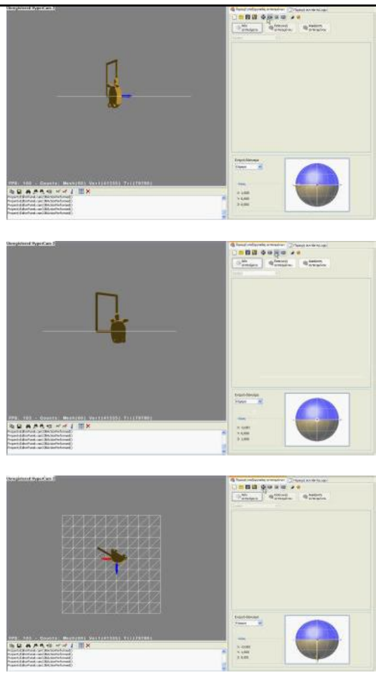

Figure 3 Episode 2 and the 3 default views of the 3d simulated space in MaLT.

These excerpts suggest that the group's instrumentation of the default views of the simulated 3D space involved the visualisation of the model in relation to the value of variable *d*. To be able to support their emergent explanations for the graphical outcome in relation to the measure of a left-roll turn command students chose the top – down view since it allowed them to observe the turtle's rolling and to specify the resulting number of distinct rectangles. By using this view the dihedral angle between the parallelograms was rather more easily discernible as it was closer to students' representation of 2D geometrical figures in the traditional setting. At an instrumentation level, students enacted certain assumptions about the 'input-output' correspondence between specific measures of turtle's turning and the position of the four constructed rectangles around the *x*-axis. Particularly, students were able to coordinate turtle's turning, one round for 360° and two rounds for 720°, with the static geometrical figure of 360° and 720° angles where the position of the two rays that form the angle coincide. Assumptions such as "with 360° or 720° they are placed together in the same line" are situated abstractions guiding this emergent scheme through which the students worked to more effectively orient their instrumented actions toward the purposeful use of the default views provided by the software.

5.2 Dynamic manipulation of variable values

The episode in this section is taken form the work of another focus group of the same classroom (group B). We look at the students' use of the 1dVT as an integral part of their attempts to achieve the simulation of 3D objects encountered in everyday situations (e.g. doors, revolving doors). From an instrumental perspective, dynamic manipulation through 1dVT constitutes one way that allows students to take control of the construction, animation and perspective taking of 3d geometrical objects in MaLT.

Early in their work when engaged in designing rectangles-windows in two vertical planes (task 3), the group B students characterised the dihedral angle drawn by the turtle as an acute and not as a right one as was the case, although they had commanded the turtle to left-roll 90 before drawing the second window. It seemed that these students had focused more on the visual characteristics of the figural representation and were confused by the 'distortion' of the dihedral angle as a result of the use of a vanishing point in the line of horizon of the TS designed to strengthen the sense of depth in the representation.

The use of 1dVT allowed the group B students to animate their construction keeping a steady perspective. Dynamic change of the model itself may in fact have minimized the 'distorting' effects of static 3D representation and prompted them to focus on the measure of the turtle's turn in the Logo code. The more the students appeared accustomed to the conventions used in the 3D simulated space the more they were able to coordinate the visual characteristics of the drawn angles with their measure related to turtle's turns. For instance during the dynamic handling of the revolving door simulation (task 5) the students of group B were able to overcome the difficulties faced earlier during task 3 and to recognize the four consecutive right dihedral angles created between the four rectangles around the common vertical side of the four rectangles (see Fig. 1). Experimenting with the variables of procedure 'slide' (see Fig. 2), they progressively became able to handle different aspects of angle simultaneously. In the next excerpt students had constructed four parallelograms around the *x*–axis by giving slider: *d* the value of 90 and S1 recognizes the need to construct four rectangles by dragging on the slider of variable *b* rather than *d* which, together with *c*, should remain constant (Fig. 1).

> S1: Wait, we should move it here first. It's the angle of the rectangle [*moves the slider :c to the value 90*], so as to become like this (*i.e. the door*) and then probably turns with this [*moves the slider :b*]. Let's see.
> S2: Yes, it definitely turns around with this [*i.e. slider :b*] as it has lr.
> S1: Yes, but we don't only want it to turn, we also want it to move even further down.
> S2: I should change here [*He puts the slider :a to the value 90 so as to have the simulation in a vertical position*].
> S1: 90 is fine.
> S2: Now, with this [*slider :b*] it turns around normally.

These successive articulations of the students' attempts to 'repair' the buggy model of a sliding door by dragging on the variable sliders, reveal steady refinements in the group's instrumentalisation of the 'slide' procedure. The meaning of the available tools emerged only through the course of their application to a specific task by the students, namely the group's effort to determine the angular relations which should be underlying a 'correct' model of a sliding door so as to prevent the distortion of the shape and confirm that the dynamic rolling of the door around the *x*-axis actually works. To achieve this students' instrumentation of the available tools hinged in particular aspects of angle as a spatial 3D concept: angle as a constitutive element of a figure which is defined and should stay fixed (variable *c*), angle as a means to move from the horizontal plane to the vertical one in relation to the viewing axis of the user (variable *a*) and should also remain constant and angle as a means of constantly changing planes in 3D space (variable *b*) around the common vertical side of the four rectangles. So in fact 'slide' should only have one variable (*b*) to work properly.

6 CONCLUSION

In our analysis, we understood much more clearly what was going on by using both IG and Constructionist Theory as tools to interpret how and why students discussed making changes to the 'slide' procedure and what kind of meanings they formed around angle in space. Our concern is that had we kept a fragmented view by using one of the two theories our interpretative lens would have been obtuse. Forging connections even at the detailed level of studying the instrumentalisation process allowed us to be more explicit in our analysis. We thus found it useful to pursue and extend prior attempts to relate Constructionism to Instrumental Theory (i.e. Hoyles et al., 2004) by taking this approach of "design for instrumentalisation" as a way to enhance connectivity between the two frameworks and to highlight the links between tool design that fosters meaning generation by means of explicit changes to the tool and students' instrumental genesis in the classroom. We particularly addressed some of the meanings generated by these 13 year-olds as they interacted with multiple interlinked representations of angular concepts in 3D space. The analysis shows how the design actually promoted the instrumentalisation processes in interaction with instrumentation in a fruitful way. Under the constructionist theoretical perspective the above incidents illustrate the relationship between the evolution of instrumental genesis and student's progressive focusing on angular relationships underlying the respective geometrical constructions and representations. The episodes presented indicate a co-emergence of the students' instrumentalisation of the available tools and their evolving success in using these resources as meaningful problem solving tools with greater utility for coordinating the figural results of the turtle's turning in 3D space. Thus, a constructionist focus on design seemed to add a productive set of tensions to the instrumental approach - namely between the design choices at the level of educator-designers of classroom tools and the students' goal-oriented use of those tools. It is indicated that technology-rich environments designed to favour instrumentalisation require evaluation of their design features and constrains in the light of empirical investigation based on the combined use of different frameworks. We thus suggest that Constructionism can also be employed in conjunction with TDS to understand in greater detail how instrumentalisation may become an integral part of meaning-making.

REFERENCES

Artigue, M. (2002) Learning Mathematics in a CAS environment: The genesis of a reflection about instrumentation and the dialectics between technical and conceptual work, *International Journal of Computers for Mathematical Learning*, **7**(3), 245-274.

Artigue, M. (ed.) (2009) *Integrated Theoretical Framework.* 'ReMath' (Representing Mathematics with Digital Media) FP 6, IST-4 026751 (2005–2009). Del. 18.

Clements, D., Battista, M. T., Sarama, J., and Swaminathan, S. (1996) Development of turn and turn measurement concepts in a computer-based instructional unit, *Educational Studies in Mathematics*, **30**, 313-337.

Cobb, P., Confrey, J., DiSessa, A., Lehrer, R., and Schauble, L. (2003) Design experiments in educational research, *Educational Researcher*, **32**(1), 9-13.

Harel, I. and Papert, S. (Eds.) (1991) *Constructionism: Research Reports and Essays*. Norwood, NJ: Ablex Publishing Corporation.

Healy, L. and Kynigos, C. (2010) Charting the microworld territory over time: design and construction in mathematics education, *Zentralblatt für Didaktik der Mathematik,* **42**(1), 63-76.

Hoyles, C., Noss, R. and Kent, P. (2004) On the integration of digital technologies into mathematics classrooms, *International Journal of Computers for Mathematical Learning*, **9**(3), 309-326.

Kynigos, C. (2007) Half-baked Logo microworlds as boundary objects in integrated design, *Informatics in Education,* **6**(2), 335–358.

Kynigos, C., Psycharis, G. and Latsi, M. (2009) Meanings for angles through geometrical constructions in 3d space. In *Proceedings of the 33rd PME Conference*, Thessaloniki, Greece, Vol. 3, 457-464.

Lagrange, J.-B. (1999) Complex calculators in the classroom: Theoretical and practical reflections on teaching pre-calculus, *International Journal of Computers for Mathematical Learning,* **4**(1), 51–81.

Lagrange, J.-B. (2005) Curriculum, classroom practices, and tool design in the learning of functions through technology-aided experimental approaches, *International Journal of Computers for Mathematical Learning*, **10**, 143-189.

Monaghan, J. (2007) Computer algebra, instrumentation and the anthropological approach, *International Journal for Technology in Mathematics Education*, **14**(2), 63-72.

Noss, R. and Hoyles, C. (1996) *Windows on Mathematical Meanings,* Dordrecht: Kluwer Academic Press.

Papert, S. (1980) *Mindstorms,* New York: Basic Books.

Pratt, D. and Noss, R. (2010) Designing for mathematical abstraction, *International Journal of Computers for Mathematical Learning,* **15**(2), 81–97.

BIOGRAPHICAL NOTES

Chronis Kynigos, is professor at the University of Athens, director of the Educational Technology Lab. He has led the pedagogical design of a set of constructionist media and employed them in research involving aspects of designing and generating socio-constructivist learning environments in the classroom. These are: E-slate, a component kit to construct microworlds, MachineLab, a programmable 3D simulator and Cruislet, a 3D navigation system over a GIS based on the Cruiser platform. He has taken part in eight European multi-organisational R&D projects and was director of the 'ReMath' project (http://remath.cti.gr).

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