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STUDYING TRAINEE TEACHER EDUCATORS' DOCUMENTATIONAL WORK IN TECHNOLOGY ENHANCED MATHEMATICS

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We address the didactical design and corresponding material developed by one trainee teacher educator in the context of an in-service program concerning the use of digital tools in the classroom of mathematics. We analyse the trainee's documentational work carried out for giving lessons to colleagues as part of his practicum. The results indicate that fieldwork activities provided a source of motivations for the development of documents.

INTRODUCTION

In this paper we study the didactical design and corresponding material developed by one trainee teacher educator in teaching mathematics with the use of digital tools. The trainee drew upon existing resources as he started to teach in teacher education classrooms. The study took place in the context of an in-service program adopting reform-oriented perspectives to train teacher educators into the use of digital tools in the classroom of mathematics. The aim of the program was to provide the participants with methods, knowledge and experience in in-service teacher education and to educate them in the pedagogical uses of digital technologies for the teaching and learning of mathematics. One of the reform aspects of the approach for teacher education (see Kynigos & Kalogeria, 2012) concerned teacher educators' and teachers' active engagement in creating their own didactical design and material as coherent part of their professional development. Taking into account that teacher educators have very few resources to draw on directly (Zaslavsky, 2008), it was critical for the trainees to get used to developing their own material. In this course, the trainees were engaged in designing and generating resources in the form of microworlds and scenarios (i.e. structured activity plans addressing critical aspects of a pedagogically sound use of technology for the teaching and learning of mathematics). A structure [1] for addressing these aspects was developed by Educational Technology Lab (http://etl.ppp.uoa.gr), which participated in the design of the course and the corresponding material. The training program took place in specialised University Centres (UC) for 350 hours. The participants were experienced qualified mathematics teachers but the majority of them had no previous experience in the pedagogical use of digital tools. The plan was to employ the newly trained teacher educators in wide-scale 96h courses to groups of teachers in specific Centres for Teacher Education Support (CTES). The trainees were given material by the trainers after each lesson and an official document (called 'Notes for teaching in CTES') containing theory and a set of twelve generic scenarios as a basis for organizing their subsequent teaching in CTES. During the course, the trainee educators gained significant experience with the pedagogical use of five categories of digital media: Computer Algebra Systems, Dynamic Geometry Systems, Programmable software, Simulations and Data Handling tools. By the end of the course the trainees had to have developed one scenario for each of these categories as well as scenarios for the practicum. Practicum was part of UC official structure provided shortly before trainees complete the course, so as to engage them in field activities and give them the experience of implementing their design in real classroom conditions and reflecting on it. Practicum was divided in two parts: teaching in school and observation - teaching in CTES. Here in focus is the second part consisted of (a) observation of other teacher educators' teaching in CTES, (b) design of a 3-hour lesson for teachers in CTES under the supervision of a mentor, (c) implementation in the classroom, (d) presentation of design and implementation in whole class special reflective sessions, (e) activity report by the trainees.

THEORETICAL FRAMEWORK

We adopt the documentational approach of didactics according to which the teacher's work is developed *with* and *on* resources in a dialectic process where *design* and *enactment* are intertwined (Gueudet & Trouche, 2009). An implication of this approach is that curriculum material is not conceived as a static body of resources that guides instruction but rather as a set of objects amenable to changes and modifications depending on the teacher's didactical design. Gueudet and Trouche (2009) use the term *resources* to describe a variety of artifacts such as a textbook, a piece of software, a student's sheet, discussions with colleagues etc. Through a class of professional situations and teachers' experience, the existing resources are modified as documents according to the formula: *Document = Resources + Schemes of Utilization.*

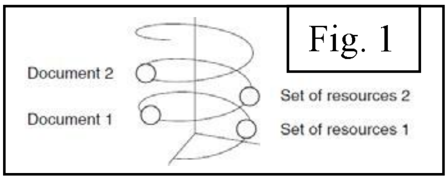

Fig. 1

Creation of documents is considered as unfolding through a dual process of instrumentation (the resources act on the teachers) and instrumentalization (teachers act upon these resources as they appropriate them). This process gives birth to a new entity, i.e. a document, which can be further transformed to a new document over time. This process is represented in Fig. 1 by a helix (Gueudet & Trouche, 2009). Describing the nature of the relation between resources and documents, Gueudet and Trouce (2009, p. 206) stress that "documentational genesis must not be considered as a transformation with a set of resources as input and a document as output. It is an ongoing process ...". However, Kieran (2009) wonders whether or not documentational genesis could be viewed as a set of transformations, albeit interrelated. According to her, a fine-grained zoom-in on a much smaller part of the helix (a point or a short arc) might also contribute to a deeper understanding of when and how instrumentation and instrumentalisation take place and how these processes influence teacher's interaction with resources. In our study a focus on the particular arc of the helix corresponding to the practicum would allow us to gain insight on the processes by which the trainees transformed existing resources to documents for their own teaching in CTES. Another theoretical construct that informs our perspective in this study is *double instrumental genesis (double IG)* (Abboud-Blanchard & Lagrange, 2006). It consists of the *personal* *instrumental genesis* (leading to the appropriation of a tool as an instrument for mathematical work at the personal level) and the *professional instrumental genesis* (leading to the construction and appropriation of the previous instrument into a didactical instrument for mathematics teaching at the professional level). Although double IG is at the core of teacher education in technology enhanced mathematics, there is very little attention given by research studies at the level of teacher educators. Therefore, we incorporated in our study a focus on the components that may influence the trainees' transition from the personal IG to the professional IG.

The general aim of this study is to shed light on the trainee teacher educators' documentational work as they began to practice teacher education themselves. We particularly explore what are the components involved in the arc connecting existing resources and new documents in these trainees' documentational work. Our focus is on how they act upon existing resources in the process of transforming them into documents for their teaching in real teacher education classrooms (instrumentalization) and how this process shaped their own activity (instrumentation). In this process we were also interested in exploring the trainees' transition from personal to professional IG.

METHOD

In order to highlight the transition from existing resources to new documents we chose Tom as an exemplary case because his teaching in CTES was based on one of the twelve official scenarios. This allowed us to view comparatively the documents he created to the existing official resources. Our role as academic trainers and mentors in the practicum provided a framework for designing our intervention which was based primarily on the making of links between knowledge-based research and practice in mathematics teacher education. In resonance with Gueudet and Trouche's (2011) principles regarding methodological aspects of research on documentational genesis we chose to (a) analyze Tom's work in time periods in and out-of-class (reflexive investigation principle), (b) address Tom's decisions taken in order to formulate his design through its use (design-in-use principle), and (c) consider his work embedded in and influenced by different collectives (i.e. peers in UC, teachers in CTES) (collective principle). We used data from different periods of time: (a) excerpts from reflective sessions which took place before Tom's design, (b) the official material and its transformations by Tom, as well as the arguments with which he documented his options, and presented in a whole class reflective session (c) Tom's activity report. Coding of discussions of the reflective sessions through constant comparative method (Strauss & Corbin, 1998) lead to a categorization in 'themes' i.e. discussions around a particular issue not necessarily in chronological order. We used parts of these transcripts in conjunction with Tom's activity report as complementary sources for analyzing Tom's design decisions in relation to the existing resources.

ANALYSIS

Before analysing Tom's documentational work we refer to the discussions that took place during the reflective sessions so as to give a flavour of the context within which

Tom's work evolved. Data analysis revealed that the most frequent discussion theme mentioned in these sessions concerned the dual role of teachers in CTES: "teacher as student" and "teacher of students". In one of the reflective sessions we took the challenge to connect this issue with current mathematics education research through the use of relevant theoretical constructs such as double IG. This idea seemed to have provided a basis for the trainees to rethink their classroom experiences from CTES until that time and interpret them through this perspective as shown in the following excerpt:

> TrainerA: What did you observe in the lesson in CTES taking into account double IG?
>
> TraineeA: I think that the teachers acted mainly as students who were trying to learn the tools rather than as teachers who would teach the scenario to their students.
>
> TraineeB: Up to a point you need to act as a student, to press keys, to understand the tools and after that to discuss with your classmates and the teacher educator what you would teach, the teaching sequence etc. There are two stages. You may stay only in the first, but this will happen only in the introductory lessons.
>
> TrainerB: Some scenarios are designed intentionally for the first stage because they reveal how mathematical ideas come to the fore through the use of the software. And it is important for the trainee to acquire such an experience so as to be able to design tasks and activities that facilitate the emergence of these ideas in the surface. I remember in my first lessons in this UC class your intense concentration on the software and nothing else. Now I see that you have started to pose pedagogic and didactic questions.

At the core of the above discussion is the difficulty inherent in teacher education courses that stems from the two roles of trainee teachers and how technology is integrated in each one of them. There are two aspects here: the first concerns the learning of technology itself and its potential for mathematics at the personal level; the second concerns the use of technology as a didactical tool. Double IG seemed to operate as a tool that helped the trainees to address the complexity inherent in teaching mathematics with digital tools as a teacher educator. Trainee B introduces the time parameter regarding the trainees' transition to the didactic level. During their early UC lessons trainees were mainly 'students' concentrated on the learning of technology while at the time of the discussion they had started "to pose pedagogic and didactic questions" (in the words of Trainer B). It seems that trainees' participation in the practicum plays a role in their transition from the personal IG to the professional IG.

Tom's documentational work

Tom's lesson in CTES was about the design of scenarios and worksheets. During his observation of lessons in CTES he had noticed that most of the teachers had difficulties in understanding the connection between scenarios and worksheets. He prepared for the teachers a worksheet corresponding to a scenario related to the study of sinusoidal function with Function Probe (FP) [2]. The specific scenario was one of the twelve official scenarios provided for the course. The indicative design of this scenario suggested the following teaching sequence in four phases. Phase 1: Highlighting the importance of converting degrees to radians for studying f(x)=sinx. Phase 2: Creating table of values in [-π, π] with step π/8, sending ordered pairs of x and y to Graph (first idea of sinusoidal curve), increasing the number of tabular values (development of conjectures), study of properties. Phase 3: Generating and filling columns in the Table with values of y=2sinx and y=3sinx, comparison to the respective values of y=sinx, construction of graphs and study of the corresponding functions, conjectures for the graph of y=ρsinx and the role of ρ. Phase 4: Performing mouse-driven transformation (vertical stretching) of y=sinx with the stretch tool, connection between graphs and algebraic representations και extraction of conclusions for the role of vertical stretch magnitude (operator) in the graph with the help of the history window, verification of conclusions through the use of Calculator.

Tom designed an original document-worksheet for the teachers in CTES aiming to teach the sinusoidal function and through this to show one way to connect scenario and worksheets. Besides, he integrated instructions within this artifact so as to facilitate the use of FP. During his presentation in one of the reflective sessions, he explained the decisions underlying his design providing also indications of the instrumentation process underlying his activity: "In my view, a complete scenario needs to have questions or issues that constitute objects for negotiation and activities designed with particular learning aims. Thus, I created a worksheet with two columns. In the left one I inserted the questions of the worksheet. In the second one the rationale, that is to say what I want to achieve with these questions. I marked with grey the questions that could be ignored. During my observation of other teacher educators' lessons in CTES I also noticed that most of the teachers did not know the software well. I resolved this by inserting footnotes at the bottom of each page." (see the part of Tom's worksheet from the Phase 3 of the teaching sequence mentioned above in Table 1).

The structure of this document integrates the issues mentioned by Tom in the previous excerpt and also signifies a process of instrumenalisation through which a document is becoming a didactical tool for CTES. From a didactic/teaching point of view Tom takes a different perspective from the curriculum as regards the use of the stretch tool. The indicated analysis of the teaching sequence provided in the official version of the scenario proposes the use of the stretch tool by the students for manipulating vertically the height of the curve y = sinx at random and through this to identify the role of ρ through the history window. Tom proposed the design of the three graphs in the same coordination system. Then he suggested that the students should stretch the graph of y = sinx until coinciding with the graph of y = 2sinx, y = 3sinx and through this to identify the role of ρ in the transformations of y=sinx kinesthetically. This approach reveals Tom's conception of the construction of mathematical knowledge according to which dynamic manipulation of mathematics objects can be a precursor mediating the transition to more formal understandings. This was also evident in the next excerpt taken from his presentation in the corresponding reflective session. Tom refers to an episode that took place during his teaching in CTES: "During the lesson we had to study functions in the form y=sin(x+a). One of the teachers said: "We can firstly construct the graph of y=sin(x+π/2) and then study its relation to y=sinx". I answered:

"Then there is no reason for using the stretch tool … I believe that in this case this tool can help students understand the exact relation". Again Tom expresses his conception regarding the integration of the stretch tool in the teaching sequence. He attaches added value to its use for mediating the targeted relationships. The other teacher's proposal does not leave space for approaching y=sin(x+π/2) through dynamic transformation of y=sinx which is at the core of Tom's didactical design.

<table>
<tr><th>Questions</th><th>Rationale</th></tr>
<tr><td>In the Table window fill in two more columns z=2sinx and w=3sinx using different colors.</td><td rowspan="6">Study of function y=ρ*sinx.
What is the role of ρ.
Multiple representations of the relation between y=sinx and y=2sinx, y=3sinx
(Table: visually, Graph: visually and kinesthetically).</td></tr>
<tr><td>Send points (x, z) and (x, w) to the Graph window.</td></tr>
<tr><td>What do you observe as regards periodicity, symmetry, monotonicity and extremes.</td></tr>
<tr><td>Design the graphs of the above functions together with the graph of y=sinx in the same coordination system.</td></tr>
<tr><td>Stretch and contract the graph of y=sinx through the use of the stretch tool (Note 1) so as to coincide with the new points of y=2sinx and y=3sinx. (The magnitudes of the transformation have to be 2 and 3 respectively). What do you observe?</td></tr>
<tr><td>Experiment with other magnitudes of transformation (i.e. vertical stretch/contract) through the use of the stretch tool. Check your conclusions through the use of the Calculator or the Table.</td></tr>
</table>

Note 1: Before answering this question you have activate the choice "Show transformations".

Table 1: Extract from Tom's worksheet for teachers.

The next step of Tom's didactical design included the exploitation of the above mentioned document-worksheet for the development of a scenario. The right column of this document brought to the fore the need for an abstract structure depicting the main issues involved in scenario design as well as the main questions that a teacher has to answer before developing a scenario. Thus he constructed a new document-'structure-for-scenario-design', which reminds Concept Maps, so as to provide a figural representation of those issues/questions and potential design trajectories. Due to space availability we provide here a brief description of a small part of this document. A cell including the phrase 'an underlying learning theory' (i.e. of a scenario) was connected with three other cells entitled (a) 'potential design' (connected to five other cells entitled 'what', 'why', 'where', 'how', 'how long'), (b) 'expected learning outcomes' and (c) 'reflection based on the implementation'. Explaining his choices as regards the specific document- 'structure-for-scenario-design' in his personal activity report, Tom mentions: "The scenario structure depicted in the map does not follow the official scenario template but it includes all the issues, processes and questions that a teacher has to consider when designing a scenario. At the same time it is closer to everyday teaching practices. It is important for the teachers to understand first what do they have to do and if they grasp it then they can move towards forming their scenario according to the official structure". Here Tom highlights that the rationale behind the creation of this specific document stems from his need to bring 'closer' teachers' everyday practices to the theoretical aspects of mathematics teaching addressed by scenarios. Working at the professional level in terms of double IG, Tom develops new didactic tools with the aim to bridge the distance between the language of official resources and the language of practitioners. At the same time he adopts a critical stance towards the official resources which allows him to develop the corresponding schemes of utilization for students and teachers according to his own perspective.

CONCLUSIONS

Three factors seem to have influenced Tom's development of the two new documents: (1) Teachers' difficulties in developing their own teaching material, (2) Teachers' difficulties in learning the affordances of the software tools and integrating them in activities with added educational value (e.g. microworlds, scenarios, worksheets), (3) Tom's knowledge, pedagogical conceptions and experiences regarding the everyday practice of teachers. The first two factors are directly linked to the observation of other teacher educators' teaching in CTES. During these lessons Tom had the opportunity to detect the above difficulties and to adopt a critical stance towards existing resources. At the same time reflective sessions seemed to have provided trainees with theoretical constructs (i.e. double IG) to reconsider teachers' dual role in CTES. Thus the above processes involved in observation and reflective sessions can be considered as part of the instrumentation aspect of Tom's documentational work. Tom constructed the two documents as didactical tools for addressing those issues and incorporated his own conceptions in his design as regards tool use for engaging students in meaningful mathematical activity. The construction of these documents signifies Tom's passage to the professional level in terms of double IG. Based on these findings we conclude that the components of the arc linking existing resources and new documents at the level of teacher educators are the activities around the practicum (observation, reflective sessions) as well as trainees' personal knowledge and epistemologies. The diagram below represents the evolution of Tom's documentational work over time in the helix:

Existing resources + { observation in CTES; design and reflective sessions in UC; personal knowledge and epistemology } → New documents

In our course the practicum was designed to take place shortly before the end of it. Our findings indicate that fieldwork activities motivated trainees to generate documents. A further implication for the design of similar teacher educator courses is that fieldwork activities should be exploited as early as possible in the course.

NOTES

[1] 1. Title, 2. Scenario's identity (author, subject area, topic), 3. Rationale (innovations, added value by the use of technology, students' learning problems addressed), 4. Context of implementation (grade, duration, location, prerequisite knowledge, social orchestration of the

classroom, goals), 5. Phases of implementation (sequence of activities, roles of the participants, anticipated teaching/learning processes), 6. Possible extension, 7. References.

[2] FP is a multi-representational software with three windows: Table, Graph and Calculator. Function graphs can be produced in a number of different ways, e.g. inserting a formula for the function, "receiving" ordered pairs (x, y) from a table ("x" and "y" columns can be generated). Particular icons allow horizontal and vertical transformations of functions (translations, reflections and stretches) made through direct actions on the graph. Stretching is carried out with the stretch tool that allows mouse-driven horizontal and vertical stretching. The corresponding magnitude of the stretches appears in the upper right corner of the Graph. A history window in the Graph allows viewing the formulas of the transformed functions.

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