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Anke M. Lindmeier
Aiso Heinze

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ACTIVITY THEORETICAL APPROACHES TO MATHEMATICS CLASSROOM PRACTICES WITH THE USE OF TECHNOLOGY

Fabrice Vandebrouck, John Monaghan, Jean-Baptiste Lagrange

Université Paris Diderot, France, LDAR; University of Leeds, England; Université Reims Champagne Ardenne, France, LDAR

This Research Forum seeks to promote further discussion on activity theoretical approaches to mathematics classroom practices with the use of technology. Its origins go back to discussions and collaborations between European mathematics education researchers whose approach to research on the use of technology in mathematics classrooms is 'informed' by activity theory (AT). These discussions resulted in a two volume special edition of a journal (see Vandebrouck et al., 2012/13). Our aim at PME is to widen this discussion.

After brief introductions to AT by John Monaghan and to recent French AT developments by Jean Baptiste Lagrange, four research teams outline their approaches and results: in the two first papers, the authors show how the use of an Activity-Theory-based framework can lead to design decisions for digital learning artefacts. In particular, Ulrich Kortenkamp and Silke Ladel use the development of an AT framework and show how a virtual manipulative environment should support the actions and operations of a child. After this, Giorgios Psycharis connects the Instrumental Approach with the Constructionism framework for exploring the construction of mathematical students' knowledge. In the two others papers, the emphasis shifts to the teachers' activity. Mirko Maracci and Maria Alessandra Mariotti elaborate on the notion of semiotic mediation, in relation to the use of artefacts to enhance mathematics teaching-learning. They provide an explicit model of the actions which are expected from the teacher in order to make the semiotic mediation process occur. Barbara Jaworski and colleagues use two AT frameworks to juxtapose different perspectives: one from those designing an innovative mathematics teaching approach and the other from the persepective of the students experiencing this teaching.

All papers focus directly or indirectly on the use of (technological) artefact as tools with a mediational goal. They draw on a range of various forms of AT addressing students' activities, teachers' activities or both. Starting from these papers, the research forum seeks to promote the way AT is connected to mathematical learning and teaching with technology; why AT seems to be so useful and therefore important in investigating the use of tools for educational purposes? What do we mean by tools and their mediational properties? What about AT and other theories?

1: ACTIVITY THEORY

John Monaghan

University of Leeds, England

AT is a cross disciplinary approach for studying human practices including teaching and learning. Its roots go back to early Soviet approaches (not just Vygotsky's, though his approach has endured) to psychology. *Activity* became a focus for Vygotsky in the 1920s in his consideration of consciousness as a *problem* for psychology:

> The major objection Vygotsky had to the mentalist tradition was that it confined itself to a vicious circle in which states of consciousness are "explained" by the concept of consciousness. Vygotsky argued that if one is to take consciousness as a *subject* of study, then the *explanatory principle* must be sought in some other layer of reality. Vygotsky suggested that socially meaningful activity (Tätigkeit) may play this role and serve as a generator of consciousness. (Kozulin, 1986, *xxiii-xxiv*)

In AT object orientated activity is the *unit of analysis*, that which preserves the essence of concrete practice. 'Activity' in education-speak is commonly used to refer to things learners and teachers do. These things can be activities in the AT sense but the 'A' in AT is 'object orientated', it has a purpose; indeed if two individuals (say two schoolchildren) are performing similar actions but have different objects, then it can be said that they are involved in different activities. The object of the activity and individuals participating in the activity are essentially interrelated. Appropriating the ideas of the last sentence has led many in mathematics education to reconsider their earlier work focused on individuals. The following quote, for example, shows three well known mathematics educators re-evaluating their view of 'context and the individual' in their first major paper on an activity-theoretic approach to abstraction:

> In the cognitivist approach, the context that may influence the process of abstraction is thus considered as a set of external factors … [in our approach] context becomes an inseparable component of the activity because participants choose to carry out actions that seem relevant to them in the given context. (Hershkowitz, Schwarz & Dreyfus, 2001, pp.197-199).

In the next two pages we outline: expositions of AT from outside the field of mathematics education which inform debates within mathematics education; learning and teaching from an AT perspective with special regard to the use of technology.

THE DEVELOPMENT OF AT

Vygotsky was particularly interested in culture, language, signs and mediation. Physical tools were not, unlike most mathematics educators, of interest in themselves, any interest was due to their mediating qualities, "the basic analogy between sign and tool rests on their mediating function that characterizes each of them" (1978, 54). The difference between signs and tools rests on:

> The tool's function is to serve as the conductor of human influence on the object of activity; it is *externally* oriented; it must lead to a change in objects … The sign, on the

> other hand, changes nothing in the object of a psychological operation. It is a means of internal activity aimed at mastering oneself; the sign is *internally* oriented. (ibid, p.55)

Vygotsky's co-worker Leont'ev introduced distinctions between *operations* (things to be performed), *actions* (conducted by an individual or group) oriented toward a *goal* and socially organized *activity* (employing a division of labour) relative to the *object* of the activity (see Leont'ev, 1979). It was Leont'ev who emphasized that all activity is motivated, even though the motive may not be explicit, and used the term 'motive-goal'. Transforming the object into an outcome is essential to the existence of an activity (and a central role of the teacher, from an AT perspective, is to work with the learner in realising the object). Subject and object form a dialectic (interrelated) unit: the object transforms the activity of the subject and at the same time the object is transformed by the psychological reflective activity of the subject.

Engeström (1987) extends Vygotsky's focus on artifact mediation to multiple forms of mediation and extends Leont'ev's frame to 'activity systems' to include the community and social rules underlying activity. Humans do not merely react to their life conditions but they are able to change the conditions that mediate their activities. Engaging in activity collectively not only increases action potential but also opens up a zone of proximal development for individual and collective learning and transformation; the study of human activity and its changes is central to understanding how individuals learn. Cole & Engeström (1993) develop a systemic model to express the complex relationships between elements mediating activity in an activity system, useful for studying the relationships that take place in teaching/learning activity with technological tools. Engeström (1999) acknowledges the "hidden curriculum" in which *rules*, *community* and *division of labour* (as well as tools) are central mediational means in activity systems, and the importance of tensions and contradictions in activity systems. Engeström and Sannino (2010) elaborate on the notion of a *cycle of expansive learning* to describe activity transformation processes which may determine a re-definition of objects, tools and the structure of the activity by participants able to promote new and possibly unforeseen conceptualization.

The work of Wertsch (1991, 1998) has attracted the attention of mathematics educators interested in tool use because he focuses on the person-tool dialectic or, as he puts it, "the *irreducible bond* between agent and mediational means" (1998, p.27); the bond in, say, a person using a calculator, is irreducible because the act of calculating with a calculator cannot be reduced to what the human alone can do or to what the calculator can do, the calculation is done by a human-with-calculator. Wertsch refers to "goal-directed action" and emphasises that "the relationship between action and mediational means is so fundamental that it is more appropriate, when referring to the agent involved, to speak of 'individual(s)-acting-with-mediational-means' than to speak simply of 'individual(s)'" (1991, p. 12). The import of this view for the dialectic between the "object of the activity and individuals participating in the activity" (discussed above) is that the tools (mediational means) which the individual(s) engage in in doing, say, mathematics arise from their cultural history.

There are tensions in these AT approaches which we do not wish to hide and we hope they will be discussed in the *Research Forum*. For example, Wertsch's unit of analysis is the 'goal-directed agent(s)-with-mediational means' whereas, for Engeström, it is the object-oriented activity system. Cole (1996, p.334), in the same vein, comments:

> Mediated action and its activity context are two moments in a single process, and whatever we want to specify as psychological processes is but a moment of their combined properties. It is possible to argue how best to parse their contributions, *in practice*, but attempting such a parsing "in general" results in empty abstractions, unconstrained by the circumstances to which they are appropriate.

LEARNING, TEACHING AND TECHNOLOGY

Using AT as a framework, we view a learning environment as constituted by the *enactment of a teaching/learning activity* oriented towards an object involving students, teacher, and artifacts; for example, the solution of a task, the reading of a document, a class discussion on a specific issue, etc. motivated by an object which might be students' mathematical knowledge development, or mathematical learning.

The object of a teaching activity from the point of view of the teacher is a didactical objective, namely the students' acquisition of a specific knowledge or skill (Bellamy, 1996). Obviously, the student's involvement in an activity can be motivated by different objectives (to understand, to get good marks, to please the teacher, etc.). These 'objectives' contribute to the overall object or motive-goal of the activity. Studying the learning environment means studying how the elements and the relationships that characterize the teaching/learning activity oriented to a didactical objective can determine the expected outcomes.

Studying the changes that learning environments undergo when technology-based artifacts are introduced means analyzing how activity changes as a consequence of tool use and how this change is meaningful for the students and the teachers (Bottino & Chiappini, 2008). This study also involves clarifying what is meant by the term 'tool' and our considerations on this close this opening section.

In place of defining a tool we make two distinctions. The first is between an artefact and a tool. An artefact is a material object, usually something that is made by humans for a specific purpose, that becomes a tool when it is used by an agent to do something. A compass becomes a tool when it is used to draw a circle (its intended purpose) but the same artefact becomes a different tool when it is used to stab someone. This establishes an irreducible bond not just between agent and tool but between agent, purpose and tool.

The second distinction is between the material and ideal forms of a tool. A tool, as an artefact, has a material form. Sometimes this materiality of a tool is not immediately apparent, as in *initiation-response-follow-up* (IRF) forms of teacher-student classroom interaction (see Wells, 1993). But behind the material form there is also an *ideal* form – its material form reflects purpose, the reason for its use (see Cole, 1996, for a discussion of this matter). The ideal form has at least two important implications for mathematics education: (i) the distinction between a tool and ways of using the tool;

(ii) interaction between material and ideal forms. To illustrate (i) consider the place value algorithm for adding natural numbers. There are a number of ways of enacting the algorithm, e.g. what are called the 'traditional algorithm' and the 'grid method' (which keeps, say, units, tens, etc quite separate). Behind these isomorphic forms of the algorithm are intentions, understandings and routines with regard to ways of using the algorithm. With regard to (ii) consider an agent using, say, GeoGebra. To carry out material actions in GeoGebra the agent needs an idea, which may be quite crude, of how to act with GeoGebra, but actions in GeoGebra provide feedback to the user which may change the agent's idea (ideal form) of how to use GeoGebra. This distinction will be considered (using different terminology) further in the next section.

2: STUDYING THE TEACHER'S ACTIVITY: DEVELOPMENTS BY THE FRENCH SCHOOL

Jean-Baptiste Lagrange

IUFM, University of Reims and LDAR, University Paris-Diderot, France

Studies of the teacher's activity in classroom use of technology by "the French school" take the teacher as a worker and the classroom as her working place. Thus it benefits from two elements in the French research context. The first element is the development of Leont'ev (1978)'s Activity Theory occurred during the two past decades in the community of research in psychology of the workplace (cognitive ergonomics). The second element is the development of studies of the mathematics teacher combining an ergonomic and a didactical approach.

ACTIVITY AT WORK AND TEACHERS' INSTRUMENTAL GENESES

For the community of research in psychology of the workplace, Activity Theory focuses on the individual as a subject and an actor in her[1] activity. In the work place task characteristics (in the context of a given situation) and existing workers' characteristics both co-determine activity (Figure 1). The dynamics of activity produces feedback effects in a twofold regulation loop. On the one hand, the object of the task is modified (effects on the performance), giving rise to new task characteristics (a new task or the pursuit of new actions for attaining the initial task goal). On the other hand, the workers' characteristics are modified: the discrepancy between expectations and results of action exerts a pressure for adapting her activity in the short term, and the resulting experience may modify her knowledge in the long term. In the *productive loop*, activity is object-oriented – toward the work process. In the *construction loop*, it is subject-oriented: the subject's object is to develop or at least preserve herself (her competence, health...). The relationships between these two dimensions differ depending on the type of situations. The possibility for the constructive dimension to be deployed is linked to the developmental opportunities open in the situation as well

[1] We use "she" and "her" due to a cultural problem native French speakers have with English language gender neutral pronouns.

as to the subject's intentions. For a teacher using technology in the classroom the first goal is that her students learn. However, teachers' skills and knowledge that make technology really contribute to learning are not acquired simply by technical instruction, but rather built in a construction loop triggered by classroom activity if suitable opportunities are provided.

In the same community of research in psychology of the workplace, Rabardel (1995) and colleagues developed a "cognitive approach of contemporary instruments" dealing with the use of artefacts for object oriented activity. Among the powerful ideas introduced by Rabardel, the instrumental genesis develops a scientific approach to understanding of how an instrument is constructed from an artefact, a human being develops intertwined knowledge about the artefact's possibilities and constraints and knowledge on the object, and the Collective Instrument-mediated Activity Situation model (CIAS) (p. 53) gives account of how an instrument mediates the relationship a subject develops towards the object, and also towards other subjects.

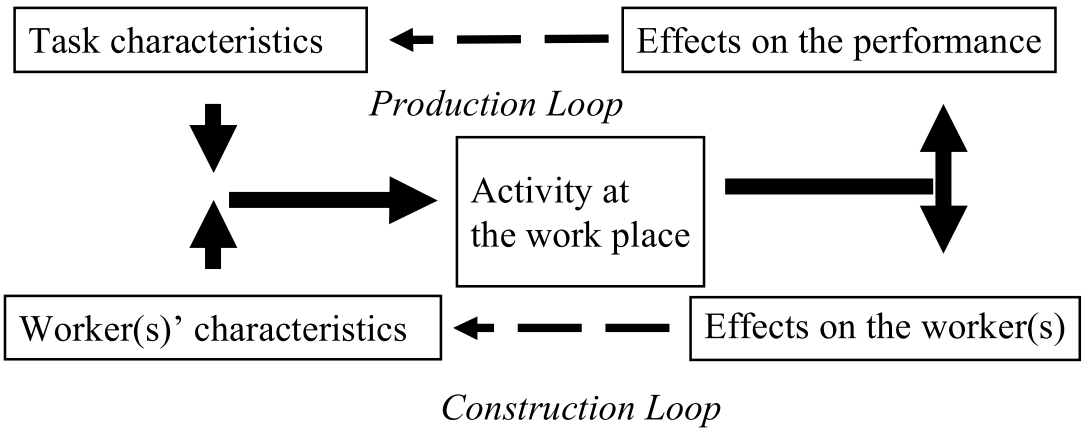

Figure 1: The five square diagram. Adapted from Leplat (1997).

Studies of classroom use of an artefact by mathematics teachers give evidence of a complex plurality of instrumental geneses which are important to distinguish in order to appreciate the teachers' position relatively to the technology. Haspekian (2005) identified two different instruments built by the teacher using a spreadsheet to teach and to learn mathematics. The teacher builds a 'personal' instrument while becoming aware of the spreadsheet's capabilities to solve mathematical problems. This instrument is not functional in itself when the teacher has to use the spreadsheet in the classroom, because it includes schemes of uses that cannot be transferred directly to students. Then the teacher has to build another instrument in a 'professional' genesis. This can be expanded using the CIAS model to show the plurality of mediations involving an artefact used for mathematics teaching/learning (Lagrange in press).

THE TWOFOLD APPROACH AND TEACHERS' GENESES OF USES

The didactical and ergonomic twofold approach was developed by Robert and Rogalski (2005) to study teaching practices in learning situations not particularly focusing on the use of technological artefacts. From a local point of view, this approach is oriented towards an analysis of students' mathematical activities in the classroom. This local point of view is extended to a global point of view, to gain access to what happen in the classroom, linking students and teachers' activities and the context of the whole activity system. Researchers like Abboud-Blanchard and Vandebrouck (2012) built their framework within this *didactical and ergonomic twofold approach*. Aiming to give account on how a teacher progresses in classroom use of technology beyond instrumental genesis of particular artefacts they introduce the notion of *genesis of technology uses* articulating three levels of organization of practice-micro, local and global- which partially resonate with the three levels of human activity elaborated by Leont'ev.

Finally, a synthesis by Lagrange (in press) highlights *productive* and *constructive* aspects of teachers' activity articulating various kinds of geneses (figure 2). In suitable conditions, students' instrumental geneses progress, allowing new uses. The teacher's reflection on situations of use involving the three levels also influences her geneses.

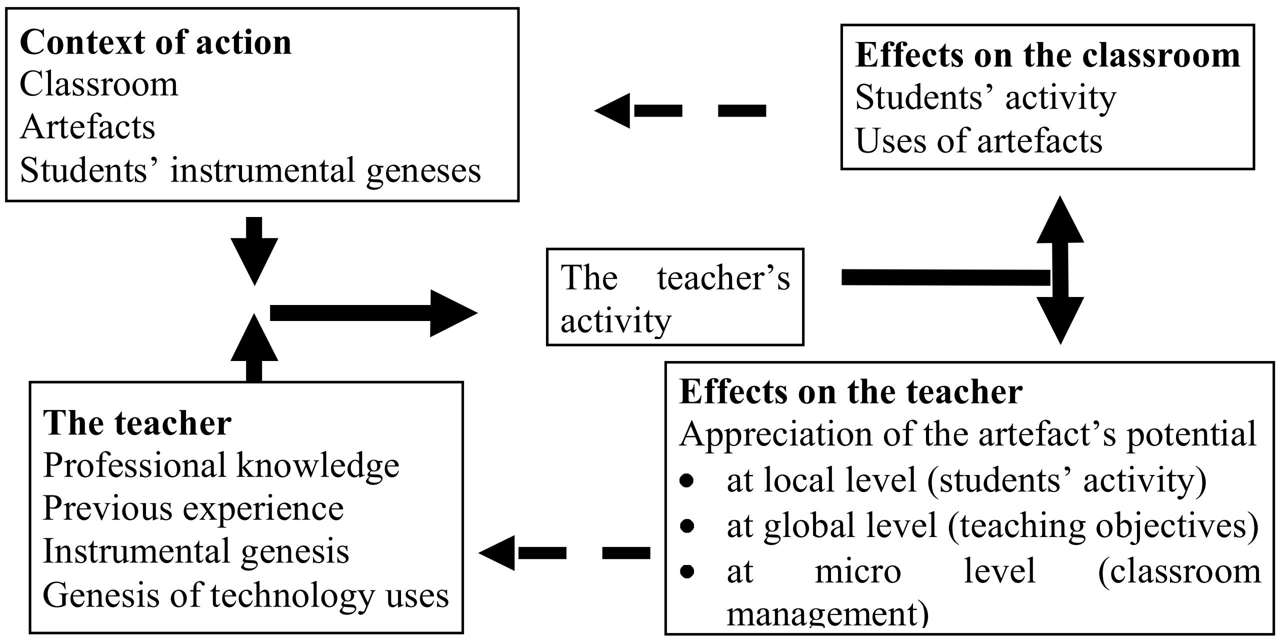

Figure 2: The twofold regulation loop diagram for the teacher using technology.

3: DESIGNING A TECHNOLOGY BASED LEARNING ENVIRONMENT FOR PLACE VALUE USING ARTIFACT-CENTRIC ACTIVITY THEORY

Ulrich Kortenkamp and Silke Ladel

Martin Luther University Halle-Wittenberg and Saarland University, Germany

Modern interactive media provides technology that might be able to enhance teaching and learning of mathematics. Mathematics education must take into account the induced changes through this technological progress, as it cannot deny the existence of smartphones and tablet computers. Today, it is far more likely that a student has such a device on their person than it that he carries paper and pencil, a ruler or even a piece of string. While it is not at all clear that this is an improvement, it is a fact that must be taken into account when designing learning environments and manipulatives for teaching and learning mathematics.

A common fear is that students are going to be reduced to “answering machines” that are only required to tick the right answer in a multiple-choice test. This is supported by the growing number of assessments that reduce the mathematical competence of a student to a score in a standardized test. A climax of this development is the categorization of exercises into difficulty levels according to their solution rate in such a test: Is an exercise difficult just because many people cannot solve it correctly? This detachment of the actual content of an exercise from its analysis is hard to justify. Additionally, it counteracts and devalues good teaching – if students can solve exercises better, the exercises become easier by definition.

Activity Theory offers an alternative to test-orientation in psychology, and through this also a methodological alternative to quantitative empirical research. As stated in the preface to (Leont’ev, 1982):

> Die methodologische Unhaltbarkeit derartiger Tests ist offenkundig. […] Es ist unschwer zu erkennen, daß sich hinter einer derartigen Überführung einer methodischen Technik in eine selbständige Disziplin, wie sie mit der Testpsychologie entstanden ist, nichts anderes verbirgt als der Ersatz der theoretischen Untersuchung durch grobe Pragmatik.

In other words: The methodology of testing is just a coarse pragmatic replacement for theoretical analysis. In our work with multi-touch technology[2] we want to find out how to improve teaching and learning in mathematics through interaction with virtual manipulatives. As a preliminary step we have to understand how the interaction with such devices takes place (Ladel & Kortenkamp, 2013). With the help of the ACAT framework (see the Methodology section) based on Engeström’s (1987) work, we were able to focus on the activities of the students with the virtual manipulatives within a social setting. Here, we will discuss the design of a virtual place value chart and how the activity with such a virtual manipulative differs from that with traditional media.

[2] Multi-touch devices are able to report several touching fingers or objects on a screen, as opposed to single-touch or mouse-driven devices that are capable of reporting one (x,y)-coordinate on the screen and the information whether a button is pressed or not. Multi-touch devices are available in all sizes, ranging from smartphones via tablet computers to large-screen interactive whiteboards.

While empirical research about the effectiveness of digital learning environments or software for teaching and learning is definitely necessary, we reject research that accepts technology and in particular software and the application design as a given fact that is unchangeable. In contrast, we still see mathematics education as a design science (Wittmann, 1992) that not only analyses but creates learning environments.

PLACE VALUE AND NUMBER REPRESENTATIONS IN MATHEMATICS EDUCATION

The (decimal) place value system is a central pillar in basic arithmetic. Arbitrary large numbers can be represented uniquely by a finite set of digits. The decimal system allows for efficient ways to

- *Count*, as we can represent an infinite number of numbers with a finite set of words
- *Compare,* as the map from numbers to numerals is unambiguous and two numerals are represented by the same word, and there exists an algorithm to decide with of two different representation is larger by comparing two numbers starting at the highest place value
- *Add, subtract, multiply and divide,* using algorithms for written methods.

The underlying process of repeated bundling is a key factor for the uniqueness. We can bundle a cardinal representation of a number by bundling in tens, and tens of tens, and tens of tens of tens, … until no further bundling is possible. This process will always lead to the same number of (less than ten for each place) bundles. The bundling activity is also reversible, which is necessary not only for written subtraction, where we "borrow" from a higher place, but also for decoding the decimal representation into a cardinal conception.

Mathematically, the repeated bundling harnesses the power of exponential growth. The representation of a quantity n is possible with $\log_{10} n$ places using 10 digits. Here we clearly see the power of the decimal system that reduces large quantities into small representations. The product of two numbers is always representable through another number.

Another important aspect of the place value chart is the flexible interpretation of numbers that is induced by grouping digits differently. The number 3247 can be interpreted as 3 thousands, 2 hundreds, 4 tens and 7 ones (3|2|4|7), but also as 32 hundreds and 47 ones (32|47), 3 thousands and 247 ones (3|247), etc. For flexible arithmetic strategies it is helpful for children to be able to do calculations that are based on such decompositions, as opposed to using only written, algorithmic arithmetic that is place-based and only considers single digits.[3]

[3] Selter (1999) proposes to distinguish Zahlenrechnen (arithmetic with numbers) and Ziffernrechnen (arithmetic with digits). Usually, mental arithmetic is within the domain of Zahlenrechnen, and written algorithms fall into the Ziffernrechnen domain. Mental arithmetic supported by written notes that do not follow the standard algorithms for place-wise calculations (in German: halbschriftliches Rechnen, semi-written arithmetic) belongs to the domain of Zahlenrechnen and can benefit from flexible interpretations of place value charts. See (Benz, 2005) for details.

For written, algorithmic arithmetic it is necessary to understand the processes of carrying over and the reversal, borrowing from a higher place. This corresponds to replacing bundles of ten objects with a single representative or vice-versa. Our final goal is to support real "understanding rather than procedural proficiency", supported by the instructional environment in the sense of Hiebert and Wearne (1992).

METHODOLOGY

We use the ACAT framework (Artefact-Centric Activity Theory, Fig. 1) to analyse the situation in which we are going to use a virtual manipulative for improving children's understanding of place value.

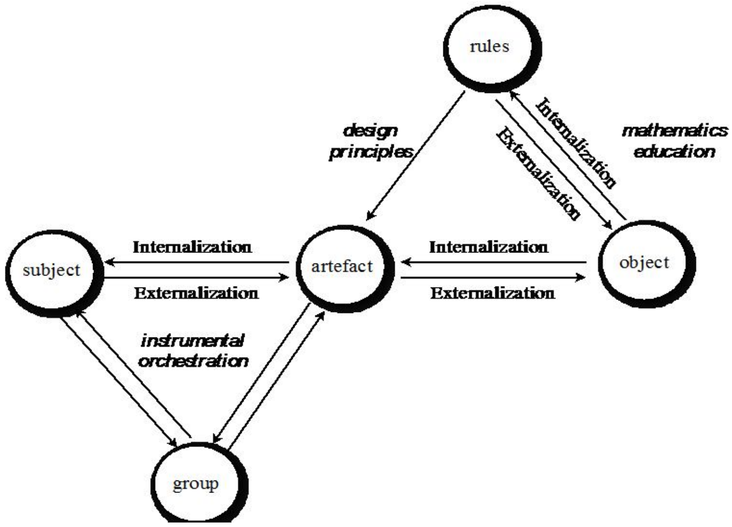

Figure 3: The ACAT Diagram

The actions carried out with that manipulative should support the mathematical design as described in the preceding section:

- Children should be able to place undistinguishable objects[4] in various places of the virtual place value chart.
- Moving an object from one place to the other should not change the value of the number represented, but initiates either a bundling or de-bundling.

Our goal is to help students to become fluent with these actions, such that they become operations for them. For a proper understanding of place value and for using flexible strategies when calculating, as well when doing written arithmetic, it is mandatory to do these operations then without the help of a supporting virtual manipulative.

[4] Gerster and Walter (1973) describe a 11-step abstraction model for the decimal system from grouping to representing numbers by sequence of digits. They emphasize the importance of the 9th abstraction which uses undistinguishable objects that differ in value only through their placement.

ANALYSIS

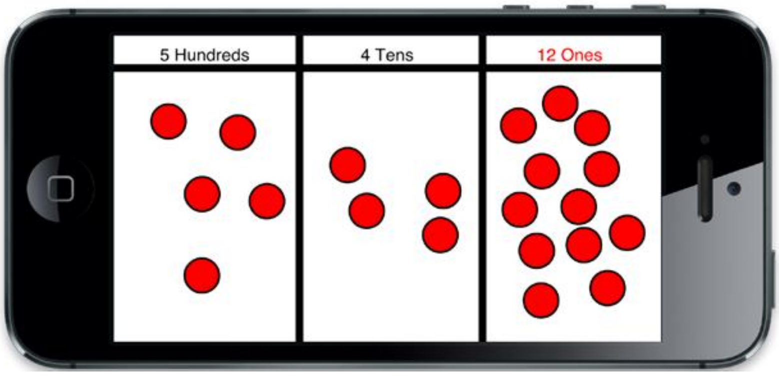

Figure 4: The virtual place value chart on the iPhone. Application available at https://itunes.apple.com/app/id568750442

Let us quickly review the main axis of ACAT (horizontal axis in figure 3): The subject – a student – manipulates an object, the number within the decimal system, by placing moving virtual tokens on the artefact. The artefact (the virtual place value chart) responds to these actions by showing the tokens and the number of tokens in each place. The properties of the decimal system are encoded into the artefact through its response when moving tokens from one place to the other: If a token is moved to a lower place, it "explodes" into the corresponding number of tokens, i.e. when a token is moved from the thousands' place to the tens' place, it will become 100 tokens. If a token is moved from a lower place to a higher place, the artefact will try to bundle the necessary number of other tokens and merge them into one, or it will refuse to move the token. Thus, the decimal system is encoded programmatically into the artefact.

A major difference between this design and traditional place value charts is the possibility to operate with the tokens *while keeping the represented number unchanged* [5] .This matches the mental operations necessary for addition and subtraction: When calculating 72 minus 47 a student should be able to use the strategy 12-7 and 60-40, which uses such a de-bundling of a ten-token into ten one-tokens. It is important to note that the virtual artefact is different to the traditional one here: When manipulating real tokens in a place value chart, children do change the represented number, which leads to questions like "when you move a ten-token to the one-place, how does the number change?" — Note that this activity helps children to understand the design of the traditional artefact, while our modern approach emphasizes the human activity and is ruled by the object (i.e. the numbers), not the artefact. The children are supported in their transition from "just moving tokens" to an operation that enables them do reach a higher level when operating within the number system.

[5] Our experience is that this difference is a key point of misunderstanding for most mathematics educator when evaluating our digital artefact. Only digital media allows for such a design, and many exercises built on the traditional place value chart no longer make sense with the new tool.

CONCLUSION AND OUTLOOK

In this brief overview we outlined how we could base the design of a digital artefact on a theory that answers some methodological drawbacks of pure quantitative methods. The view through activity theory is necessary to change the design from the traditional, non-digital design that emphasizes the tool and its response to human actions into a design that supports the cognitive processes of the children and respects the mathematical foundation of the tool.

As a next step we are currently designing and implementing a virtual manipulative that supports adding and subtracting in a subdivided place value chart, which gives rise to other possible actions that should become operations. In particular moving between the summands in addition should lead to discordant change (German: *gegensinniges Verändern)* and moving between the operands in subtraction should lead to concordant change (*gleichsinniges Verändern)*, as this will leave the sum (respectively difference) unchanged.

4: ABSTRACTION THROUGH INSTRUMENTALIZATION

Giorgos Psycharis

University of Athens, Greece

THEORETICAL FRAMEWORK

In this contribution we aim to connect work in activity theory with the international discussion concerning the networking between theoretical perspectives with respect to using digital technologies in mathematics education. The present contribution concerns an attempt to extend existing discussion on possible connections between Constructionism and Instrumental Theory. Particularly, Kynigos and Psycharis (2013) adopted an approach with the aim to promote connectivity between the two theories by addressing the design of digital artefacts so that learners' uses of them may happen in particular ways conducive to the generation of meanings. Building on this approach, this contribution aims to address instrumentalization as a framework for exploring the construction of mathematical knowledge in terms of mathematical abstraction with respect to using digital technologies designed to facilitate students' meaningful engagement with mathematical ideas. Constructionism and Instrumental Theory share some common characteristics that can provide a basis for networking. A key consideration of the two theories is the two sided relationship between tool and learner (cf. Lagrange, 1999, Hoyles et al., 2004). Also both theories have a strong focus on design in order to support students' instrumentalization when integrating digital tools in their learning activities. A general design principle of Instrumental Genesis (IG) as regards the design of technology is that the artefacts should be designed to support their efficient transformation to instruments while enabling flexible user modifications to the artefact, particularly through instrumentalization (Kaptelin & Nardi, 2006). In a similar way, constructionist design approaches aim to tackle the possibilities afforded by expressive digital media to build and rebuild virtual structures and to describe the ways in which students' interact with them (Pratt & Noss, 2010).

In this chapter, I build upon existing research work to explore further networking between Constructionism and IG by connecting the idea of "reciprocal shaping" between the learner and the tool on the one hand, and the IG of mathematical instruments on the other hand. Hoyles et al. (2004) pointed out that situated abstraction has the potential to complement the idea of a process of instrumentation, "shaped by the tool", as means to precisely state how mathematical knowledge is constructed in computer-based settings. Kynigos and Psycharis (2013) considered the parallel between instrumentalization and the idea of "shaping of the tool by the learner" to explore issues of design and IG. Under a constructionist approach, my general aim here is to explore the nature of the constructed knowledge in terms of mathematical abstraction when the main focus is on students' instrumentalization, i.e. meaning generation directly linked to changes students make to digital artefacts that were designed to provide learners with further opportunities for instrumentalization. In this approach, instrumentalization refers to all aspects of students' interaction with the artefact – including actions on particular representations and tools (e.g. Logo code) or means available to perform an action (e.g. sliders). My main theoretical account of abstraction is *situated abstraction* (Noss & Hoyles, 1996), which addresses how mathematical abstraction is scaffolded within computational media. In this perspective, abstraction (or abstracting) is seen as a meaning generation process in which mathematical meanings are expressed as invariant relationships, but yet remain tied up within the conceptual web of resources provided by the available computational tool and the activity system. In the analysis I complementarily use theoretical tools provided by *abstraction in context* (AiC) (Hershkowitz et al., 2001) to capture the details of connections between existing and new mathematical knowledge in students' vertical mathematization by means of situated abstractions. Particularly, I use the three stages of abstraction posited by AiC: (1) a need for new mathematics; (2) construction of new mathematics through reorganization of prior mathematical knowledge; (3) consolidation of new mathematics through further use. To illustrate this approach, I use empirical results of students' instrumental genesis in the context of two research studies designed and implemented under a constructionist perspective.

EPISODE 1: EXPRESSING PROPORTIONALITY IN GEOMETRIC TASKS

In this episode, 13-year-old students worked in their classroom with Turtleworlds (Kynigos, 2002), a Logo-based Turtle Geometry software which affords dynamic manipulation of geometrical objects through the use of a specially designed tool (called Variation Tool, VT). The main part of the VT consists of 'number-line'-like sliders, each corresponding to one of the variables used in a Logo procedure. Dragging a slider has the effect of the figure dynamically changing as the value of the variable changes sequentially. The students were engaged in a project to build enlarging-shrinking models of capital letters with one variable. The students had to connect formal and graphical descriptions of geometrical figures continually, and through manipulating variable segments or angles, to appreciate the inappropriateness of additive strategies.

Since the inclusion of a wrong relationship (e.g. an additive one) in a procedure would result in a ‘distorted’ figure, the students had to identify the need to build appropriate proportional relations with the same independent variable. The study with Turtleworlds took place in a secondary school in Athens with two 13-year-old students’ classes (first grade of the secondary level, 16 teaching sessions).

A group of students (Alexia and Christina) completed an enlarging-shrinking model of N in Turtleworlds during 6 successive phases (4 classroom sessions) (see Table 1). In phase 6, the students were able to specify appropriate proportional relations in order to construct different enlarging-shrinking models of N (e.g. 25°, 30°). The students realised that the use of additive algebraic expressions constituted an erroneous strategy (phase 3), as confirmed by the graphical distortion of the figure when dragging on the VT. The students then (phase 4) attempted to test the multiplicative correlation of the two lengths for N (45°), which emerged as a ‘translation’ into formal notation of the situated abstraction “the tilted one is nearly one and a half times the other in the original pattern”, posited by Alexia. When they inserted the relation 1.5*:r in their code the graphical feedback revealed that the side length did not exactly coincided with the horizontal line that they had drawn at the letter base. This seemed to have created a basis for the students to continue further instrumentalization of the symbolic expression according to the graphical feedback resulting from the use of the VT in order to prevent the distortion of the figure.

to letterN fd 200 rt 135 fd 295 lt 135 fd 200 end	to letterN :r :t fd :r rt 135 fd :t lt 135 fd :r end	to letterN :r fd :r rt 135 fd :r+40 lt 135 fd :r end	to letterN :r fd :r rt 135 fd :r+50 lt 135 fd :r end	to letterN :r fd :r rt 135 fd :r+45 lt 135 fd :r end	to letterN :r fd :r rt 135 fd :r*1,5 lt 135 fd :r end	to letterN :r fd :r rt 135 fd :r*1,45 lt 135 fd :r end	to letterN :r fd :r rt 135 fd :r*1,42 lt 135 fd :r end
Phase 1: Original pattern	Phase 2: Two variables	Phase 3: One variable, additive strategy			Phase 4: One variable, specification of multiplicative relations		Phase 5: Appropriate relation

Table 1: The Logo code(s) in the first five construction phases.

At the level of instrumentalization, we see that the modification of Logo code can provide a useful lens to capture the evolution of students’ abstraction of the proportional relations involved in the task. The students needed (stage 1 of AiC) to find a strategy to dynamically enlarge and shrink models of capital letters. This strategy emerged as part of the students’ ongoing experimentation to avoid graphical abnormalities in the constructed geometrical figures through vertical reorganisation of prior knowledge of proportionality (stage 2 of AiC). Finally, the students were able to connect this strategy to the official mathematics and to use it to construct enlarging-shrinking models of different letters (stage 3 of AiC).

EPISODE 2: EXPRESSING GENERALITY IN FIGURAL PATTERN TASKS

In this episode, 14-year-old students worked with eXpresser (Noss et al., 2009), a microworld which affords the creation of different coloured patterns by repeating a building block of several tiles (‘unit of repeat’). In eXpresser, students have tools to edit properties that specify the construction of the pattern as well as properties related to the number of coloured tiles needed. The students can work with 'icon-variables' (i.e. pictorial representations of an attribute of a construction such as the number of repetitions) can be used to represent the total number of tiles of a certain colour. Through the use of icon variables, students can also create relationships between two patterns of different colours based on dependencies (e.g. the number of red tiles in one pattern is 2 more than the number of green tiles in the other pattern). Icon-variables appear in a pink frame and they can be copied, deleted or used in operations (e.g. addition). This process encourages students to select appropriate independent (icon) variables (i.e. 'unlocked' numbers) and to build general expressions for their patterns. Another feature of eXpresser is that of 'General Model' window: when the user animates a pattern the system shows the construction as the pattern unfolds for different (randomly chosen) values of repetitions. In case students did not choose the correct dependencies between two patterns of different colours (i.e. based on the same independent variable), the unfolded pattern in the 'General Model' appears to be distorted ('messed-up') and it is not coloured. In order to colour their pattern in the 'General Model', the students have to construct a general expression (i.e. the Model Rule) that always gives the total number of all tiles for the model through the use of an independent (icon) variable. In the study with MiGen [1], three case study groups of 14-year-old students (6 sessions for each group) were asked to construct and validate patterns through general expressions that underpin them.

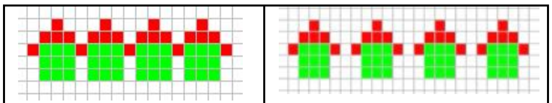

Fig. 5a Fig. 5b

After an initial familiarisation with eXpresser, the students were asked to find the general rule of the pattern shown in fig. 5a and then to modify it so as to create the pattern shown in fig. 5b. A group of students created three building blocks: one constant (i.e. the red tile on the left part of the first house, fig. 5a) and two general ones with the use of respective icon-variables (i.e. one building block for the roof with 5 red tiles and another one for the green square with 9 green tiles). By animating their pattern, the students realised that the construction was 'messed-up' since the icon-variables representing the number of repetitions in each pattern changed according to different (randomly chosen by the system) values. In order to avoid 'messing-up' the students recognised that the two variables had to take the same value during the animation. Then, students' instrumentalization evolved as follows: First, the students replaced the one icon-variable with the other through dragging and the use of the command 'replace'. Then, in order to colour their pattern in the General Model for any repetition, they constructed the general expression of their model through the use of icon-variables in the General Rule window (fig. 5c shows an immediate instantiation of this expression for three repetitions of the model). Thus, this window seemed to have operated as a template that eased students' instrumentalization of the general rule and supported them in linking the numeric, the symbolic and the visual. Finally, the students were able to express this rule with paper and pencil (i.e. 5x+9x+1) and to implement this strategy for creating the pattern of fig 5b.

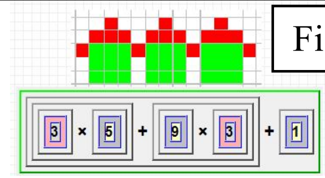

Fig.5c

At the level of instrumentalization, the creation and further manipulation of icon-variables to build general rules reflects the purposeful way in which the computational setting provided a *structure*, which students could exploit in shaping the available resources in a way that remain connected to their views of the general pattern. In terms of AiC, the students needed (stage 1 of AiC) to find a strategy to construct a pattern and to animate it correctly. 'Messing-up' provided a mechanism that challenged students to generalise through appropriate manipulation of icon-variables (stage 2 of AiC). Finally, the students were able to implement this strategy to animate different patterns (stage 3 of AiC).

CONCLUSION

The above episodes describe students' mathematical abstractions emerging from the need to address generality so as to carry out particular tasks. The analysis reveals that students' actions emerged in activity and that meaning generation and students' instrumentalization were irreducibly linked. Forging connections between Constructionism and IG at the detailed level of studying the instrumentalization processes allowed us to be more explicit in our analysis about the role of tools in students' reshapings of meanings during their activity and the correspondent vertical mathematizations by means of situated abstractions. The analysis indicates also three main areas of connectivity between the two theories on the basis of addressing mathematical abstraction: (a) the properties of instruments in terms of action and feedback; (b) the nature of instrumentalization in each theory; and (c) how instrumentalization is related to the constructed mathematics.

Notes

1. The research took place in the context of Angeliki Zoupa's dissertation thesis (in press). I owe her many thanks for allowing me to analyse part of her data.

5: SEMIOTIC MEDIATION AND TEACHER'S ACTIONS

Mirko Maracci*, Maria Alessandra Mariotti**

*Department of Mathematics, University of Pavia, Italia

** Department of Mathematics and CSCI, University of Siena, Italia

Activity Theory (AT) can be regarded as a "multi-voiced" theory (Engeström, 1999; Kuutti, 1995): a framework gathering together a number of different studies carried out by researchers across the world and from different disciplines, that develop at different levels a core of shared "ideas" or "concepts" – the hierarchical structure of activity, and tool mediation among others – whose original initial expression and elaboration can be found in the studies of Vygotsky (1978) and Leont'ev (1964/1976). We will *discuss a specific elaboration of the notion of mediation* in relation to the use of artefacts to enhance mathematics teaching-learning *with a specific focus on the teacher's actions.*

ARTEFACTS, SEMIOTIC MEDIATION AND TEACHING-LEARNING

The term *mediation* has become widely present in the mathematics education literature on the use of artefact, specifically to convey the general idea that artefacts are intermediary entities able of establishing links between the artefact user and the object towards which the artefact's use is directed (Meira, 1995; Rabardel, 1995; Borba and Villarreal, 2005). The study of the mediating function of the artefact is often limited to the study of its role in relation to the accomplishment of tasks, while the complexity of the relationship between the student's accomplishment of a task and mathematics learning risks to remain in the shadow. Rooted in a Vygostkyan framework and combining a semiotic and educational perspective, the Theory of Semiotic Mediation (TSM) (Bartolini Bussi & Mariotti, 2008) addresses specifically that complexity.

When accomplishing a task, students develop personal meanings related to the actual use of the artefacts. But in order that such meanings become truly meanings for the students, the students have to gain awareness of these emerging meanings; such consciousness-raising is fostered by semiotic processes (Leont'ev, 1964/1976). Hence students have to be involved in semiotic tasks regarding the artefact use and leading to the explicit formulation of the meanings they have developed. In addition, though the relationship between artefact use and mathematical meanings can be obvious for mathematics experts, the process which leads from personal to mathematical meanings is not a spontaneous one but deserves *the teacher's intentional intensive mediation*.

The assumed perspective raises the issue *how an artefact can be used by the teacher to mediate mathematical teaching-learning process*. Following Hasan (2002) we consider mediation a complex process in which the *mediator* is not the artefact but it is the person who intentionally takes the initiative and the responsibility for the use of the artefact to mediate a specific content. Hence in a teaching-learning context *the mediator is the teacher*, who introduces the artefact to mediate students' appropriation of mathematical knowledge. The teacher's mediation is directed towards *the students - conscious mediatees - who are asked to actively take part in the mediation process, and on whom the success of mediation depends*. The artefact is one of the constitutive elements of the *circumstances for mediation*, that entail also the modalities of use of the artefact, the tasks to be accomplished, the whole organization of the classroom work, the classroom interactions among students and between them and the teachers. *Mediation is semiotic* for at least two main reasons:

- the artefact is used because of its potential to refer both to meanings emerging from accomplishing the task and to the mathematical knowledge to be learnt;
- social interaction is crucial for learning and it is based on semiotic processes centred on the artefact use, involving both the teacher and the students.

THE DIDACTICAL CYCLE AND TEACHING-LEARNING ACTIVITY

In order to mediate the mathematical teaching-learning process the teacher has to design and realize specific *circumstances* to assure that the semiotic mediation process takes place as desired. That means that the teacher has to set up specific modalities of use of the artefact, both for the students and for herself, that are aimed to foster the expected semiotic mediation process. The study of the *circumstances*, which can foster semiotic mediation, led to the elaboration of a model of the *teaching-learning activity* centred on the use of an artefact, encompassing an *explicit model of what is expected from the students and the teacher.* Following Bartolini-Bussi (1996) we consider mathematical teaching-learning as a single, collective, possibly poly-motivated activity carried on by students and teachers. The very idea of activity is characterised by the existence of a heterogeneous community of individuals performing different actions related to the motives of the activity.

The model elaborated within the TSM consists in the iteration of *didactic cycles*, each one encompassing different components: (a) the start of a cycle is based on the *accomplishment of tasks that require the use of an artefact*; (b) then students are involved in specific semiotic processes encompassing *the production of signs related to the artefact use*, that can be referred to in the following collective work; (b) finally the whole class is engaged in *collective discussions* grounded on the previous components and aimed at fostering the evolution towards mathematical signs.

Each component of the didactic cycle contributes differently but complementarily to develop the complex process of semiotic mediation; each of them has its own objectives, the meaning of which becomes fully understandable when considered with respect to the others within each cycle and in the wider context of the iteration of different didactical cycles.

The analysis of the semiotic mediation process through the didactical cycles have to take into account different levels: (a) the whole cycle or iteration of cycles, that constitutes the overall activity with its educational objective; (b) the single components that take their sense in the context of the cycle (or cycles) they are part of; (c) the actions intentionally and purposefully performed in each step of the didactical cycle; and even (d) the actual realization of the actions depending on the concrete conditions in which they are carried out. Hence, the hierarchical structure of activity in *activity, actions and operations* – originally elaborated by Leont'ev – can provide a suitable frame for studying the *circumstances* favourable to the occurrence of the desired semiotic mediation process.

THE TEACHER'S ACTIONS IN THE CONTEXT OF DIDACTICAL CYCLES

A didactical cycle, or an iteration of didactical cycles, can be seen as an activity whose motive is to promote the generation of students' personal signs related to the accomplishment of a task through an artefact and their evolution towards desired mathematical signs. The fundamental components of the activity are those *actions* and *sub-actions*, enacted during the activity, and *intentionally and consciously* aimed at pursuing the motive of the activity (Pontecorvo, Ajello, & Zucchermaglio, 2004).

Students' production of signs related to the use of artefact.

The first goal towards which the teacher's actions are directed is to foster the *students' production of signs* expressing the personal meanings developed through the artefact use: the so-called *unfolding of the semiotic potential* of the artefact. Hence the teachers' actions have to be directed towards the design of tasks (a) which could lead students to develop personal meanings related to the artefact use having the potential to evolve towards mathematical meanings and (b) through which the students' personal meanings become conscious and explicit, expressed through personal signs. The goal of promoting students' production of signs is pursued through actions which concern mainly, but not exclusively, the first two components of a didactical cycle.

Evolution of personal signs towards desired signs

The evolution of personal signs is the crucial issue. It can pass through the collective construction of shared signs related to both the use of the artefact and the mathematics to be learnt, that can occur through classroom discussions. Thus, one can recognize two distinct classes of goals in a classroom discussion: *the joint construction of shared signs* related to the use of the artefact and *the evolution of these signs towards mathematical signs*. These two goals constitute two complementary components of the motive of the teaching-learning activity centred on the use of an artefact, and need to be shared by the teacher and the students, who are asked to assume their responsibility with respect to the achievement of these goals and to actively participate in the classroom discussion.

Different typologies of actions have to be put into practice by the teacher during a classroom discussion for pursuing these goals. In particular one can identify two pairs of actions serving respectively each of the two goals: the "back to the task" / "focalization" pair, and the "ask for a synthesis" / "provide a synthesis" pair (see also Mariotti, 2009; and Mariotti & Maracci, 2010).

"Back to the task" and "focalization". The *back to the task* actions consist in the request for the students to recall the task faced in previous sessions and to report on how they accomplished it. A productive utilization of the back to the task actions may have the effect of provoking a large number of contributions, resulting in a rich net of shared signs related to the use of the artefact. However, contributions may generate a lot of spurious elements, and consequently there is the need for the teacher to select the pertinent aspects of the shared meanings in respect to the development of the mathematical signs that constitute the final educational goal. This need can be satisfied through *focalization* actions, that consist in drawing the students' attention towards specific aspects of their experience with the artefact, or towards specific signs produced and used to refer to those aspects.

"Ask for a synthesis" and "provide a synthesis". The repeated alternate mobilization of the two actions described above can lead to the joint construction of shared and stable signs. Though retaining key elements pertinent in respect to the development of the desired mathematical signs, those signs are still anchored to the artefact actual use. In order to promote the evolution towards the mathematical signs there is the *need to re-elaborate on those signs (a) promoting their de-contextualization from the use of the artefact by the students, (b) maintaining (in the de-contextualization process) those aspects which are related to the use of the artefact but are recognized as pertinent to the target mathematical signs, and (c) promoting their generalization with respect to*

the specific tasks. These goals can be achieved through the request to students to *produce syntheses* concerning what has been done and discussed up to a certain moment. Symmetrically, the teacher herself can provide *a synthesis* with the goal of making explicit connections between the artefact and the mathematics context. Through this pair of actions it is possible to set up a shared semiotic environment within which mathematical signs might be consciously produced and put in relation with the artefact signs already shared.

CONCLUSION

The TSM provides an explicit model of the teacher's role in order *to mediate mathematical teaching-learning process through the use of an artefact*. When articulated within the frame of AT, the actions that the teacher can perform are identified by their goals and related to the motive of teaching-learning activity. In order to be effective, the actions described can and should be performed keeping in mind the need of provoking the participation of all the students. In fact the goals in a classroom discussion cannot be pursued without the active participation of all the students. To this end the teacher can enact different "semiotic acts", which depend on the actual conditions in which the discussion is realized. Such acts can be conceptualized as *operations* with respect to the teaching-learning activity.

We are convinced that the model developed may contribute to the issue of teachers' professional development, specifically it may contribute to teachers' consciousness-raising about their role and about the decisions they have to make.

6: USING ACTIVITY THEORY TO MAKE SENSE OF DIFFERENCES IN PERSPECTIVES ON MATHEMATICS TEACHING

Barbara Jaworski, Carol Robinson, Janette Matthews, Tony Croft

Loughborough University, UK

ACTIVITY THEORY ANALYSES OF RESEARCH FINDINGS

We use Activity Theory to make sense of findings from the design and study of an innovative approach to teaching a mathematics module to first year (university) engineering students[6]. The innovation was designed to promote students' conceptual understandings of mathematics and included use of inquiry-based questions and tasks, a GeoGebra medium for exploring functions, small group tutorial activity and a small group project (assessed). Significant in the findings were the differences between teaching aims in design of teaching and student perspectives on their experiences and learning goals (Jaworski, Robinson, Matthews & Croft, 2012). The teaching-research team designed tasks and approaches for lectures and tutorials to engage students and

[6] The ESUM Project – Engineering Students Understanding Mathematics, Jaworski & Matthews, 2011. Funded by the HE STEM Programme through the Royal Academy of Engineering – For two Case Studies from the project see http://www.hestem.ac.uk/resources/case-studies/engineering-students-understanding-mathematics-1 and http://www.hestem.ac.uk/sites/default/files/esum_2.pdf

promote students mathematical meaning making, their conceptual understanding. The students engaged with tasks in lectures and tutorials and developed their own perceptions of this experience. It is relevant to quote students' words from focus group interviews held after the teaching had finished:

> I found GeoGebra almost detrimental because it is akin to getting the question and then looking at the answer in the back of the book. I find I can understand the graph better if I take some values for x and some values for y, plot it, work it out then I understand it … if you just type in some numbers and get a graph then you don't really see where it came from.

> Understanding maths – that was the point of Geogebra wasn't it? Just because I understand maths better doesn't mean I'll do better in the exam. I have done less past paper practice.

Activity (in Activity Theory terms) in this project is *the whole* with which we work and in which we participate. 'We' are the teachers and researchers, the students, and other stakeholders, administrators, policy makers and so on. Included also are interlinking and interacting conditions, and the issues that are generated through practical interpretation of theoretical goals and their interaction with the cultures involved. Thus the Activity is everything, and not just the sum of all the parts. According to Leont'ev (1979), "Activity is the non-additive, molar unit of life … it is not a reaction, or aggregate of reactions, but a system with its own structure, its own internal transformations, and its own development" (p. 46). Thus, one reason for employing activity theory is to capture complexity in the wholeness described, as well as to examine specific elements and their contribution to the whole. We recognize that different groups within this constituency act in different ways towards the whole: they have different 'motives' for activity or 'goals' for their actions (e.g., Leont'ev, 1979). In Engeström's (e.g., 1999) terms they have different 'objects' within activity. We distinguish here between *Activity* as in Activity Theory, and the *activity* that students and teachers engage in locally with tasks in a lecture or tutorial. We rely on context to make this distinction clear.

We use Activity Theory specifically to address issues that we see *between* the intentions of the approaches to teaching and use of resources (in the innovation) *and* students' responses, engagement and performance. The institutional context is central to analysis, but hard to factor in. So, one purpose of the use of AT is to try to make sense of the relationship *between* the purposes of the innovation and associated findings *and* the aspects of context in which the innovation is embedded.

USING ACTIVITY THEORY FRAMEWORKS TO MAKE SENSE OF THE FINDINGS

We express these findings first, using Engeström's (e.g., 1999) expanded mediational triangle to explore conflicts and contradictions, and second, using Leont'ev's three levels of activity: *activity–motive*, *actions–goals*, and *operations–conditions* to aid characterization of activity. In the first, due to the differences (or tensions or contradictions) which have emerged in the ways in which the teaching team and the students perceive the activity as a whole, we hypothesise two activity systems operating side by side – the activity as experienced by the students in contrast with activity as experienced by the teaching team. There are apparent areas of overlap between them which we need to explain. This framework emphasizes differing *objects* for activity. We start from the triangular representation of Engeström, and use our own tabular form as a more effective way of presenting our data. The central double arrow representing outcomes of activity is of especial interest as we discuss below.

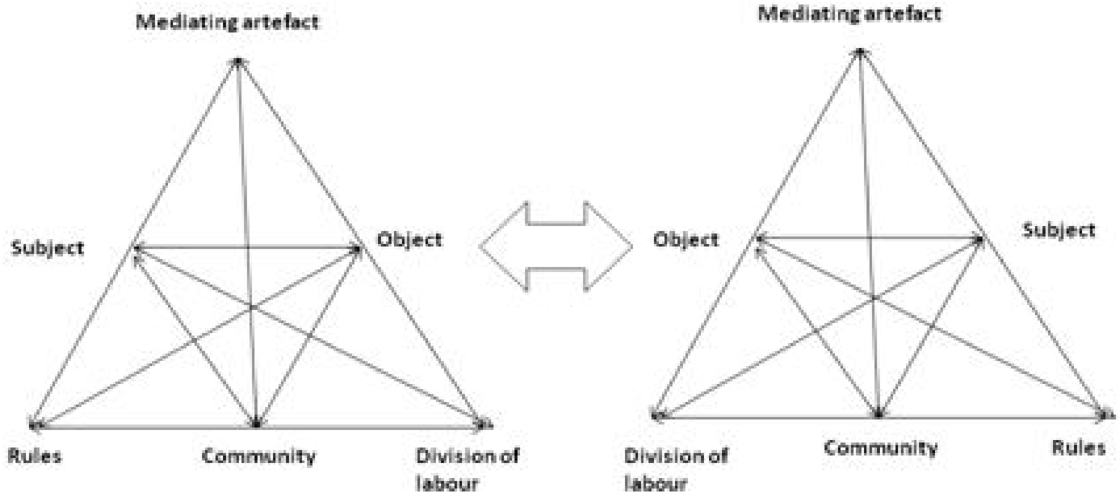

Figure 6: Two versions of Engeström's expanded mediational triangle (EMT) representing teachers' (left) and students' (right) perspectives of the teaching-learning environment as shown in Table 1.

EMT	Teaching Activity	Student Activity
Subject	Teacher or teaching team	Student or student cohort
Object	Engaging students conceptually with mathematics so that they learn in a conceptual/relational way rather than an instrumental way; understand the concepts involved in a way that they can use mathematics flexibly in relation to engineering tasks.	To participate in what is offered in the module to some degree and with a range of objectives related to desired outcomes (passing the exam), perceptions of what it means to study and learn (practicing past papers, plotting graphs by hand), and the amount of effort they will give.
Mediating artefacts	GeoGebra, inquiry-based questions, small groups, project. Theoretical concepts underpinning the innovation.	The lecturer, GeoGebra, i-b questions, small groups, project, demands of other modules which inhibit their devoting time to mathematics, other students, social life
Rules	Curriculum, assessment, university regulations, norms & expectations. Nature of discipline - what it means to 'understand' mathematics. Time allocation, e.g. in lectures, where concepts often have to be rushed.	University programme, curriculum, assessment, university regulations and norms/expectations; expectations of peers, what is needed to be successful (e.g., to pass the exam). Grading system.
Community	Academic, university and education communities, the wider world, and the cultures that permeate these communities	Student, academic, and university communities, the wider world, and the various cultures that permeate these communities
Division of labour	There are things that teachers do and that students do, usually different. Teachers have expectations of students' activities and roles.	There are things that teachers do and that students do, usually different. Students have expectations of teachers' activities and roles.

Table 1: Elements of Engeström's triangle expanded for the two systems

This tabular form emphasises some of the differences (such as the objects of activity of each group) but suggests that certain aspects are in common (such as the academic and university community). Important here is that it is not the objective nature of these communities that is in question but the perceptions of them held within the two groups. Teachers' perceptions of community see relationships within the communities with respect to academic practice, conceptual learning within a discipline, in our case the nature of mathematics, and so on. Students' perceptions of community see relationships in terms of what is required of them, what they are prepared to contribute, and how they discern their position in relation to official authority in contrast with the demands of their own culture. These differences of perception extend to division of labour and how labour within the two groups is perceived very differently, both in terms of own labour and of labour in the other group. Seen in these terms it is not surprising that outcomes seem quite different in relation to perceptions within the groups, although, in objective terms, measures of achievement have similar value for both groups (i.e. students who get the highest score get the highest grades).

In the second case, in Leont'ev's three levels, we contrast the activity of teaching with the activity of students' learning: all *activity* is necessarily *motivated* (level 1) and can be seen in terms of *actions* that are explicitly *goal*-related (level 2). Actions can be seen to be mediated by certain *operations* which are *conditioned* within prevailing circumstances and constraints (level 3). This framework emphasises ways in which the nature of activity is actually different for the two constituencies or cultures involved, that of the teachers and that of the students.

Level	Teaching Team	Students
1	*Activity* is *teaching- learning of mathematics*. For the teacher(s) it is *motivated* by the desire for students to gain a deep conceptual-relational understanding of mathematics. We might in this case call it "teaching-*for*-learning". We design tasks and approaches carefully to promote the desired learning	*Activity* is *learning within the teaching environment and with respect to many external factors* (youth culture, school-based expectations of university etc.) and is (probably) *motivated* by the desire to get a degree in the most student-effective way possible with a perception of understanding but little concern with the *nature* of understanding.
2	Here, *actions* are design of tasks and inquiry-based questions – with *goals* of student engagement, exploration and getting beyond a superficial and/or instrumental view of mathematics. *Actions* include use of GeoGebra with the *goal* of providing an alternative environment for representation of functions offering ways of visualizing functions and gaining insights into function properties and relationships. *Actions* include forming students into small groups and setting group tasks with the *goals* to provide opportunity for sharing of ideas, learning from each other and voicing mathematical ideas	For students, *actions* involve taking part in the module: attending lectures & tutorials; using the LEARN VLE system; using specially designed workbooks and other materials etc., doing coursework, revising for tests – with *goals* related to student epistemology. So *goals* might include an intention to attend lectures & tutorials because this is where you are offered what you need to pass the module; clear views on what ought to be on offer and what you expect from your participation; wanting to know what to do and how to do it; wanting to do the minimum amount of work to succeed; wanting to understand; wanting to pass the year's work.

3	Here we see *operations* such as the kinds of interactions used in lectures to get students to engage and respond, the ways in which questions are used, the operation of group work in tutorials and interactions between teachers and students. The conditions include all the factors of the university environment that condition and constrain what is possible – for example, if some tutorials need to be in a computer lab, then they all have to be; lectures in tiered lecture theatres constrain conversations between lecturer and students when tasks are set, limitations on time constrain what can be included.	*Operations* include degrees of participation – listening in a lecture (while texting a mate?), talking with other students about mathematics, reading a HELM book to understand some bit of mathematics, using the LEARN page to access lecture notes, Powerpoint etc. The conditions in which this takes place include timetable pressure, fitting in pieces of coursework from different modules around given deadlines, balancing the academic and the social, getting up late and missing a lecture. They also include the organization of lectures and tutorials and participating within modes of activity which do not fit with your own images of what should be on offer.

Table 2: Leont'ev's levels of activity expanded for the two systems

The above juxtapositioning adds strength to our hypothesis that we have two different activity systems here within (apparently) the same environment with common elements. However, in most cases the common elements are perceived/experienced differently. Perhaps the most important difference is the *object* of activity (Engeström) or *the motivating force* (Leont'ev) for the two systems. Both are valid, but the fact that they are different means that along with other factors – values placed on forms of understanding (the *rules* of the enterprise) or whether GeoGebra is positively helpful in promoting learning (mediating artefacts) – they result in the tensions observed.

What is the value of seeing the whole in these terms? What implications do we find? Having expressed our intention to work within a sociocultural frame, taking account of context and culture is fundamental. Here "we" are both the teaching team and the research team. As researchers we employ theory to synthesise from our findings. As teachers we seek to know more about how we can achieve our teaching-learning goals. Continuing teaching approaches as things stand is likely to perpetuate the position characterised above. Changing cultures (mathematical culture, student culture …), and some aspects of context (allocation of time to lectures, use of laboratories …), is difficult or impossible. Working within culture and context focuses attention on the local situations in which teaching and learning take place since this is where change is more possible. The innovation itself was itself such a change (quite a dramatic one!). In conceptualising new approaches, in making such changes we have to keep coming back to the global perspectives revealed through the analysis above. We shall be reporting further from our ongoing questioning about how to develop student's mathematical meaning making within the complexity we have revealed.

7: TWO COMPLEMENTARY VIEWPOINTS AROUND ACTIVITY

Fabrice Vandebrouck

LDAR, University Paris-Diderot, France

Starting from these papers, the research forum seeks to promote the way AT is connected to mathematical learning and teaching with technology; why AT seems to be so useful and therefore important in investigating the use of tools for educational purposes? What do we mean by tools and their mediational properties? What about AT and other theories?

Answers are related to a distinction between two points of view regarding Activity with artefacts (Norman, 1991): the “subject” view and the “system” view. In the system view, the artefact enhances the performance of the system whereas in the subject view the artefact changes the nature of a task the subject is facing. From the subject view, students as well as teachers are studied as actors of their activity. For instance, adopting the system view, Jaworski and colleagues show important differences between the Teaching Activity and the Students Activity (both at the three levels of Leont’ev and the poles of Engeström’s model). Does it mean that learning doesn’t occur for some students during the innovative project? No. As recalled by Jaworski and colleagues, one can distinguish between “Activity” and “activity” that students (and teachers) engage with tasks (personal view). Activity as in AT is different from the classroom activity involving tasks and tools: Activity is the whole which goes beyond activity (and activities) to encompass the teaching-learning system in its sociocultural entirety. In this sense, the cultural historical origin of AT doesn’t conflict with Piagetian-inspired individual origin ideas (Lerman 2013) as some other AT theorists claim.

In both viewpoints, the tool is central for both teaching and learning. Tools embed meaning which are culturally and historically established, and which are made concrete in the tool design and use. In Jaworski and colleagues’ paper, the mediating role of the tool – Geogebra- is experienced differently by students from the intentions and expectations of their teachers in designing mathematical tasks to encourage mathematical understanding. AT enables juxtaposing of students' and teachers' worlds in relation to the tool use and the teaching-learning interface. But with respect to the “subject” view, Maracci and Mariotti propose a complementary perspective according to which students and teacher can (are expected to) share the motive of mathematics teaching -learning activity centred on tools use.

At last, central to the idea of mathematics teaching-learning activity with tools is the accomplishment of tasks that require the use of the artefact – or the tool, see Monaghan’s chapter for the precise distinction. In Psycharis’ paper for instance, the task given to students is to build enlarging-shrinking models of capital letters with one variable. The tasks, as well as the artefact, carry the necessary mathematics with it that allows the learning. The use of artefacts for accomplishing the tasks shapes students’ – and also teachers’ – learning. For instance, in Psycharis’s and in Kortenkamp &

Ladel's paper, some properties of artefacts are analysed in terms of actions and feedbacks.

The tasks also carry a semiotic potential regarding to the artefact use. However, as recalled by Maracci and Mariotti, the relationship between the students' accomplishment of tasks and their mathematical learning often remains in the shadow. Several papers, in particular the one of Maracci and Mariotti of course, contribute to our understanding of the teacher's role and the tool use to investigate this shadow.

In the French school of activity theory (Vandebrouck, in press), as detailed by Lagrange, the potential of the task is considered with regard to the possibility for the constructive dimension of the activity to be deployed. In some sense, the collective discussion during the semiotic mediation is devoted to promote students' constructive activity. In the twofold approach, also mentioned by Lagrange, the teacher-to-students discourse aims to promote student constructive activity (this is called "constructive help"). In the theory of instrumental genesis, constructive activity is referred to the instrumentation and instrumentalization processes (Rabardel, 1995) as in Psycharis's or Silke & Ladel's papers. The instrumentation process may lead the student to internalize the schemes of use of the instrument. As Monaghan pointed out, in Wertsch's work the focus is also more on the subject-tool dialectic (or even subject-tool-task dialectic) than on the whole Activity system. Psycharis writes in the same way that a key consideration of the two theories - both the instrumental approach and the constructionism – is the two side relationship between tool and learner.

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