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mascil

DZLM

Conference Proceedings

Educating the educators: international approaches to scaling-up professional development in mathmatics and science education

15 -16 December 2014
in Essen, Germany

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Conference Proceedings in Mathematics Education 2

Teachers are the decisive factors in ensuring the achievement of creative and sustainable learning outcomes in mathematics and science education, in fostering young peoples' competences and enabling them to become critically thinking, responsible and active citizens. Teacher professional development (PD) is the key to effecting change in mathematics and science education throughout Europe. Whilst the relevance of teacher PD is beyond dispute, there is still too little knowledge about how to scale-up professional development successfully and sustainably.

Initiating discourse and leveraging international exchange about concepts and experiences concerning this essential topic were the main aims of the conference 'Educating the Educators – International approaches to scaling-up professional development in mathematics and science education', which took place in December 2014 in Essen, Germany.

The conference was hosted by the EU-funded project mascil (mathematics and science for life), coordinated at the University of Freiburg, Germany, and the DZLM (German Centre for Mathematics Teacher Education) and brought together researchers and practitioners: teacher educators, multipliers and teachers who are all engaged in the field of maths and science education. This volume reflects particpants' conference experiences.

Further, we are pleased to provide here background information and aims of the conference issue, as well as conference conclusions and an outlook on future needs and trends. The papers relate to four conference tracks, each addressing a different approach to scaling-up professional development in maths and science education:

(1) Scaling-up with multipliers in face-to-face professional development courses

(2) Blended learning concepts and e-learning support

(3) Disseminating and scaling-up through materials

(4) Professional learning communities.

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Conference Proceedings in Mathematics Education

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EDUCATING THE EDUCATORS: INTERNATIONAL APPROACHES TO SCALING-UP PROFESSIONAL DEVELOPMENT IN MATHEMATICS AND SCIENCE EDUCATION

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Integrating inquiry-based tasks and the world of work in mathematics and science teacher education -

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1 The main focus of the paper

In this paper our goal is to explore how tasks aiming to promote inquiry-based learning (IBL) and the integration of the world of work (WoW) in mathematics and science teaching, developed in the context of Mascil project (see: www.mascil-project.eu), have been embedded into prospective teachers' (PTs') mathematics and science education. Our focus is on PTs' engagement in designing, implementing and reflecting on Mascil tasks or tasks developed by them in the same spirit. In particular, we present examples of such tasks, their transformation in actual teaching and PTs reflections on their experience. Finally, we discuss the emerging issues as regards the PTs' engagement in working with tasks that support IBL in workplace contexts and the nature of teacher education that promotes the integration of IBL and the WoW in the teaching and learning of mathematics and science.

2 Theoretical framework

Research informed teacher education programs have been considered as contexts for prospective teachers' professional development. For instance, some of the main approaches adopted in mathematics teacher education programs are: engagement in mathematics-related tasks to develop awareness of the mathematical structure and power of tasks; designing, analysing, and trying out tasks with learners; observing and analysing task implementation in the classroom; and using theory to explain teachers' experiences (Watson & Mason 2007). Through the above approaches teachers and prospective teachers challenge existing knowledge and perceptions of teaching and learning they had developed as students themselves and recognize the potential of inquiry and experimental teaching. In general, tasks have been recognised as important resources in mathematics and science teacher education that are conceptualised in different ways by teachers (Remillard 2005) and also transformed through the process of designing and implementation (Henningsen and Stein 1997).

The dynamic character of tasks as amenable to changes is strongly related to the process of inquiry in learning and teaching. Our theoretical approach prioritizes the issue of inquiry in students' learning considering as important "… to abandon efforts to teach by inquiry in favour of teaching for inquiry"

(Settlage 2007, p. 204), that is to say, to help students' practice and develop the skills of inquiry. Jaworski (2006) proposes three forms to address the inquiry practice: inquiry in mathematics; inquiry in teaching mathematics; and inquiry in researching the process of using inquiry in mathematics and in the teaching of mathematics. In this study PTs are engaged in all these forms of inquiry practice through working with tasks and resources as students, transforming these tasks in the process of designing for teaching, observing the process of inquiry based learning through students' engagement with the tasks and researching the inquiry process in actual teaching. In this perspective, tasks are considered as means for making links between theoretical ideas and research-based findings to the actual teaching of mathematics and science.

The situation becomes more complex when the tasks refer to realistic and workplace contexts that pose prospective teachers new challenges related to the adaptation of these tasks in the school context, to classroom management and to the process of making links between the context and the underlying mathematical and scientific concepts and processes (Ainley, 2011; Roth, Mc Ginn & Bowen, 1998). In this direction, research in science and mathematics education indicates the need of a situational perspective in integrating inquiry in teaching (Crawford 2007; Friedrichsen and Dana 2005; Hogan and Morony 2000; Nicol 2002; Windschitl 2004) and promoting teachers' professional development (Borko, 2004). WoW may function as a source of valuable situations, meaningful contexts for mathematics and science teaching, as they can support learning through interaction in the social and material world (Derry and Steinkuehler 2006). Research evidence concerning teaching and teacher education on the basis of WoW contexts is rather limited.

Since Mascil tasks are not content specific and are based in vocational and WoW contexts, it is important to explore what kind of issues may emerge as regards the integration of these tasks into curriculum and teaching through PTs' engagement in designing, implementing and reflecting. Prospective teachers have to cope with more complex situations both at the level of defining their mathematical or scientific goals and at the level of managing the use and transformation of the tasks in the classroom. Our study aims to contribute in understanding this complexity.

3 The context of the study

Our perspective in this study is country-specific as we refer to three postgraduate courses in Greece and target-group specific as the participant teachers are prospective teachers who are going to teach in secondary general and vocational school mathematics, applied science and engineering subjects. The first and the second course are parts of a two-year master's programme in mathematics education offered in the Department of Mathematics of the University of Athens (UoA). This programme is offered mainly to mathematics graduates. The third course is part of an one-year long post-graduate teaching certificate program that focuses on teaching and learning issues, models, strategies and tools that can be applied in different secondary school subjects. This programme is offered for all higher education graduates in the School of Pedagogical and Technological Education of Patras (ASPETE). As the teacher educators are the members of the Greek Mascil team, during the academic year 2013-2014 they encouraged PTs to use Mascil classroom tasks or develop their own in the same spirit as part of their teacher education activities. Below, we provide in section 4 a description of the methodology of the study including a brief discussion about the context of each course, the teacher education tasks and the process of data collection and analysis. In section 5 we present some preliminary findings while in section 6 we summarize emerging issues for mathematics and science teacher education.

4 Methodological issues

4.1 The context of the courses

The first and the second course emphasize theoretical issues in mathematics education and familiarize PTs with main research findings. An ultimate goal of the two courses is the linking of these findings to the actual teaching through different actions such as reading and discussing research papers, designing and analysing tasks, conducting didactical interventions in groups of students, designing and teaching a lesson in the classroom and analysing students' work on the basis of research findings. Instructional practice in the teacher education sessions aim to support PTs' reflections on their experiences with the use of the designed tasks in teaching as well as to link emergent issues with existing research. A distinct feature of the second course is the integration of technology in the teaching and learning of mathematics. In this course, the cycle design-implementation-analysis is based on tasks addressing a pedagogically sound use of technology for the teaching and learning of mathematics with an emphasis on the role of tools and context in students' conceptual understanding. We note that the first course is offered in the winter semester and the second one in the spring semester.

The third course focuses on teaching and learning issues, models, strategies and tools that can be applied in different secondary subjects. It emphasizes interdisciplinary and visual approaches in teaching. Its main goal is to prepare PTs for classroom teaching and lesson design based on an inquiry approach.

4.2 The teacher education tasks and PTs' engagement

Concerning the first two courses, the PTs were asked to collaborate in groups and prepare an account on designing, implementing and evaluating a teaching intervention with a focus on students' mathematical thinking. Initially, they had to select a Mascil task, or to develop their own. By reading research papers in mathematics education related to IBL and WoW as well as to students' understanding of specific mathematics concepts related to the task, the PTs were engaged in the process of defining their teaching/research goals and transform their tasks accordingly. Their designs were discussed in the course and the final versions of the tasks were used in the classroom or outside the classroom with small volunteering groups of secondary school students. Finally, the analysis of the interventions and the PTs reflections were presented in the course. As regards their main aims, one difference of the two courses concerned the integration of IBL and WoW in PTs' activity in their teaching intervention. In the first course, the WoW was seen as a context of the task and the PTs were asked to focus on IBL in analysing students' mathematical thinking and make connections to mathematics education research. In the second course, the PTs were directly asked to explore how the WoW was connected to IBL and students' mathematization of the problem. This change was due to the fact that when the second course was offered (spring semester) the teacher educators had already had an initial experience through their participation in Mascil and they had started to form a clearer view of the program's critical questions and challenges such as the making of connections between the WoW and mathematics and science teaching in schools.

During the third course, PTs from different disciplines were asked to study curriculum documents and textbooks in their field and identify units showing elements of integration of mathematics and science with other disciplines and/or the WoW. They had also to read a number of papers referring to the meaning of interdisciplinarity and how it can be connected to the WoW. For their final assessment they had to create their own video or select one from the internet which would be used as a basis for designing a scenario for teaching a specific unit of mathematics or science. PTs prepared written accounts of their scenario and the related tasks, its uses with students, as well as their arguments for their choices. Similar approaches and examples had already been discussed in the course. Some PTs applied their scenarios in the classroom.

4.3 Data collection and analysis

Fourteen PTs in the first course, eleven in the second and a hundred and twenty five in the third worked with Mascil tasks or other in the same spirit. The data consisted of: (1) PTs' written accounts in which PTs had to describe the rationale of their designs and their experience from their implementation, (2) the material constituting the PTs' designs for their teaching interventions (scenarios, worksheets, power point presentations, etc.), (3) selected videos from the course discussions, and (4) PTs' journals. For the data analysis, we adopted a broadly data grounded approach (Straus and Corbin 1998) focusing on qualitative content analysis of students' design and implementation on classrooms in terms of how inquiry realized, how the WoW has been integrated and what aspects characterized PTs' tasks. Moreover, in order to capture the evolution of PTs' design of tasks integrating IBL and WoW, we concentrate on examples selected from their work.

5 Findings

5.1 PTs' design and implementation

5.1.1 PTs' design and implementation in the first course

Here, we focus on the work of one group of PTs who participated in the first and second course. This group worked on the same task designing different types of intervention in the two courses. The task was transformed by the PTs according to the requirements of each course, but our common focus on the integration of IBL and WoW allowed us to view comparatively the designs they created and thus to capture the potential differences in the ways they addressed the integration of IBL and WoW in their teaching. The classroom task was chosen by the PTs from the Mascil site and concerned the covering of a backyard with circular pave-stones that leaves the minimum uncovered area. It is an optimisation problem and the aim is to find the configuration of circles with the maximal density. As regards the WoW, arranging objects in this way is a skill needed in many professions and everyday life situations. As regards IBL, students have to: (a) model the problem from the real world to the mathematical world, (b) identify possible solutions through the use of geometry, (c) select the best solution and (d) interpret the outcomes and reflect on the appropriateness of the solution.

The PTs transformed the context of the task and referred to the work of an architect/designer of exterior places. The problem was formulated as follows:

"You are working as an architect/designer of exterior places. One of your clients has a rather difficult taste: he doesn't like vertices; therefore, he would like his backyard to be covered with pave-stones in the shape of circular discs. He also wants the pave-stones to cover the maximum possible area, so as the grass that grows between the stones is as little as possible. Your job is to find a configuration for the circular pave-stones that leaves the minimum possible empty space between them".

The PTs designed the following teaching sequence in four phases:

Phase 1 (Experimenting with different arrangements through the use of circular objects): The students are given a sheet of paper and a number of particular circular objects (coins, backgammon checkers etc.) (Fig.1). Different pairs of students are given circular objects with different radius. The students are expected to experiment with different configurations and to realise that each one of them is based on a pattern created by a particular unit (a polygon) that can be seen as extended to the infinite plane.

Phase 2 (Recognizing the pattern in each arrangement): The students are asked to explore two particular arrangements through a model based on euros (Fig. 2) by focusing on the centre of the coins for each one of these, draw the polygons that are formed with vertices the centres of the coins and identify the emerging. The students are expected to design various patterns generated by repetition of polygons and, through this, to identify the structural unit of the pattern.

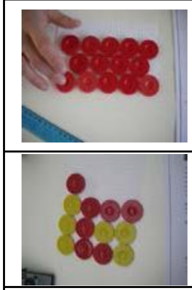

Figure 1. Students' arrangements with checkers.

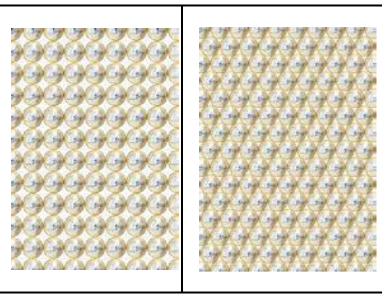

Figure 2. The two patterns of arrangements with euros.

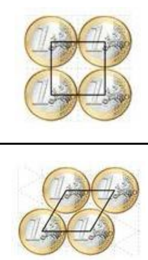

Figure 3. The units of the arrangements in Fig. 2.

Phase 3 (Calculating the percentage of the covered area): The students are asked to calculate the percentage of the area that is covered with coins for each one of the various patterns in their previous arrangements. The students are expected to: (a) conclude that this percentage depends on the type of arrangement and (b) identify that the percentage of the covering does not depend on the length of the radius.

Phase 4 (Proving the conjecture of the best arrangement): The students are given the pictures shown in Fig. 3 and they are asked to compare the uncovered areas when the unit of the pattern is a square and a rhombus, which have equal sides. Then they are asked to justify which arrangement is the best.

The task was implemented for two teaching hours in one class of 16 year-old students (10 boys, 22 girls) in one experimental upper secondary school in Athens. The implementation took place in the context of the Geometry course and the students worked in groups of three to five. Basic concepts needed for being engaged in the task (e.g., calculation of the area of various shapes such as polygons, circular sectors) had already been taught and the lesson was conducted by the group of the PTs. PTs wanted to investigate: "*(a) the process of modelling and possible students' difficulties with the mathematical concepts and (b) the students' dispositions towards IBL"*.

5.1.2 PTs' design and implementation in the second course

In their new design, the PTs kept the same problem and teaching sequence but integrated the use of digital tools in all phases and they transformed the questions of the students' worksheet accordingly. In particular, they designed dynamic geometrical figures (with Geogebra) to be distributed to the students in each phase with the aim to provide further opportunities for students to engage in experimentation and to enhance IBL. Below, we provide two examples.

Example 1: In the first step of the phase 1 of the activity the students are engaged in exploring the arrangement of a specific number of equal circles through the use of Geogebra (Fig. 4). By moving the circles freely in the plane the students are expected to identify potential solutions of the problem at the perceptual level. In the next step of the same phase, the students are provided with a slider for changing dynamically the radius of the circles with the aim to recognize if and how the changes in the radius influence the best arrangement. Also the number of the provided circles on the screen is not enough for covering all the available space so as to trigger students' attention to the fact that the pattern can be potentially extended to the infinite plane.

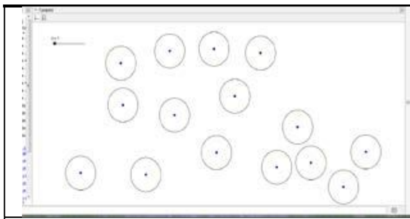

Figure 4. Moving circles freely in Geogebra.

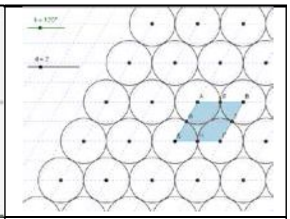

Figure 5. Dynamic manipulation of arrangement.

Example 2: In the last phase of the teaching sequence the PTs designed an arrangement that can be manipulated with the use of two sliders (Fig. 5). The first slider changes the angle created by the lines linking the centres of the equal circles and the second one its radius. Through dynamically changing the values of this angle the students can observe the way that the area of the figure changes in relation to the angle. By comparing the different arrangements, the students are expected to realise that in each pattern a circle is created by the different parts of the pave-stone within the unit of a pattern (the rhombus or the square). This realisation can help them to recognise that the best solution to the problem corresponds to a rhombus with angles 60 and 120 degrees. Then they can be engaged in proving this conjecture based on their existing knowledge of geometry.

The task was implemented with one group of two 16 year-old students since it took place near the end of the school-year and there was not enough time for the organisation of an intervention in the classroom setting. Before students' engagement with the task there was an introductory phase during which the students were familiarised with the available tools and functionalities (e.g., designing basic geometrical figures and polygons, defining circular sectors, measuring areas). PTs' goals in this case were: *"(a) to describe the mathematization of the problem by the students including constrains and opportunities for meaning generation and (b) to analyse how inquiry and WoW appear in students' activity"*.

5.1.3 PTs' design and implementation in the third course

Phase 1. The philosophy of Mascil was discussed with so as to study the extent that such philosophy is present in the school curriculum and textbooks. The analysis of their reports shows that their reference to connections of content to the WoW usually had a general character, while existed educational approaches dominated their references, e.g. cross-curricular teaching, the method of project, etc.. PTs often referred and suggested actions like specialists' visits in classrooms and students' visits at outdoors places and factories.

Phase 2. Students designed scenarios that aimed to link teaching and the WoW. On the basis of given examples they were asked to create or inquire appropriate video for connecting the WoW to the curriculum. For example, PTs watched the video in the following link (http://thefutureschannel.com/videogallery/an-engineer-and-her-robot/) and started to design students' tasks. Almost all PTs were involved in inquiry, searching for available resources and teaching materials. Three characteristic examples are presented below:

Example 1: The scenario of a PT, who was a mechanical engineer, is based on a video (https://www.youtube.com/watch?v=Zrp0RC3XTpw) explaining how a homemade windmill works and produces electric current as well as how can be constructed. The idea of this scenario is not strongly connected to the WoW, while the PT seems not to be able to discern the WoW from the practical activities. Its main task poses the question "What type of studies or professional knowledge should a scientist have in order to participate in a project aiming to construct a wind generator?" which is related to the WoW but in a general way.

Example 2: The scenario of a PT, who was a pharmacist, is based on a video (http://www.youtube.com/watch?v=3a4lwvve9A8) that presents how a Nuclear Magnetic Resonance center works. The main idea is interesting and it is accompanied by detailed worksheets for the students with differentiated questions for a group of physicists, a group of biologists, and a group of chemists. It also suggests a final common task, where all groups have to work together on a protein, and a chemical compound that is attached to it in order to contribute in the completion of this process. It includes analytic design and description of implementation, but it is extremely specialized and probably time demanding and unrealistic for applying in the classroom.

Example 3: The scenario of a PT, who was an electrical engineer, is also based on a video showing the installation of solar panels on the roof of a house in Greece (http://www.youtube.com/watch?v=SPGY9eqSvR0). The complete scenario can be found in the Mascil repository. The task asked students to propose how solar panels will be installed on a flat roof of a house taking into account a number of factors. The PT had professional experience on this type of installation. He implemented his scenario in a classroom of 10 students, 11th grade of a vocational school.

Figure 6. PT's Power Point presentation of photovoltaic unit.

In the first teaching period, the students watched the video and the PT presented different information about photovoltaic units through a power point presentation (Fig. 6), while during the second teaching period the students worked in three groups on the questions of the task. The PT was able to handle the task satisfactorily and had established good communication with the students. However, only one of the groups managed to proceed to the first set of questions. The others met difficulties with the basic mathematical knowledge like the calculations of the sides of a right triangle and the trigonometry and were reluctant to follow and try teacher's suggestions. They also met difficulties even with the simple arithmetic calculations. Their interest was mainly around practical issues, e.g. companies, programs for business financing.

5.2 PTs' reflections

In the first course, the PTs were enthusiastic about their collaborative engagement in the design and implementation of the teaching intervention and they felt that their knowledge about teaching and classroom reality was enriched. In their presentation in the course and in their journals' accounts they referred to: their expectations in relation to students' mathematical activity, the mathematization process and its connection to the mathematics content; difficulties in classroom management inherent in IBL teaching approaches and the importance of linking research to teaching. Some characteristic extracts from their power point presentation are the following: *"how can we take into account the authenticity of the context in our design and teaching?", "it is difficult to manage classroom interaction if you are the only teacher in the classroom", "the students find difficult to understand the language in this type of problems", "it is difficult to follow your plan concerning the duration of the intervention"*.

In the second course, the PTs felt again that they had a chance to design an innovative teaching intervention and they reported that their knowledge about integrating the use of digital tools in their teaching was widened. In their presentation in the course and in their personal accounts of their work the PTs reflected on their experience and indicated the new elements of their knowledge about integrating IBL and workplace contexts in mathematics teaching as follows:

(a) The complexity of the cycle design-implementation-analysis in practice when it concerns the integration of IBL and WoW in the classroom of mathematics. The PTs appreciated the difficulty to make links between the original task and reality. Based on their experience with the task, they recognised that if the backyard had been designed to have a specific shape the problem could become rather demanding for the students in terms of mathematical exploration. The PTs mentioned in their final report: "*A realistic problem connected to a particular workplace provides a challenge for the students to explore it. However, students can be trapped as they have to take into account the real constraints as it was the case with the borders of the backyard that it was too difficult for the students to manage … thus, what we learned from this experience is that design, implementation and analysis are too difficult and complex in practice"*.

(b) The critical role of the visual representations provided through digital tools in favouring students' engagement in conjecturing-inquiring-proving. The PTs appreciated the use of technology in influencing students' activity to more exploratory directions (*"The digital tools helped students to work with the problem visually, to make conjectures and prove them. When the students formulated one conjecture through the use of the tools, we asked them to prove it algebraically so as to strengthen their findings"*.

(c) The PTs' role as teachers in the intervention. The PTs were able to reflect on their own interventions to the group of students justifying their choice with references to the students' lack of familiarization with the functionalities of the software (*"In contrast to our previous intervention in the classroom, in this case our role was more directed because students were not familiar with the use of the software"*.

In the third course, despite the fact that PTs did not manage to provide scenarios that successfully linked content to the WoW, they were involved in an inquiry-based design of teaching scenarios. At the beginning, they complained that it is difficult to find a good video to use in their teaching. Finally they managed to produce plans of scenarios and tasks based on videos, that certainly were much more interesting than the usual routine teaching and felt a degree of satisfaction. However, their arguments for the value of their videos and scenarios reveal limited awareness of how school knowledge can be meaningfully connected to the WoW. However, some of the PTs came up with findings that showed that the school content is presented in a conventional form with small fragments of connections between subjects and the WoW and identified these missing elements. A characteristic extract from a PT's journal follows: "*The subject, Electrotechnology I, which we teach in the 11th grade, involves many topics related to other subjects, like physics and mathematics, and also to our everyday life. Specifically, when we talk about real technological establishments, e.g. solar panels, in our teaching, reference* *to the co-operation of electrical engineers, civil engineers and economists, as well as meteorological knowledge, whose participation is important for a successful establishment, is absent. These important aspects are omitted from their textbooks and the subsequent teaching.*" In another assignment the PT reports: "*I studied 9th grade biology curriculum and school textbooks. DNA helix is one of the most important topics. However, the school context presents the knowledge isolated from the importance of this knowledge for science, for specific workplaces, and professionals like criminologists. Identification of DNA in detection of crimes is a crucial procedure. Such a context could add students' meaning making concerning DNA.*"

5.3 Interpretation of PTs' activities

5.3.1 Commentary on the first and the second course

As regards the first course, in the initial part of their work the PTs read papers about IBL and students' geometrical thinking. In the design phase, the PTs had difficulty to link the theory and research to their intervention, but in the process of the course they realised its importance and were able to ground their research goals on existing research findings. However, the link between content specific studies and IBL was rather vague both in the design and in the analysis of the intervention. For instance, the arrangement of circles was seen by the PTs mainly as a way of covering a potential infinite plane rather than as a 'real' context providing constrains and limitations. Thus, in the design of the task and its implementation the PTs did not take into account the possibility to engage students in working with a model of a backyard whose sides have specific lengths. The targeted learning aim posed by the PTs concerned students' identification of the least uncovered area within the arrangement of four intersecting circles that constitute the unit of a particular arrangement. In their personal accounts of the implementation of the task, the PTs mentioned that their main aim was to help students conceptualise that a specific polygon created within circles (i.e. the unit of the arrangement) underlies the covering of the whole plane. As one PT put it:

"We wanted the students to understand the generalization to infinity, that is to say, the group of four circles is actually the constituting element of the whole (plane) that can be covered in the same way".

Also, IBL was not always evident in PTs' attention to classroom events in the analysis of their intervention. Overall, their main focus was on the modelling process and its relation to mathematics content as well as on students' motives to be engaged in inquiry. In their analysis and reflection the PTs seemed to pay attention to students' conceptual difficulties while IBL was investigated through students' written responses in a questionnaire given to them after the classroom intervention. Also the connection of the task to the WoW was rather weak as it remained only at the level of attributing the role of a designer to the students. This is probably related to the nature of the original task and to the directions given by the teacher educator.

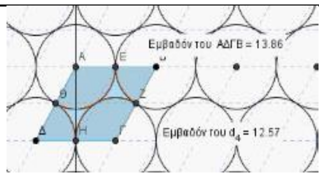

Figure 6.

As regards the second course, at the initial part of their work the PTs read papers about the relation between school mathematics and workplace mathematics. Some of these papers were presented in the class and in the discussion followed issues about the tensions inherent in the connection between school mathematics and WoW emerged. As mentioned earlier, the PTs decided to keep the same problem in their design but to integrate in all phases the use of digital tools. In their analysis, they provided a detailed account of students' engagement in IBL in terms of horizontal and vertical mathematization emphasizing the role of the digital tools in meaning generation. For instance, in the initial part of their exploration in phase 4 to answer which arrangement was the best one (i.e. based on a square or a rhombus) the students conjectured that a square and a rhombus have the same area. One PT intervened and asked the students to test that conjecture. One student had the idea to design the height of the rhombus and realised that there is a difference between the two areas (Fig. 6). Then, through dynamically changing the angle created by the lines linking the centres of the circles the students were able to observe that the height was decreasing and, as result, the area of rhombus was decreasing. Finally, the students continued to prove this finding through their existing knowledge of geometry. In their analysis the PTs concluded: *"We conclude that the use of tools here helped students' vertical mathematization".*

As regards the WoW, the PTs in the second course appeared to be more sensitive in integrating aspects of WoW and the context of the problem in their implementation of the task and the respective analysis. For instance, after describing students' experimentation to construct perceptually a conjecture for the best arrangement (phase 1), they enriched their analysis by referring also to one student's making of links between his experimentation and the real context of the problem (phase 2). Particularly, after selecting the first arrangement (figure 2) the student commented that this choice is better *"if there is not an economic problem"* implying that this type of arrangement would be more expensive since more circular pave stones need to be used. Overall, the PTs kept an open eye to the emergence of the workplace throughout the students' activity. In their analysis they mentioned that the students worked from the beginning having in mind one particular ('real') backyard and often they expressed concerns about the accuracy of their covering. For instance, the students wondered if particular arrangements they tested were appropriate for backyards of different shapes. Thus, through this kind of students' experimentation the PTs realised that the problem could become rather complicated for the students if the backyard had specific dimensions (e.g., a rectangle with specific length and width) since in this case the best arrangement could be different according to the shape of the available space.

5.3.2 Commentary on the third course

Engaging PTs in inquiry based activities especially in courses with large groups is a demanding issue. A few of them produced scenarios similar to the examples that had been discussed in the course. However, through the organized tasks of this course PTs obtained experiences of inquiry for their own teaching design and managed to find sources and materials in order to produce innovative teaching scenarios. Not all of them, though, managed to develop inquiry based activities for their students. WoW is easily taken to be a practical activity or an everyday application in science teaching. The learning goals of their scenarios usually were general, while tasks required general knowledge with weak connections to the WoW and to the meaning of specialists' contribution. This seems to be PTs' main difficulty, on one hand to identify appropriate contexts for connecting the WoW with the school content and on the other the development of meaningful tasks promoting curriculum learning through the use of the WoW. In terms of classroom implementation, basic knowledge needed for tackling tasks that use situations related to the world of work, when is distant from current teaching, creates obstacles in students' successful engagement. PTs are not always able to support inquiry learning, even if inquiry tasks are involved in their designs. Finally, the use of videos in classrooms attracts students' attention and adds authentic aspects of the WoW that are missing from described contexts of the WoW.

The analysis of their scenarios revealed a number of limitations in terms of the presence of WoW, although their ideas were generally interesting in terms of teaching. The scenarios' context was often too broad, while the suggested videos were not always closely related to the learning of aimed concepts. In terms of inquiry, PTs' scenarios put limited emphasis on how students are involved in inquiry based learning. Tasks coming along videos usually were of a general nature and not closely connected to the context.

6 Emerging issues

In the three courses, the PTs seemed to have developed their professional learning about integrating IBL and WoW in mathematics and science teaching. The integration of WoW appeared to be more difficult for the prospective teachers than designing IBL tasks and developing a more inquiry stance in teaching. This is expected as the introduction of the WoW in teacher education courses is new while IBL is an approach that is more broadly supported throughout their studies. The use or resources is another issue that emerged in all the three courses. PTs used research-based resources (e.g., research papers), hands on materials (e.g., coins, in the first course), digital tools (e.g., dynamic environments, in the second course) and internet resources (e.g., videos, in the third course). To identify or develop these resources and to explore their possibilities in mathematics and science teaching was not easy and required support from the teacher educators. However, PTs started to recognise their role in supporting mathematical and scientific inquiry and make learning meaningful.

Our findings pose new questions related to the nature of teacher education that promotes the integration of IBL and the WoW in the teaching of mathematics and science. PTs' courses need to be enriched with more specific examples of IBL activities related to the WoW. Normal curriculum and teaching neglect this dimension and PTs have to be enculturated in the teaching of mathematics and science that is meaningful for life and work. Teacher educators are demanded to offer clear guidance in order to support PTs to relate content knowledge to specific workplaces and life activities. This issue highlights two challenges that teacher education need to address: what school mathematics and science are necessary for life and work, but also what is the mathematical and scientific knowledge embedded in real life and work situations that can be involved in classroom teaching.

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References

Ainley, J., 2011, "Developing purposeful mathematical thinking: A curious tale of apple trees". In B. Ubuz (Ed.), Proceedings of the 35th Conference of the International Group of the Psychology of Mathematics Education (PME), Vol. 1, pp. 1-16, Ankara: Turkey.

Crawford, B. A., 2007, "Learning to teach science as inquiry in the rough and tumble of practice", Journal of Research in Science Teaching, Vol. 44, pp. 613 – 642.

Derry, S. J., and Steinkuehler, C. A., 2006, "Learning and Instruction, Cognitive and Situative Theories", Encyclopaedia of Cognitive Science, Wiley Online Library.

Friedrichsen, P. M., and Dana, T.M., 2005, "Substantive-level theory of highly regarded secondary biology teachers' science teaching orientations", Journal of Research in Science Teaching, Vol. 42, pp. 218 – 244.

Henningsen, M., and Stein, M. K., 1997, “Mathematical tasks and student cognition: Classroom-based factors that support and inhibit high level mathematical thinking and reasoning”, Journal for Research in Mathematics Education, Vol. 28, pp. 524-549.

Hogan, J., and Morony, W., 2000, “Classroom teachers doing research in the workplace“. In A. Bessot & J. Ridgway, (Eds.), Education for Mathematics in the Workplace, Dordrecht: Kluwer Academic Publishers.

Jaworski, B., 2006, “Theory and practice in mathematics teaching development: Critical inquiry as a mode of learning in teaching”, Journal of Mathematics Teacher Education, Vol. 9, pp. 187-211.

Nicol, C., 2002, “Where's the math? Prospective teachers visit the workplace“, Educational Studies in Mathematics, Vol. 50, pp. 289-309.

Remmilard, J., T., 2005, “Examining key concepts in research on teachers‘ use of mathematics curricula“, Review of Educational Research, Vol. 75, pp. 211-246.

Roth W.-M., Mc Ginn, M. K., and Bowen, G. M., 1998, “How prepared are pre-service teachers to teach scientific inquiry? Levels of performance in scientific represent action practices", Journal of Science Teacher Education, Vol. 9, pp. 25–48.

Watson, A., Mason, J. 2007, “Taken as shared: a review of common assumptions about mathematical tasks in teacher education", Journal of Mathematics Teacher Education, Vol. 19, pp. 206–215.

Windschitl, M., 2004, “Folk theories of “inquiry”: How preservice teachers reproduce the discourse and practices of an atheoretical scientific method”, Journal of Research in Science Teaching, Vol. 41, pp. 481 – 512.