

Studying the process of becoming a teacher educator in technology-enhanced mathematics

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Abstract In this paper, we explore the process of becoming a teacher educator in the pedagogical use of digital tools in mathematics teaching. The study took place in the context of an in-service program during the trainees' engagement in their practicum fieldwork activities including the process observation-reflection-design-implementationreflection. We explored the features of this context that facilitated the trainees' transition from the level of trainee educator to the level of teacher educator as well as the nature of the trainees' documentation work for teachers. The results showed that observation of other teacher educators' teaching in conjunction with reflection during the program's respective sessions facilitated the trainees' transition to the professional level. The identified operational invariants underlying the trainees' designs concerned the focus of their observation in teacher education classrooms, the importance they attributed to the constraints and opportunities provided by the wider educational context and epistemological issues regarding the teaching and learning of mathematics with technology. The analysis of trainees' designs revealed three kinds of documents ("explanatory," "instructive" and "facilitative") and corresponding roles of trainees during the implementation. These documents targeted different aspects of TPACK depending on the trainees' conceptualizations of teachers' roles either "as students" or "of students."

Keywords Teacher educators' education · Documentational approach · Teacher knowledge and epistemologies · Technological-pedagogical-content-knowledge

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Introduction

In this paper, we study *the process of becoming a teacher educator (TE)* in the context of an in-service, reform-oriented professional development (PD) program within the Greek educational system aiming to educate TEs into the pedagogical use of digital tools in the teaching of mathematics. The program was implemented in University Centers (UCs), and the plan was to employ the newly trained TEs in wide-scale courses to educate groups of teachers in specific Centers for Teacher Education Support (CTES). In Greece, the ones that typically undertake the role of TEs are qualified teachers of secondary education. After being educated about the aims of specific teacher education programs, they can also work as TEs in parallel to their teaching in schools.

The present study aims to address a gap in mathematics teacher education research that is indicated by the literature review in the next section: While there is plethora of research studies related to different aspects of teachers' professional development, we know very little about TEs education and development especially when technology is integrated in the teaching of mathematics. Our aim is to contribute in the field of TEs' education by analyzing the processes that took place in the program's practicum (observation–reflection–design–implementation–reflection) with an emphasis on the trainee TEs' didactical design. Specifically, we investigate how practicum processes facilitated the trainee TEs' transition to the professional level of a TE as well as their documentation work (DW) in this context. What we term as trainees' DW includes "looking for different kinds of resources, selecting/designing mathematical tasks, planning their succession, managing available artifacts, etc." (Gueudet and Trouche 2009, p. 199).

Literature review

During the last decade, there has been a growing interest in the role of TEs for improving the quality of mathematics teaching (Ball and Even 2009; Even 1999, 2008; Jaworski and Huang 2014; Jaworski and Wood 2008; Pope and Mewborn 2009; Zaslavsky and Leikin 2004). While there is a huge body of research on teachers' knowledge and practices (Ball et al. 2008; Cochran-Smith and Lytle 1999; Ponte and Chapman 2006; Shulman 1986; Sullivan and Wood 2008), the knowledge required or targeted by TEs received relatively little attention, but is an emerging field of research (Beswick and Chapman 2012, 2013; Jaworski and Huang 2014; Even 2014). Most of the studies in this area focus on competencies or forms of knowledge required by TEs as well as on the mechanisms of knowledge construction and development (Even 2008; Smith 2005; Zaslavsky 2008; Zaslavsky and Leikin 2004). Collaboration among TEs, teachers and researchers in joint ventures with a focus on teaching and the TE as researcher constitute dominant trends of relevant research facilitating co-learning and self-understanding the (Jaworski 2001, 2003, 2008; Chapman 2008; Special Issue of ZDM Vol. 46, 2014). Based on Schön's (1987) term of reflective practitioner, the existing research emphasizes the importance of TEs' reflection on-action and in-action in all phases of their work (e.g., planning, interacting with teachers, observing teachers' work) (Zaslavsky 2008). In order to describe TEs' knowledge, some scholars built on existing models developed for teachers' learning. For instance, Zaslavsky and Leikin (2004) extended the teaching triad idea (Jaworski 1994) to describe TEs' learning. The technological pedagogical content knowledge (TPACK) framework (Koehler and Mishra 2008; Mishra and Koehler 2006) has been used to investigate the skills and knowledge preservice and in-service teachers need to develop so as to integrate the use of digital resources into their classroom practice (e.g., Niess 2005; Lee and Hollebrands 2008; Drijvers et al. 2013). However, TPACK has not been studied in relation to the forms of knowledge that TEs target when teaching mathematics teachers and our study aims to contribute in this domain.

Existing research also shows that there are few formal PD programs for TEs' education; therefore, our knowledge of becoming a TE is limited and is mainly based on self-studies of university-based researchers acting as TEs (e.g., Krainer 2008; Tzur 2001). Even (2014) calls for additional research on "curriculum, pedagogy and structure of programs aiming at didacticians' preparation" (p. 332). Reform-oriented PD programs that promote specific mathematical and pedagogical concepts, skills and dispositions have much to do with understanding the teachers' conceptions of mathematics, teaching and learning as they evolve from the traditional ones toward the goals of the reform (Simon 2008). Thus, teacher's and TE's epistemology¹ is considered as an important factor in changing the traditional teaching model. However, although teachers' epistemology has been an object of study for more than three decades (Thompson 1992; Ernest 1989), TEs' epistemology has received very little attention. An example of research on this topic is the study of Kynigos (2007) that illustrated the role of digital artifacts in challenging trainee TEs' knowledge and epistemologies of teaching and learning mathematics during a PD course. Also, recent literature reviews on TPACK (Voogt et al. 2013; Chai et al. 2013) identified the need for further research on the interrelation between TPACK, teacher practical knowledge and teacher beliefs as well as the effects of PD courses on the development of TPACK. Taking into account the above research challenges, our study focuses on epistemological issues underlying the trainees' DW and their relation to the forms of TPACK knowledge targeted for teachers in the context of a reform-oriented program.

Our choice to analyze trainees' DW stemmed from the importance attributed to the design of tasks and artifacts in current PD programs in which teachers' interactions with various resources facilitate their learning and collective work (Gueudet et al. 2013; Pepin et al. 2013). The key role of tasks is described by Zaslavsky and Leikin (2004), who developed a three-layer model of growth-through-practice of all the participants (mathematics TEs, TEs' educators and teachers), when designing tasks as members of a community of practice (Lave and Wenger 1991). Zaslavsky (2007, 2008) also considers the interplay between the TEs' role as task designers with their role as researchers or facilitators of teachers' learning as a driving force of effective learning through reflective practice. Under this perspective, trainees' engagement in developing their didactical design, implementing it and reflecting on their experience was central key in our study. Our approach is informed by current trends concerning the central role of practicum (in teacher education programs) in connecting coursework and content with actual classroom practice (Bergsten et al. 2009; Bahr et al. 2014).

Summing up, in the present paper we address two research challenges:

- 1. We investigate how the context of the practicum of an in-service reform-oriented PD program (consisted of observation-reflection-design-implementation-reflection) facilitated the trainees' transition toward the role of a TE.
- 2. We study the nature of the trainees' DW focusing on the implicit knowledge and epistemologies embodied in resources as well as on the trainees' practices during the

¹ Teachers' personal theory of knowledge and knowing (Hofer 2004).

implementation. Through this, we attempt to connect the TPACK framework to TEs' epistemologies and practices.

Our study is expected to inform existing research concerning the structure and content of reform-oriented PD programs for TEs (in the area of technology-enhanced mathematics) and their impact on trainees' design. Also it is expected to address the connection between TPACK and other theoretical frameworks, a need that has also been indicated (Voogt et al. 2013; Chai et al. 2013).

Theoretical framework

Teachers' knowledge, epistemologies, and practices

A number of researchers have elaborated on Shulman's (1986) categorization of teachers' knowledge so as to connect it to different aspects of the mathematics teachers' profession, such as epistemology (Ernest 1989), practice (Cochran-Smith and Lytle 1999) or teaching (Ball et al. 2008). A common feature of all these approaches is their focus on how the knowledge of subject matter can be transformed in teaching practice. In order to describe the knowledge needed for integrating technology in teaching, Mishra and Koehler (2006) built upon Shulman's work and developed the TPACK framework taking into account the interplay between content, pedagogy and technology:

- Technology knowledge (TK) concerns knowledge about technical aspects of hardware and software;
- Technology content knowledge (TCK) refers to the ways that technology can be interrelated with subject knowledge;
- Technological pedagogical knowledge (TPK) describes a broader knowledge of technology in relation to pedagogical strategies that can be adopted in different teaching and learning settings. TPK concentrates on the knowledge of tools and its functionalities, as well as on the interrelation between specific tools and tasks;
- Technological pedagogical content knowledge (TPCK) describes an emergent form of knowledge that goes beyond content, pedagogy and technology. TPCK requires understanding of concepts, pedagogical techniques for communicating mathematical content in constructive ways, knowledge of students' difficulties in learning particular topics, knowledge of students' prior knowledge, and how all the above can be readdressed through the use of technology.

Another dominant issue of teachers' professional development concerns the link between teachers' knowledge and their epistemologies. Scholars working on the integration of technology in mathematics teaching argue that teachers' designs based on the use of technology reveal aspects of their epistemology (Hoyles 1992). In the PD program we refer to, trainee TEs' learning was expected to be developed through their engagement in designing resources in the form of digital artifacts and scenarios for the trainee teachers in CTES. The course designers considered these resources to be improvable or fallible² rather than as static curriculum material that teachers simply had to strictly implement (Kynigos 2007).

Many researchers have stressed the importance of interrelating knowledge and epistemology, the difficulty of distinguishing between them and their influence on the everyday

² From a broader epistemological perspective, this view is closer to fallibilism that considers a mathematicians' activity as a process including conjectures, refutations and new proofs (Lakatos 1976).

teaching practice (Nespor 1987; Thompson 1992; Ernest 1989; Pajares 1992). The relation between teachers' epistemologies and practices is complicated and not linear as it is strongly affected by constraints and opportunities afforded by the social context (Ernest 1994). For example, a teacher's conceptions regarding the teaching of mathematics may be rooted in the epistemological paradigm of absolutism³ while the teacher's practices might be closer to the paradigm of fallibilism (Footnote 1) and vice versa. Interconnections between teacher's practices and epistemologies formulate the following different roles in teaching (Ernest 1994): instructor (i.e., targeting skills' mastery and correct performance); explainer (i.e., targeting conceptual understanding with unified knowledge); and facilitator (i.e., targeting problem posing and solving). At the same time, each role signifies a different stance toward the curriculum and the corresponding material and leads to three patterns of curriculum use (ibid): the strict following of a text; the modification of the textbook approach—enriched with additional problems and activities; and the construction of the mathematics curriculum by the teachers themselves. In our study, we aim to identify the roles that the trainees adopted for themselves in their practice and their perceptions of teachers in CTES as well as to connect both of them to the TPACK forms of knowledge they targeted (for teachers in CTES) in their designs. Through this, we attempt to connect the TPACK framework with the trainees' epistemologies and practices.

Documentational approach and double instrumental genesis in TEs' education

Existing research indicated a shortage of resources for mathematics teacher education (Zaslavsky 2008). So, TEs face the challenge of creating their own material for teaching teachers by developing a dynamic relationship with curriculum resources (Remillard 2005). We adopt the documentational approach of didactics (Gueudet and Trouche 2009) according to which the teacher's work is developed with and on resources (Adler 2000) in a dialectic process where design and enactment are intertwined. An implication of this approach is that curriculum material is not conceived as a static body of resources that guides instruction, but rather as a set of objects amenable to changes and modifications depending on the teacher's didactical design. Through a class of professional situations and teachers' experiences, the existing resources can be modified as documents according to the formula:

A scheme of utilization of a set of resources incorporates "*practice* (how to use selected resources for teaching a given subject) and *knowledge* (on mathematics, on mathematics teaching, on students, on technology)" (Gueudet and Trouche 2011, p. 401). While practice entails observable parts of teachers' stable behavior for a given class of situations (called usages), knowledge embodied in resources is implicit, intertwined with teachers' beliefs, and can be inferred from the usages. The constituent elements of this kind of knowledge are the *operational invariants* built through different contexts of using the resources. Thus, the formula can be modified as follows:

Document = Resources + usages + operational invariants

Under this perspective, our use of the term "document" in this paper incorporates both the material component of documents/resources and the essentially implicit knowledge/

³ Mathematics as the science of typical deduction from axioms to theorems (Davis and Hersh 1981).



Fig. 1 Schematic representation of a documentational genesis (Gueudet and Trouche 2009, p. 206)

epistemology "piloting" their usages, i.e., the operational invariants inferred from the trainees' activities in the context of the practicum.

Extending the instrumental theory of Vérillon and Rabardel (1995), Gueudet and Trouche (2009) considered the creation of documents as unfolding through a dual process of instrumentation (the resources act on the teachers) and instrumentalization (teachers act upon these resources as they appropriate them) (Fig. 1). Starting from a set of existing resources, this process gives birth to a new entity, i.e., a document, which can be further transformed to a new one over time in an ongoing process. Although the documentational approach is adopted widely to analyze teachers' interaction with resources, it has not been exploited yet at the level of TEs and our study aims to contribute in this direction.

Another theoretical construct that informs our perspective is double instrumental genesis (double IG) (Abboud-Blanchard and Lagrange 2006; Haspekian 2011). It consists of the personal instrumental genesis (personal IG) leading to the appropriation of a tool as an instrument for mathematical work at the personal level and the professional instrumental genesis (professional IG) leading to the construction and appropriation of this instrument into a didactical instrument for mathematics teaching at the professional level. Although double IG is at the core of teacher education in technology-enhanced mathematics, research has given little attention to it as regards TEs. In this paper, we used double IG in two levels: (a) to study the trainees' transition from personal to professional IG in the context of the practicum; (b) to identify how they used these instruments in their designs targeting an analogous transition for the teachers in CTES.

Taking an overall view of our main theoretical frameworks, double IG and the interrelation between TPACK and the trainees' epistemologies transcend our analysis in both before and after implementation in CTES. The documentational approach is used complementarily with double IG, TPACK and the trainees' epistemologies to gain insight into the operational invariants underlying their DW.

Research methodology

Context of the study and participants

The Greek educational system is highly centralized with a single curriculum that the teachers have to follow strictly. Also teachers have limited opportunities for in-service development due to the lack of long-term teacher education structures. The objective of the PD in-service program we refer to was the integration of digital technologies in teaching

practices (for more details about the context, see Kynigos and Kalogeria 2012). It was implemented in UCs all over Greece for 350 h spread over 8 months. The design of the program included two kinds of courses addressed, respectively, to TEs (in UCs) and to teachers (in CTES). Traditional teacher education programs focusing on the integration of technology usually have limited impact on teachers' practices, and one reason for this is related to the fact that teachers usually analyze ready-made tasks (Trgalova et al. 2011). Thus, one of the reform aspects of this program is the conceptualization of TEs (and teachers) as active partners in the integration of technology in teaching practice (Noss and Hoyles 1996) through their engagement in creating their own material and implementing them in real teacher education classes, as part of their practicum.

Enrolling a UC course, the trainees were engaged in designing resources in the form of scenarios⁴ based on the use of technology. UC curriculum included five categories of digital media: Computer Algebra Systems, Dynamic Geometry Systems, Programmable software, Simulations and Data Handling tools. Throughout the course, the trainers gave trainees material related to their lessons. At course begin, the trainees received an official document called "Notes for teaching in CTES" which contained theory and a set of twelve generic scenarios based on the use of the above digital tools (we will refer to these materials as "existing resources"). These scenarios constituted the core of UC and CTES curriculum, and they were expected to be exploited by both the trainee TEs (as a basis for organizing their subsequent teaching in CTES) and the teachers in CTES (as material to be implemented in their classrooms).

The reported study took place in a UC course at the University of Athens during the trainees' engagement in their practicum that was provided 3 months before the end of the course in parallel with the UC curriculum. Practicum involved: (a) 4-h teaching in school; (b) observation of other TEs' teaching in CTES; (c) design of a 3-hour lesson for teachers under the supervision of a mentor and implementation in CTES; and (d) ten 3-hour reflective sessions at UCs throughout the duration of the practicum. Thus, it was based on the sequence observation–reflection–design–implementation–reflection. Here our focus is on the part of the practicum concerning the trainees' activities in CTES.

Trainees could select one of the existing CTES courses for (a) observing official trainers' teaching and (b) implementing their designs. The official trainers had undergone successfully a previous UC course, and they had diverse teaching experiences in CTES. During observation, the trainees were expected to experience very different practices, since CTES structure allowed trainers to organize their teaching according to their own methods/ approaches. The trainees had to describe in an "observation form" specific teaching aspects.⁵ They also had to select a critical event, provide a reflective account on it and discuss it in the reflective sessions. For their own teaching, the trainees had to design lesson plans and to provide a reflective account on their implementation. Instructional practice in

⁴ Scenarios are activity plans addressing critical aspects of the teaching and learning process and the corresponding materials (digital, e.g., microworlds, or non-digital, e.g., worksheets). A scenario structure was developed by the Educational Technology Lab (http://etl.ppp.uoa.gr) (National and Kapodistrian University of Athens) which participated in the design of the program and the course materials. This structure included: (1) title; (2) identity (i.e., author, subject area, topic); (3) rationale (i.e., innovations, added value by the use of technology, students' learning problems addressed); (4) context of implementation (i.e., class year, duration, location, prerequisite knowledge, classroom social orchestration, goals); (5) phases of implementation (i.e., sequence and analysis of activities, participants' roles, anticipated teaching/learning processes); (6) possible extension; and (7) references.

⁵ The aspects involved in the observation form were: topic and aims of the lesson; trainees' levels concerning the use of technology; resources used; classroom organization; teaching methods/processes; and classroom interactions.

the reflective sessions at UCs aimed to support trainees' reflection on their recent field experiences. Trainees' typical activities in the reflective sessions were to: (a) present a critical event and the content of their "observation forms"; (b) present their own design and teaching in CTES and reflect on it; and (c) discuss emerging issues and suggest possible changes.

Our role as academic trainers and mentors of trainees allowed us to communicate the main principles of the course by challenging trainees to inquire into the design and use of resources based on digital tools. The 16 participants in the UC course that we study here were experienced, qualified mathematics teachers working in secondary education. Five of them had a doctoral degree in mathematics, one was a Ph.D. student in educational technology, and the rest held master's degrees in applied mathematics or mathematics education. When entering the course, they had limited previous experience in the pedagogical uses of digital tools and no experience in teaching teachers.

Main aim and research questions

Our aim was to study the trainees' DW through shedding light on the operational invariants that influenced their design and practice in the context of the practicum with a parallel focus on the features of this context that facilitated their transition from the level of trainee educator to the level of TE.

Before analyzing trainees' DW, we stepped back and explored the practicum processes that had taken place before the trainees' engagement in developing their own didactical design. More specifically, we focused on how observation in CTES and the subsequent reflective sessions facilitated their transition from personal to professional IG. Thus, our first research question is:

• How did the practicum context before the trainees' engagement in developing their own didactical design facilitate their transition from personal to professional IG?

Next, we investigated the trainees' DW during their practicum. Given that it was the trainees' first attempt to produce their own material as TEs, it was important to gain insight into the operational invariants underlying their DW. We were interested in: (a) highlighting the role of observation in their designs and (b) identifying the interrelation between trainees' epistemologies and the TPACK forms of knowledge targeted for teachers in CTES. Thus, our second research question and its sub-questions are:

- What is the nature of the operational invariants underlying the trainee TEs' DW during the process of observation-reflection-design-implementation-reflection?
 - How did observation influence the trainees' DW?
 - What is the relation between trainees' perceptions of teachers in CTES, the roles they adopted for themselves and the TPACK forms of knowledge targeted for teachers in CTES?

Data collection and analysis

The data consisted of: (1) the researchers' notes from the discussions that took place during the UC reflective sessions; (2) the trainees' designs for their lessons in CTES (scenarios,

worksheets, PowerPoint presentations, etc.); and (3) the trainees' activity reports⁶ filled out-of-class.

The first source of data allowed us to analyze how trainees conceptualized their experiences from observation and reflected on them so as to identify evidence indicating their transition from personal to professional IG (first research question).

The analysis of the second source of data was conducted in conjunction with the other two sources in order to answer the second research question. In this process: (a) Our notes from UC reflective sessions (constituted of verbal comments) informed us about the trainees' reflections on their fieldwork experiences. (b) The activity reports provided information concerning the trainees' own written evaluation of their design–implementation. Both sources of data helped us to analyze their DW over time and to identify the interrelation between the targeted TPACK forms of knowledge, the trainees' perceptions of teachers in CTES and the roles they adopted for themselves during the implementation. Thus, we were able to clarify and triangulate the trainees' choices and the reasons supporting them so as to explore the operational invariants underlying their DW.

In analyzing trainees' DW and reflection, we followed the principles proposed by Gueudet and Trouche (2011) regarding research methodology on documentational geneses: (a) it was developed in time periods in and out of class, including trainees' continuous follow-up on their work before and after the implementation (reflexive investigation principle); (b) it evolved along instrumentation/instrumentalization processes through the actual implementation of resources in teacher education classrooms (design-in-use principle); and (c) it was embedded in, and influenced by, different collectives, i.e., how trainees' designs were influenced by their participation in the UC collective, as well as by their participation in the CTES classrooms as observers and TEs (collective principle).

The analysis was carried out in two steps corresponding to our research questions:

1. First, we analyzed the discussions during the reflective sessions so as to gain insight into the trainees' transition from personal to professional IG. Initially, each one of us (the two authors) open coded these discussions focusing on their content indicating provisional categories (Charmaz 2006). By comparing our categorizations, we kept the common ones, merged or deleted some others. Then, we developed rules conveying the meaning contained in our provisional categories (Lincoln and Guba 1985) and we re-examined our data. Through the constant comparative method (Strauss and Corbin 1998), we concluded on our final categorization with common names and definitions (Table 1) until reaching an 88% level of intercoder reliability (Miles and Haberman 1989). Each category included discussions of different lengths around a specific dominant issue which we called "theme of the discussion." Thus, in each category we could have discussions with different "themes." For example, the category "teaching teachers" included themes such as "How we teach a scenario," "Interrelating digital tools and mathematical concepts," "The integration of teaching/learning theories in teachers' didactical design."

Further refinement of these discussions brought to the fore themes which provoked trainees' reflection about their personal experiences in CTES (observation, design, or teaching) and/or favored the emergence of new perspectives for interpreting/reconceptualizing phenomena of teaching practice. We characterized these themes as

⁶ Templates in which trainees described aspects of their designs and their experiences from the implementation (e.g., distance between design and actual implementation, teachers' participation/difficulties, potential changes in case of redesigning a scenario).

Code names	Brief definitions
TS-teaching students	Teaching mathematics in school
TC-theoretical construct	Interpreting classroom practices through the lens of theory
TT—teaching teachers	Teaching at the teacher education context
MC—mathematical content	Mathematics curriculum (concepts, tasks, etc.)
RT-resources for teaching	Scenarios, worksheets, digital tools, etc.
SI—systemic issues	Constrains/opportunities/rules of the wider educational context

 Table 1
 Categorization of discussions

"crucial reflective themes" (CRTs). Often, CRTs seemed to operate as generators of new discussions belonging to the same or different categories.

Three identified CRTs that helped us to answer our first research question were "The interrelation between scenarios and worksheets," "The integration of technology in trainees' didactical design" and "Distinguishing 'teachers as students' from 'teachers of students'." These CRTs are presented in the first part of the Results Sect.

2. Next, we analyzed our second source of data (i.e., trainees' designs) taking into consideration our first and third sources of data (i.e., the reflective discussions and the trainees' activity reports) so as to support our interpretations. Specifically, we sought elements of the three above-mentioned CRTs as mirrored in trainees' designs: How existing resources (scenarios) were transformed into new documents (e.g., worksheets); how technology was integrated (i.e., the TPACK forms of knowledge targeted for teachers in CTES); the trainees' conceptions concerning the role of teachers in CTES ("teachers as students," "teachers of students").

In this paper, we present the analysis of the designs of three trainee TEs (Tom, Ian, and Jim) that we considered as representative of all participants' designs. Tom and Ian had a master degree in mathematics education, while Jim held a Ph.D. in pure mathematics. We chose to analyze Tom's DW as an exemplary case of developing original documents and Ian's and Jim's cases because they built their design upon the same existing resources, and it was important for us to take a comparative view of the factors that influenced their designs.

Results

Since the discussions of the reflective sessions constituted a crucial aspect of the practicum and helped us to answer our research questions, we initially present an overview of these discussions. Then, we present our analysis in two parts. The first one concerns the discussions between observation and design related to CRTs compatible with our first research question. In the second part, we provide illustrative examples of trainees' DW in order to address our second research question.

Through the analysis of the reflective sessions, 45 discussions and 9 CRTs were identified and categorized as shown in Table 2.

According to these data, 31 out of 45 discussions (69%) concerned the trainees' new professional role as TEs and focused on issues related to teaching teachers and the

Code names	Number of discussions	Number of CRTs
TS—teaching students	8	0
TC-theoretical construct	2	1
TT—teaching teachers	14	3
MC-mathematical content	2	2
RT—resources for teaching	17	3
SI—systemic issues	2	0
Total	45	9

 Table 2
 Quantitative data of the discussions and CRTs in UC

corresponding resources. In a similar way, the majority of the emergent CRTs were related to the above issues while SI⁷ and TS had no CRTs.

Illustrative examples of discussions between observation and design facilitating trainees' transition from personal to professional IG

Below, we analyze three CRTs that helped us to answer our first research question.

The interrelation between scenarios and worksheets

Observation in CTES was expected to help trainees in designing and implementing a scenario in CTES. Thus, it was critical for them to be able to transform a scenario into a teaching sequence through worksheets. We intentionally brought for discussion the interrelation between scenarios and the respective worksheets during the reflective sessions preceding the trainees' teaching in CTES, as it is shown in the following discussion:

Trainee A: Personally, I see the creation of a worksheet as a prerequisite for a structured teaching sequence with specific learning aims.

Trainee B: However, a scenario might not be detailed. In this case, how can it be transformed to worksheets?

Trainer A: The less detailed parts of a scenario might be intentionally designed so as to challenge teachers to develop their own activities.

Trainee C: But when you design a worksheet, you need to be precise.

Trainer A: UC scenarios should be seen as objects amenable to changes by different TEs in the process of creating their own agenda. One particular scenario can be transformed into a number of different worksheets.

Trainer B: In CTES, I always engage teachers in creating worksheets because designing a worksheet requires a clear idea of the structure of the lesson. It can operate as a structure that precedes the full version of a particular scenario.

The discussion here revolved around the "by-design" characteristic of scenarios which intentionally provide teachers and TEs with many degrees of freedom when planning their teaching. It seemed to have perturbed the trainees' conceptions for design, since most of

⁷ The category 'SI' concerned primarily the institutional role of CTES within the national educational system (e.g., number of teachers in CTES classrooms, timetable), and it seems to be of less importance for the trainees at this phase of their training.

them were used to implementing activities following the strict didactical instructions and texts of the official curriculum. The trainees appear to need more detailed instructions on how to create the worksheets corresponding to these "incomplete" scenarios. Taking the CTES scenarios as a starting point, the discussion addressed the multiplicity of didactical designs that can be generated by the same resources, which presupposes an active involvement of the teacher in creating own material.

Similar discussions took place in which the course trainers challenged the trainees' conceptions of tools/resources for teaching in CTES. Therefore, the reflective sessions seemed to favor the trainees' transition toward the professional IG facilitating the transformation of existing materials (such as scenarios) into didactical instruments.

The integration of technology in trainees' didactical design

This CRT brought to the fore a number of issues, such as those mentioned in the following excerpts:

Trainee D: The TE I observed complained about a number of issues: "There is limited time... Do I have to teach the software to the teachers? Is it necessary to teach them how particular mathematical concepts can be approached through the use of different tools? To what extent can scenarios be taught in parallel to teachers' familiarization with specific software?"

Trainer A: Through scenarios, we aim to teach structures of mathematical concepts rather than particular concepts, along with the teaching of the software itself.

Trainee E: Thus, teaching scenarios and software at the same time can help us to overcome time restrictions. It can also be used to combine the technological and the pedagogical aspects of the tools.

This discussion echoes the critical issue of the integration of technology in the teaching of mathematics and the corresponding dilemmas of both TEs and trainee TEs in managing the complexity of teaching in CTES. Time restrictions, teaching sequence, targeted mathematical concepts, the integration of tools within scenarios and teachers' familiarization with technology are some of the parameters that influence the decisions of a TE. The learning of technology itself by the teachers in CTES seemed to have posed another layer of complexity that had to be taken into account when planning a lesson for them. The above discussion reflects trainee TEs' emergent dilemmas about the TPACK forms of knowledge that they should target through their teaching in CTES. As evidenced by the above excerpts, the reflective sessions' context was rich in opportunities for trainees to address the complexity of decisions that need to be taken by a TE. This can be considered as a step taken at the level of trainees' professional IG since the trainees at this phase seemed to have focused on how to exploit the potential of technology for their teaching in CTES.

Distinguishing "teachers as students" from "teachers of students"

In one of the reflective sessions, which concerned a trainee's observation in CTES, we—as trainers—deliberately brought to the foreground the dual role of teachers in CTES and connected it with current mathematics education research through the use of relevant theoretical constructs, such as double IG. This idea contributed to the emergence of the

above CRT which challenged trainees to rethink their observation in CTES and interpret it through this lens:

Trainer A: Can you rethink your observation in CTES taking into account double IG? Trainee C: It seemed to me that the teachers in CTES were students who had to learn the software during the teaching of the selected scenarios. I now realize that this point was not clear to me before the present discussion.

Trainer A: The TE might have targeted only the learning of the tools in the specific lesson.

Trainee D: Up to a point, you need to act as a student, to press keys, to understand the tools and after that, to discuss with your classmates and the TE what you would teach, the teaching sequence, etc. There are two stages. You may stay only in the first, but this will happen only in the introductory lessons.

Trainer B: I remember in my first lessons in this UC class your intense concentration on the software and nothing else. Now, I see that you have started to pose pedagogic and didactic questions.

The introduction of the theoretical construct of double IG seemed to operate as a tool that helped the trainees address the complexity inherent in teaching mathematics with digital tools. Trainee D mentioned that acting "as students" is a prerequisite for trainees' passage to the didactic level. During their early UC lessons, trainee TEs were mainly students concentrated on the learning of technology, while at the time of the discussion, they had started to concentrate on pedagogical/didactical issues. It seems that trainees' observation of lessons in CTES, coupled at the theoretical level with double IG, started to play a role in their transition from the personal to the professional IG. We see here that there is an analogy between trainee TEs in UC and trainee teachers in CTES: In both levels, they initially tended to act "as students" interested in the learning of technology per se (TK, TCK), while the pedagogical aspects of the integration of technology in teaching occurred later and signified the trainees' role as "trainers of teachers" and "teachers of students," respectively, focusing on more "complicated" forms of knowledge (i.e., TPK, TPCK).

Taking an overall view of the discussions around the above CRTs, it seems that the phase of observation in conjunction with the reflective sessions allowed trainees to reconsider phenomena of teaching practice in CTES and take a deeper view of the complexity inherent in their future profession. In terms of double IG, they started to question how they would use/transform the existing resources into didactic tools and they expressed their dilemmas about their classroom management and the forms of TPACK knowledge that they should target in CTES. Moreover, they became aware of the dual role of teachers in CTES ("as students" and "of students") and they related these roles to different kinds of design and corresponding forms of knowledge. In this process, the practicum context provided a basis for trainees to conceptualize their role and its requirements at the professional level and seems to have facilitated their transition toward professional IG.

The nature of the operational invariants underlying the trainees' DW: three illustrative cases

In order to address the nature of operational invariants underlying the trainees' DW during the practicum, we analyze Tom, Ian, and Jim's design cases. These cases indicate the

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	-1.9	6 -0.92						ł				
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Fig. 2 Sending pairs of (x, y) from table to graph

transformation of existing resources into new ones with the use of Function Probe⁸ (FP) (Confrey 1991-2002).

Tom's case

Tom's lesson in CTES was about the design of scenarios and worksheets. During his observation, he noticed that most of the teachers had three main difficulties: (a) to understand the connection between scenarios and worksheets; (b) to get familiar with the FP functionalities; and (c) to conceive and describe the aspects of the teaching/learning process suggested by a scenario. Thus, Tom developed two original documents: The document "worksheet" and the document "structure-for-scenario-design."

The document "worksheet" was based on one of the twelve official scenarios provided in UC related to the study of sinusoidal function with FP. The UC curriculum suggested the following teaching sequence:

- Phase 1: Converting degrees to radians for studying f(x) = sinx.
- Phase 2: Creating table of values in [-π, π] with step π/8, sending ordered pairs of x and y to graph (first idea of sinusoidal curve) (Fig. 2), increasing the number of tabular values (development of conjectures), and study of properties.
- Phase 3: Creating and filling in columns in the table with values of $y = 2\sin x$ and $y = 3\sin x$, comparison to the respective values of $y = \sin x$, construction of graphs in the graph window by inserting the formulas in the "Input field" and study of the

⁸ Function Probe is a multi-representational software with three windows: Table, graph, and calculator. You can produce function graphs in a number of ways, e.g., inserting a formula for the function in the graph ("Input field"), "receiving" ordered pairs (x, y) from a table ("x" and "y" columns can be generated). Particular tools allow horizontal and vertical transformations of functions (translations, reflections and stretches) through direct actions on the graph. The graphs of the transformed functions are depicted in the same window.



Fig. 3 Table values of $y = \sin x$, $y = \sin 2x$, $y = \sin 3x$ and their graphs (by clicking on each graph, it becomes bold and its corresponding formula appears in the "Input field")

corresponding functions (Fig. 3), conjectures for the graph of $y = \rho \sin x$, and the role of ρ .

• Phase 4: Using the stretch tool⁹ to manipulate vertically the height of the curve $y = \sin x$ at random (Fig. 4) to identify the role of ρ through the history window (Fig. 5).

The document "worksheet" (see part of it in Table 3) had the form of a two-column table. In the left column, Tom inserted the worksheet questions, and in the second, the rationale. In a reflective session, he explained his choices stressing that "A complete scenario needs to have questions or issues that constitute objects for negotiation and activities designed with particular learning aims." He also mentioned that during his observation in CTES, he had noticed that most of the teachers did not know the software well. Thus, he inserted footnotes on each page (Footnote 1 in Table 3).

Tom modified the teaching sequence proposed by the official UC version. Instead of identifying the role of ρ through the history window after stretching the curve $y = \sin x$ at random, he proposed to have the three graphs in the same coordination system. Then, he suggested that the students should identify the role of ρ kinesthetically by stretching the graph of $y = \sin x$ until coinciding with the graph of $y = 2\sin x$ and $y = 3\sin x$ (Fig. 6). In Tom's approach, students' conceptualization of ρ is inextricably linked to the use of the stretch tool. Tom attached added value to the dynamic manipulation tools for mediating the meaning of the targeted relationships. His design seems to target the development of trainees' TPCK by interrelating mathematical content (i.e., the role of ρ) with technology (i.e., stretch tool) and pedagogy (i.e., facilitating understanding through dynamic

⁹ The stretch tool allows mouse-driven horizontal and vertical stretching of the graph. The corresponding magnitude of the stretches changes dynamically during the stretching and appears in the right corner of the "Input field." A history window in the graph allows viewing the formulas of the transformed functions.



Fig. 4 Stretching $y = \sin x$ with the magnitude of the stretch appearing in the "Input field" (a) and the produced graphs (b)



Fig. 5 Formulas of the transformed functions (Fig. 4b) and the magnitude of the stretches in the history window

manipulation of the graphs). This approach also reveals Tom's conception of the construction of mathematical knowledge according to which, dynamic manipulation of mathematical objects can be a precursor mediating the transition to more formal understandings.

The document "structure-for-scenario-design" was in the form of a concept map providing a figural representation of the issues that a teacher has to rethink before developing a scenario. For instance, one concept map cell that included the phrase "a scenario needs a teaching design" was connected with arrows to five other cells entitled "what," "why,"

Table 3 Extract from	Tom's worksheet	for teachers
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Questions	Rationale
In the table window, fill in two more columns $z = 2\sin x$ and $w = 3\sin x$ using different colors. Send points (x, z) and (x, w) to the graph window What do you observe as regards periodicity, symmetry, monotonicity and extremes? Design the graphs of the above functions together with the graph of $y = \sin x$ in the same coordination system	Study of function $y = \rho^* \sin x$. What is the role of ρ ? Multiple representations of the relation between $y = \sin x$ and $y = 2\sin x$, $y = 3\sin x$ (table: visually, graph: visually and kinesthetically)
Stretch and contract the graph of $y = \sin x$ through the use of the stretch tool (Note 1) so as to coincide with the new points of $y = 2\sin x$ and $y = 3\sin x$. (The magnitudes of the transformation have to be 2 and 3, respectively.) What do you observe?	
Experiment with other magnitudes of transformation (i.e., vertical stretch/contract) through the use of the stretch tool. Check your conclusions through the use of the calculator or the table	

Before answering this question, you have to activate the option "Show transformations"



Fig. 6 Stretching $y = \sin x$ to coincide with $y = 2\sin x$ and $y = 3\sin x$

"where," "how" and "how long." These cells were further linked to cells including more specific questions such as: "What do we plan to teach?" "What is new about our approach?" "For whom are we designing?" "What kind of tools are we using?" and "How much time is needed for implementation?" In his activity report, Tom explained his choices: The scenario structure depicted in the map does not follow the official scenario template, but it includes all the issues that a teacher has to consider when designing a scenario. It is closer to everyday teaching practices. It is important for the teachers to understand clearly what they have to do and then to form their scenario according to the official structure.

Tom's rationale stemmed from his need to bring teachers' everyday practices "closer" to the theoretical aspects of mathematics teaching addressed by scenarios. Working at the professional level in terms of double IG, he developed new didactic tools with the aim of bridging the distance between the language of official resources and the language of practitioners.

Tom's DW aimed to help teachers conceptualize the demanding way of design required by the course. He seemed to consider teachers "as students" who needed familiarization with technology, while at the same time he targeted their transition to "teachers of students" by introducing directly the link between tools and pedagogic/didactic aims. He did not intend to engage trainees in designing lessons directly for their students, but to provide them with appropriate tools for designing their own lessons in the future and targeting complicated forms of knowledge (i.e., TPK, TPCK). He aimed to explain to the teachers from his own perspective—issues related to the efficient integration of technology in the teaching and learning of mathematics through worksheets and scenarios. He seemed to take the role of "explainer" (Ernest 1994) when targeting teachers' understanding as regards the connections between scenarios and worksheets and the unified knowledge for meaningful mathematics teaching.

The operational invariants underlying Tom's design were related to: (a) his epistemology for the teaching and learning of mathematics; (b) his observation in CTES concerning trainees' difficulties with (i) FP, (ii) understanding the relationship between scenarios and worksheets, and (iii) constructing their own scenarios; (c) his conception of trainees "as students" who need particular supporting structures/tools and "explanations" (by the TEs) to reinforce their TPK and TPCK in order to make the transition from their personal IG to their professional IG.

Ian and Jim's cases

Ian and Jim chose to follow the agenda of the TEs they observed. Their lessons in CTES concerned the teaching of linear functions with FP. Their teaching was based on one of the twelve official scenarios provided for the course. The problem was the following: "A salesclerk sells three products A, B, and C at different prices. After every sale, the clerk records the quantity x (in kg) and the amount y of money received. After 20 completed sales, the clerk entered the values in two columns in the FP Table window so as to check: (a) how many sales were made of each product and (b) if mistakes had been made in price computation. In how many ways can the clerk conduct the check?" The official scenario's indicative design suggested a three-phased teaching sequence. Below is a description of phase 1.

• Phase 1 (Exploring proportional relations): A ready-made table containing 20 pairs (weight, price) of 20 sales in two corresponding columns is provided. Teacher can chose or discuss with students the possibility of filling in one more column defined by the ratio z = y/x (Fig. 7). In this case, students are expected to notice that this ratio takes three particular values (0.6, 1.2, 2.1)—unless a value corresponds to a mistaken

Fig. 7 Weight-price pairs of 20 sales in the table window

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8	۲	
Х	γ	z=y/x
weight	price	
0.8	1.68	
0.2	0.24	
1.6	1.92	
1.3	0.78	
5.5	11.55	
1.9	3.99	
11.5	6.9	
3.8	4.56	
3.4	2.04	
6.2	4.96	
2.2	2.64	
3.5	7.35	
5.3	3.18	
6.8	4.08	
5.7	6.84	
4.7	9.87	
15	9	
3	6.3	
8.1	9.72	
5	3	
4		

computation. These values can be an indication for the existence of y = ax relations. Then, the students send the points (x, y) to the graph window (Fig. 8) and they are expected to notice that the three values of the ratios correspond to three different groups of collinear points. Also, it is suggested that they construct the graph y = x and stretch it vertically so as to coincide to the existent groups of points (Fig. 9). The tool provides the corresponding formula for each transformation of the graph. The students are expected to explore the problem either algebraically (i.e., by linking the corresponding ratios to linear functions) or geometrically (i.e., by dynamically manipulating the graph of y = x) and to connect the three ratios encountered in the table to the three values of a (of y = ax), respectively.

Ian's DW Ian decided to take into account in his design the following parameters that he observed: (a) the different levels of teachers' familiarization with digital technologies; (b) difficulties with particular functionalities of FP; and (c) difficulties in conceiving the links between the different representations of FP. Thus, he started his lesson in CTES by demonstrating in an interactive whiteboard particular functionalities of FP. Then, he gave the teachers a worksheet that he had developed including the ready-made table of the official scenario. This worksheet was structured in the form of step-by-step instructions ensuring the correct use of FP tools for the requested activities. For instance, these are the instructions concerning phase 1:"Table window: Fill in the 3rd column with the ratio y/x and press enter"; "Write down the resulting values and explain what they show"; "Table window: Send points to the graph"; "Graph window: Graph \rightarrow graph choice- $s \rightarrow$ Tick 'Show transformations' \rightarrow Click on the icon 'y = ...' to create the graph of y = x."

In his activity report, Ian described his choices:



Fig. 8 Points sent from table to graph



Fig. 9 Stretching vertically y = x to pass through the three groups of points. Point (6.2, 4.96) will be left out

My design follows the official material, but my worksheet is more instructional due to better time control... Though the worksheet should be designed to provide learners with freedom to pose and answer their own questions, I chose to make it more instructive – probably more than needed – so as to support teachers' familiarization with the available tools while solving the problem... Additionally, the

trainer of this particular CTES classroom had a similar approach in his own teaching...

During a reflective session, Ian again justified his choices:

I would prefer a less instructive design. But I was anxious about time. Through my choice, the worksheet was completed by all teachers in time. They liked the environment and seemed to have learned the FP functionalities better.

Teachers' and students' engagement in exploratory activities with digital tools was a central idea underlying the UC course design. Ian seemed to initially share this view, but he finally designed a rather instructive lesson. He considered teachers exclusively "as students," as he gave priority to the correct use of tools. Also, he exploited the corresponding scenario as a vehicle for teaching FP functionalities, rather than as a means to engage the teachers in reflecting on how to transform the scenario into their own teaching plan. Thus, Ian targeted the development of trainees' TK and TCK by emphasizing the ways that the available tools could be interrelated with subject knowledge. As regards the connection between Ian's epistemology and practice, we note that although his conception of the mathematics learning with the use of technology was probably closer to less instructive approaches, in practice he moved to the role of "instructor" (Ernest 1994) targeting skills' mastery and correct performance through a strict following of the worksheet. While working at the level of his professional IG, Ian seemed to have tried to secure teachers' familiarization with tools and content (teachers' personal IG) as a prerequisite for their future passage to professional IG taking the role of "instructor."

The operational invariants underlying his design were related to: (a) his emphasis on constraints and opportunities afforded by the context (e.g., time restrictions, technological environment) which led him to follow instructive design and practices, respectively, regardless of his (possibly different) epistemological considerations for the teaching and learning of mathematics; (b) his observation in CTES concerning: (i) the different levels of teachers' familiarization with digital tools, (ii) their difficulties with FP and (iii) the previous teaching model adopted by the trainer in CTES; and (c) his conception of trainees mainly "as students" who need detailed instruction in order to overcome their difficulties.

Jim's DW Jim observed another trainer's lessons in CTES that included teachers' initial familiarization with FP tools. In his observation form, he noted the need to introduce teachers to the use of the stretch tool. In a reflective session, Jim mentioned that he initially demonstrated on the interactive whiteboard technical issues related to FP and then introduced the stretch tool. His worksheet included the following open tasks for the teachers (without any kind of instructions, even for aspects of the software): (1) The teachers were asked to work in groups of two and provide both an algebraic and a geometrical solution for the given problem; (2) group discussions for the potential findings of a group of students and ways to approach them; (3) whole class discussion of the above findings; (4) whole class discussion for the added value of FP; (5) design of indicative questions for the students by each group of teachers; and (6) presentation of groups' design in the classroom.

Jim described his lesson in a reflective session:

I gave the teachers a table with two columns corresponding to the weights and the prices for 20 sales of the three products. There was also a third empty column. I did not tell them to calculate a ratio or to send the two columns to the graph. I was curious to see how many groups would be able to find them... I asked them to solve the problem algebraically and geometrically... I organized the class in groups of two

following their trainer's model... All groups found at least one solution. I passed through the groups and encouraged them to find another one... They discussed in their groups, they explored, I showed them how to stretch y = x... I was stressed by the possibility of not being able to use the FP tools. Thus, I had prepared another worksheet including more detailed instructions for the tool use. Finally, it was not needed... Generally, I believe that the level of instruction should be gradually decreased. When we first introduce a kind of software we need to prepare a more instructive worksheet. But in the next lessons the worksheets should be as open as possible. The aim is to support teachers' thinking. You cannot tell them continuously 'press this' or 'press that'...

Jim's design was closer to the ideas favored by the UC course. Exploration of mathematical ideas, connections between different representations and collaborative work had a central position in Jim's design and revealed his epistemological conceptions of the teaching and learning of mathematics. He took the risk of implementing the specific scenario in the form of open worksheets and of confronting potential teachers' difficulties with FP tools on the spot. His design reveals that Jim initially considered teachers "as students" and through a social process which evolved in six phases, he progressively came to view them as "teachers of students." Particularly in phase one, he engaged teachers in exploring the problem "as students" (geometrically and/or algebraically), while in all subsequent phases, he asked the teachers to adopt the role of "teachers of students" through a series of activities. In terms of Ernest (1994), Jim adopted the role of "facilitator" by targeting problem posing and solving, and favoring the development of meaningful material by the teachers themselves. He took into account the fact that the main functionalities of FP had already been taught in previous lessons by the official trainer. Facing the challenge to balance the correct use of technological tools and their integration in creating meaningful mathematical representations, Jim did not seem to be interested in technological skills per se, but rather how these would be incorporated by the teachers in their own didactical design. Thus, his choices revealed that Jim targeted trainees' more complicated forms of knowledge (i.e., TPK, TPCK), as his design intertwined the mathematical content of linear functions and its pedagogy with technology (approaching y = axalgebraically or geometrically through ratios and dynamic manipulation of y = x with the stretch tool, respectively).

The operational invariants underlying Jim's design were related to his: (a) epistemological conceptions of the ways mathematical knowledge should be approached though the use of technology; (b) observation concerning the teaching agenda of the official trainer in CTES; and (c) conception of trainees initially "as students" who work in groups to explore different solutions of the given problem with the use of tools, and subsequently as "teachers of students" who are facilitated by his mediation to move gradually toward their professional IG.

Discussion

First research question: Features of the practicum context facilitating the trainees' transition from personal to professional IG

Our analysis revealed that the trainees' transition from personal to professional IG took place during the phase of observation in CTES in conjunction with the discussions in the reflective sessions. According to the structure of observation, each trainee could select a critical event and bring it to the discussions. Thus, a multiplicity of issues emerged directly from their own classroom experiences, the main of which were: the importance of transforming existing materials into didactical instruments and the different ways by which this could be achieved; the dilemmas stemming from the integration of technology in the teacher education courses and the targeted forms of TPACK knowledge for teachers; the trainees' conceptualization of their dual role "as students" in UC and "as trainers of teachers" in CTES, as well as of the dual role of teachers' in CTES "as students" and "as teachers of students"; the trainees' awareness that each of these dual roles is related to different targeted forms of knowledge ranging from TK to TPCK. The structure of the practicum based on the interconnection between observation in CTES and reflection in UC facilitated the emergence of these issues. In conjunction with our interventions (e.g., by introducing theory), these factors seemed to have supported the trainees' passage to the professional IG (Fig. 10). Thus, our findings corroborate existing research revealing that TEs' growth appears interrelated to practice.

Second research question: Operational invariants underlying the trainees' DW

The analysis revealed that the formula Resources + usages + operational invariants \rightarrow New document for each one of the three trainees' DW can be configured as shown in Figs. 11, 12 and 13. Each figure informs Fig. 1 about distinct documentational geneses characterized by different schemes of utilization (usages + operational invariants) leading to three kinds of documents: explanatory, instructive and facilitative.

The analysis showed that trainees' observation in CTES influenced differently their DW according to their focus: Tom and Ian focused on teachers' difficulties (with tools or more general), while Jim decided to confront potential teachers' difficulties with FP tools on the spot. Ian and Jim adopted the official CTES trainers' teaching model and continued their didactic agenda, respectively.

Besides, in practice trainees adopted different roles for themselves as TEs ("explainer," "instructor," "facilitator") and through their designs they targeted different forms of TPACK knowledge depending on their conceptions of the teachers' role in CTES. It seems

	 Discussions during the <i>reflective sessions</i>: issues emerging directly from the trainees' classroom experiences trainers' interventions 	
Observation +	 need for transforming the existing materials dilemmas from the integration of technology – targeted forms of knowledge for teachers dual role of teachers' in CTES "as students" and "of students" 	⊣ Trainees' DW

Fig. 10 Interrelation between the context of the practicum and trainees' DW

Existing resources (official documents): - One scenario from Notes for teaching in CTES - The structure		 Operational invariants (inferred from the usages) 1. Focus of his observation in CTES on trainees' difficulties with: FP the relationship between scenarios and worksheets construction of own scenarios 2. His epistemology for the teaching 	The new documents: <i>Explanatory</i> documents targeting TPK and TPCK
of scenarios		 and learning of mathematics with technology: students' construction of mathematical meaning through manipulation of representations, teachers' learning of technology through connecting tools and pedagogic aims 3. His conception of trainees "as students" who need supporting tools and "explanations" in order to be able to design their own lessons for students 	

Fig. 11 Tom's case: documentational genesis of an explanatory document



Fig. 12 Ian's case: documentational genesis of an instructive document

that the role of "explainer" is related to conceptions of teachers initially "as students" who can be able to become teachers "of students" through DW targeting TPK and TPCK. In contrast, the emphasis on contextual constraints and opportunities (e.g., time restrictions,

Existing resources	Operational invariants (inferred from the usages)	The new documents:
(official document): + One scenario from Notes for teaching in CTES	 Focus of his observation in CTES on: the teaching model of the official trainer in CTES the official trainer's agenda His epistemology for the teaching and learning of mathematics with technology: creating meaningful representations through problem posing and solving, intertwining the mathematical content and its pedagogy with technology His conception of trainees initially "as students" who collaboratively explore different solutions of the problem with the use of tools and subsequently as "teachers of students" who are facilitated by his mediation to move towards professional IG 	<i>Facilitative</i> documents targeting TPK and TPCK

Fig. 13 Jim's case: documentational genesis of a facilitative document

technological environment) and the conception of teachers mainly "as students" seem to constitute features of the role of "instructor." In this case, the "instructor's" DW targets mainly teachers' TK and TCK and bypasses a focus on pedagogical aspects (the P aspect of TPACK). "Facilitator" at first considers teachers "as students" who will become "teachers of students" through the mediation of a DW that targets mainly teachers' TPK and TPCK. But one critical difference is that the "facilitator" views this process to be interrelated with the social orchestration of the classroom involving collaborative group work and joint reflection and Jim's mediation is adapted according to the differentiated teachers' or groups' needs/difficulties. This indicates another difference between the "facilitator" and the "explainer": The former is flexible enough to address these needs/difficulties that are expected to emerge through the teachers' interactions with the open worksheet, while the latter predefines the teachers' needs and the ways to overcome them through the explanatory documents.

The analysis also indicated different ways by which the course philosophy and the trainees' epistemologies affected their DW—or not. It seems that the emphasis on constraints and opportunities afforded by the technological context is related to more instructive DW focusing mainly on the T and C aspects of TPACK, targeting skills' mastery and correct performance. On the contrary, a DW emphasizing the P aspect is related to epistemologies that view knowledge as a unified body. In this case, the role of a trainer is to help teachers think and interconnect the different features of knowledge so as to be able to enrich the curriculum with additional problems and activities. Also, the emphasis on the P aspect seems to be connected to conceptualizations of knowledge as a process of inquiry based on posing and solving problems and the aim of a trainer is to facilitate teachers to adopt an active role in constructing the mathematics curriculum. Additionally, the inclusion of the P aspect in trainees' DW appears to favor teachers' transition to professional IG since it helps teachers consider the use of technology in mathematics teaching through the lens of pedagogy.

OPERATIONAL INVARIANTS								
Categories	Specific characteristics							
Contextual constraints and opportunities	- Time restrictions - Technological environment							
Observation	 Teachers' difficulties (with tools or more general) Official trainers' teaching model and agenda 	Teachers' difficulties (with tools or more general)	Official trainers' teaching model and agenda					
Epistemology for the teaching and learning of mathematics with technology	The correct use of tools as a prerequisite for linking tools and content	-Students' learning through manipulation of representations -Teachers' learning through connecting tools and pedagogic aims	Teachers' learning through: -Creating meaningful representations and problem posing/ solving -Intertwining the mathematical content and its pedagogy with technology					
Knowledge,	Kinds of documents							
practices	Instructive	Explanatory	Facilitative					
TPACK forms	TK and TCK	TPK and TPCK	TPK and TPCK					
Teachers' roles	Teachers as students → Detailed instruction	Teachers as students → Instruction providing supporting tools and 'explanations' by the TE → Teachers of students	Teachers as students \rightarrow Instruction based on collaborative group work and joint reflection \rightarrow Teachers of students					
Trainee TEs' practices	Instructor	Explainer	Facilitator					

Table 4 A synthesis of the three emerged documentational geneses

An overall description of the above findings is presented in Table 4 facilitating a comparative view on the three kinds of documentational geneses.

Conclusions

Responding to Even's (2014) call for further research on programs aiming at TEs' education, our analysis showed that the practicum context facilitated the trainees' transition to the professional level. Double IG was used as a tool for analyzing the trainees' passage from the level of personal familiarization with the digital tools to their integration in teaching as part of the TE's profession. It was also used as a theoretical construct that helped the trainee TEs to become aware of and target (through their designs) teachers' transition from the role of "teachers as students" to the role of "teachers of students." Observation was crucial in this process since it mediated the connection between theoretical knowledge and practical aspects of teaching.

Documentational genesis was used to analyze the three cases of trainees' DW. The identified operational invariants were related to: (a) the trainees' focus of observation in CTES classrooms (either on trainers' teaching model/agenda or on teachers' difficulties); (b) the importance they attributed to the constraints and opportunities provided by the wider educational context; and (c) epistemological issues regarding the teaching and learning of mathematics through the use of technology and how the trainees' conceptualized the role of teachers in CTES. These operational invariants contributed to the geneses of "explanatory," "instructive," and "facilitative" documents. Each kind of document emphasized different aspects of TPACK while entailed distinct roles of TEs in teaching practice and different TEs' conceptualizations of the teachers' role.

By focusing on trainee TEs' didactical design and implementation, our study has three main contributions in the field of TEs' education in technology-enhanced mathematics.

First, it responds to the need for proposing categories of operational invariants underlying teachers' documents designed for their teaching in different contexts (Gueudet and Trouche 2009). Specifically, we offer an in-depth analysis of trainee TEs' designs in the context of their practicum, leading to a categorization of the operational invariants underlying their documents, an issue that has not been studied before. Our findings also reveal how these operational invariants lead to the genesis of different kinds of documents.

Another contribution of our study is the connection between TPACK and other theoretical frameworks, a research challenge that has also been suggested (Voogt et al. 2013; Chai et al. 2013). The documentational approach in conjunction with the theoretical construct of double IG provided a basis for connecting the TPACK framework to TEs' epistemologies and practices. This allowed us to gain insight into the factors that influence the teaching of TPACK and lead to various kinds of documents addressing different aspects of TPACK. Thus, our theoretical approach can be integrated into the field of TEs' education to study issues of knowledge and epistemology in trainees' DW that is based on the use of technology.

Finally, our study can be exploited broadly in the design and structure of reformoriented PD programs for teachers and TEs in the area of technology-enhanced mathematics. The identified operational invariants seem to play an influential role in both the trainees' development of documents and their practices since they reveal how factors related to knowledge/epistemology are connected to different targeted forms of TPACK. Trainees' engagement in designing/implementing resources in the form of scenarios and digital artifacts seems to help them to rethink a number of issues related to technology and favor more complicated forms of knowledge such as TPCK. This kind of activity seems to facilitate reflection on and questioning of the context and the content of teaching and supports what Tzur (2001) calls "a shift in the kind of reflective analysis" (p. 280) in which TEs are engaged. As regards the structure of a PD program, our analysis showed that practicum procedures consisted of observation-reflection-design-implementation-reflection should spread throughout the duration of a TEs' program. Early introduction of observation, reflection on emerging issues and negotiation through the lens of theory provides rich opportunities for educators to help trainees readdress issues of content and pedagogy with the use of technology.

In this paper, we provided a detailed analysis of three trainee TEs' DW as representative of the produced documents in the context of their practicum. Further research is needed in order to more deeply explore other aspects of TEs' education, such as the relation between the teaching model of the observed trainers and the trainees' design as well as trainees' practices and norms during their implementations in teacher education classrooms.

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