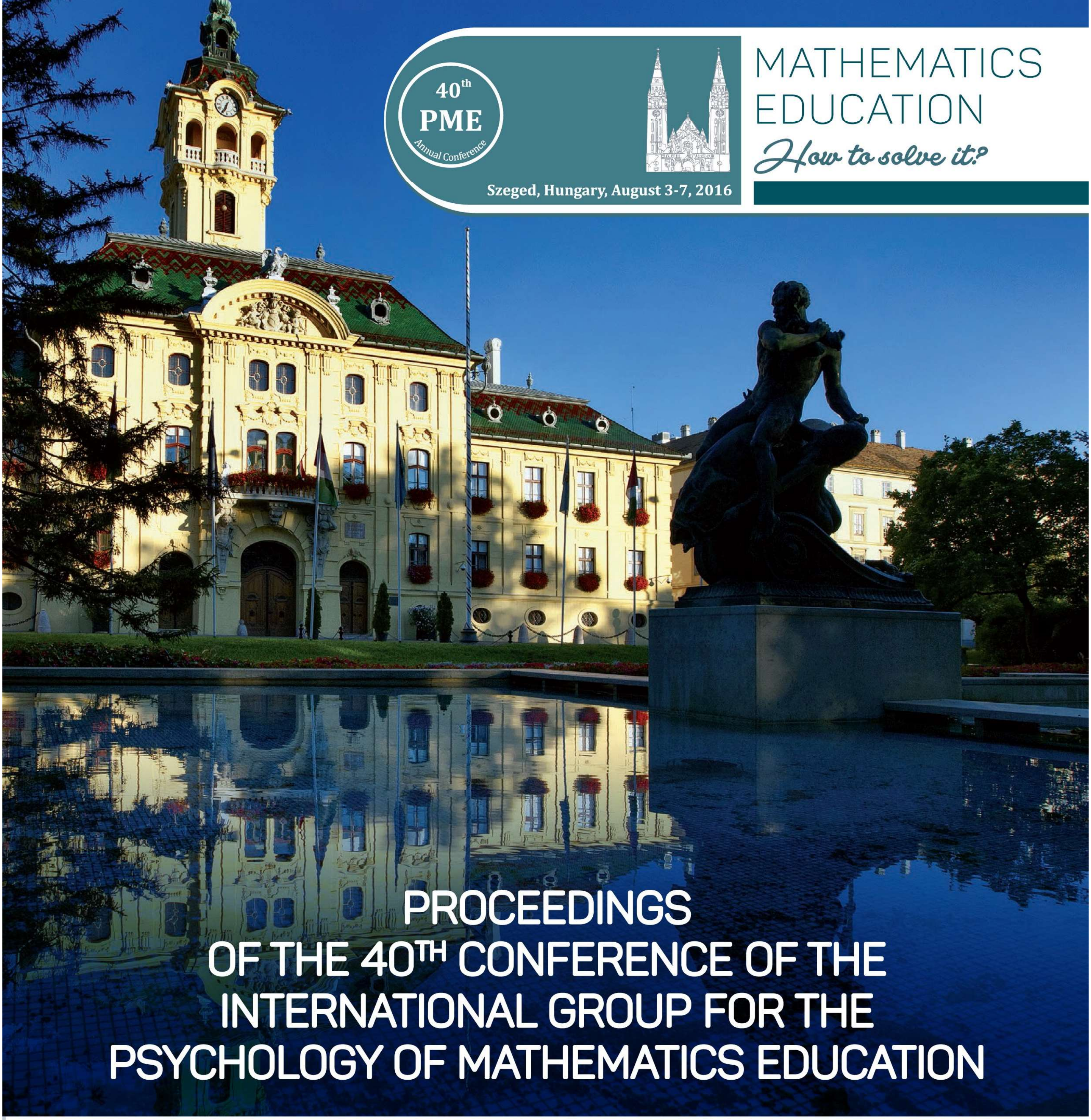

40th PME Annual Conference

Szeged, Hungary, August 3-7, 2016

MATHEMATICS EDUCATION

How to solve it?

PROCEEDINGS OF THE 40TH CONFERENCE OF THE INTERNATIONAL GROUP FOR THE PSYCHOLOGY OF MATHEMATICS EDUCATION

EDITORS: CSABA CSÍKOS • ATTILA RAUSCH • JUDIT SZITÁNYI

VOLUME 4

PME40 / SZEGED / HUNGARY

3-7 AUGUST, 2016

Proceedings of the 40th Conference of the

International Group for the Psychology of Mathematics Education

PME40, Szeged, Hungary, 3–7 August, 2016

MATHEMATICS AND SCIENCE TEACHERS' COLLABORATION: SEARCHING FOR COMMON GROUNDS

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This paper focuses on the collaboration between one mathematics teacher and three science teachers during a school year in a professional development context supporting inquiry-oriented approaches and connections with the world of work. Through an Activity Theory perspective it addresses contradictions and convergences that emerged in this collaboration as well as interactions between the activity systems of mathematics and science teaching when the mediating tool is the notion of function and its graphical representations. The results indicate the development of shared understandings for the different perspectives that function and graphs are viewed in mathematics and science teaching and shifts in the teaching activity of the teachers in the direction of connecting meaningfully mathematics and science.

INTRODUCTION

The issue of communication between science and mathematics in school classrooms has been acknowledged as crucial for a deeper understanding of common or related conceptual domains. Consequences of the lack of this communication are spread in different directions as on the textbooks' rationale (Triantafillou, Spiliotopoulou & Potari, 2015); students' understanding (Planinic, Susac & Ivanjek, 2012); teachers' classroom discourse and activities (Shirley et al., 2011). The need to provide teachers opportunities to build connections between mathematics and science teaching into their classrooms is more than evident. For example, Berlin and White (1995) argue that this collaboration provides opportunities for students to have less fragmented, and more learning stimulating experiences. However, the undertaken research needs to be strengthened, while more evidence on the actual context of mathematics and science teachers' collaboration could be emerged. Frykholm and Glasson (2005) suggest that authentic contexts could provide fertile ground for this collaboration, while King, Newmann and Carmichael (2009) introduce the idea of 'rich tasks' that involve inquiry-oriented activities in the context of real world scenarios.

This paper refers to a study that took place in the context of a European project, Mascil (see: www.Mascil-project.eu), that aims to promote the integration of inquiry-based learning (IBL) and the world of work (WoW) in the teaching and learning of mathematics and science. To achieve these goals, teacher education and professional development activities have been designed where science and mathematics teachers collaborate in groups to design, implement and analyse lessons in the spirit of lesson study approaches (Hart, Alston & Murata, 2011). A critical issue to consider is in what ways these collaborative activities challenge teachers from different disciplines to

2016. In Csíkos, C., Rausch, A., & Szitányi, J. (Eds.). *Proceedings of the 40th Conference of the International Group for the Psychology of Mathematics Education,* Vol. 4, pp. 91–98. Szeged, Hungary: PME.

explore the integration of mathematics and science into their teaching and recognize epistemological and didactical issues related to these different practices. To address this issue, we adopt an Activity Theory (AT) perspective to focus on the teaching activity of mathematics and science teachers and on its development in the context of collaboration. We focus on the notion of function and its graphical representation that appeared to be central in the teachers' collaborative activities and we address the following research question: How do mathematics and science teachers' collaborative efforts enrich their teaching activity and enhance connections between the different epistemological and didactical issues on functions and graphs?

THEORETICAL FRAMEWORK

We adopt Engeström's (2001) approach to investigate the process of mathematics and science teachers' professional learning when they are challenged to integrate IBL and the WoW into their teaching. We consider two activity systems, the activity of teaching mathematics and the activity of teaching science, to study the contradictions and convergences that emerged between the two systems, when teachers attribute meaning to the notion of function and its graphical representation. Functions and their graphs are approached from different perspectives in the two disciplines. The teaching of function in school mathematics is mainly formal and the focus is on its definition and its properties. Function is a multifaceted object playing a central role in the development of other mathematical ideas. In science teaching, functions are formulated on the basis of experimental data and are tools for describing, explaining, and predicting real world phenomena (Michelsen, 2006). As regards graphs, making sense of a graph in mathematics means "gaining meaning about the relationship between the two variables and, in particular, of their pattern of co-variation" (Leinhardt, Zaslavsky & Stein, 1990, p.11). In physics, the role of the context in which a graph is used takes a significant role in its meaning (Roth & McGinn, 1997). Below, we provide some main theoretical concepts related to our AT perspective.

The "activity system" is a basic concept of AT in the way that is approached by Engeström (2001). It is collective, tool-mediated and it needs a motive and an object. Individual and group actions are studied and interpreted against the background of entire activity systems. Activity systems are transformed over lengthy periods of time when the object and the motive of the activity are reconceptualized to embrace a radically wider horizon of possibilities than in the previous mode of the activity. Central to the process of transformation are contradictions within and between activity systems emerging when a new element comes from the outside. The idea of movement across borders appear in what Engeström (2001) describes as third generation of AT. Figure 1 shows a

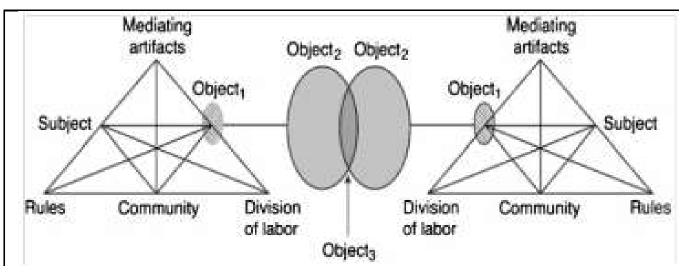

Fig. 1. Interacting activity systems (Engeström 2001, p. 136)

representation of a third generation activity in the form of two interacting activity systems represented by an extended mediational triangle. The two triangles indicate the basic dimensions of the second generation AT with elements the subject and the object of the activity that is constructed through the mediation of tools, but it is also framed by the community in which the subject participates, its rules and the division of labor. Object 1 moves from an un-reflected and situationally given goal to a collectively meaningful object constructed by the activity system (object 2) and to a potentially shared or jointly constructed object (object 3). By studying contradictions and convergences between the two activity systems, we examine how the notion of function and its graph (mediating artifacts) mediate the teaching actions of mathematics and science teachers (subjects) to form shared meanings and goals (object 3).

METHODOLOGY

In mascil implementation, 12 groups of in-service secondary teachers from mathematics, science and technology have been established. Each group, supported by a teacher educator, participated in two or three cycles of designing, implementing and reflecting during a period of a school year. Before and after each implementation of the designed lessons professional development (PD) meetings took place. During PD meetings teachers collaborated in designing together inquiry-based tasks, shared their experiences from the implementations and discussed emerging issues. Besides, interviews were arranged with a number of participants from each group in order to further address the impact of the PD experience on their professional learning.

In this study, we focus on three science teachers (sctA, sctB, sctC) and one mathematics teacher (mtA) who worked in the same upper secondary school and were members of the same mascil group (7 teachers). These teachers collaborated in the design and implementation of three tasks (*Elasticity of Ropes*, *Biodiesel* and *Drug Concentration)* integrating mathematics and science in the context of three cycles of designing-implementing-reflecting. Here, we analyze data from the first four out of six PD meetings, the classroom implementation of the first task and the teachers' interviews. In these PD meetings, the teachers started to exchange ideas about co- designing, discussed about the design of the first task and reflected on its implementation. The classroom implementation of this task lasted 3 teaching sessions (45 minutes each) and it involved: introduction to the task through short videos of situations where ropes broke; discussion about the importance of exploring these phenomena; experimentation with weights and springs to conjecture Hook's law; experimentation with weights and wires in non-linear situations where the elasticity is destroyed and the material breaks; construction of graphs of the Hook's law by the students based on their measurements; comparison of weight-elongation graphs for different materials (e.g., glass, rubber); classroom discussion on emerging issues about the elasticity of materials and the functional relations used to model the relevant phenomena. The science teachers orchestrated mainly the experimentation phases while the mathematics teacher had the responsibility to manage the classroom activity related to functions and graphs.

The data were audio-recorded and transcribed. Under a grounded theory approach (Charmaz, 2006) we analysed the different sources of data related to the three phases of the first cycle and to the teachers' interviews. Initially, we identified parts of the data concerning the activity of mathematics teaching and the activity of science teaching. Then we focused on common themes that cross the two practices (e.g., inquiry in mathematics and science teaching, the role of context in mathematics and science teaching, the nature of concepts and processes in mathematics and science). Episodes indicating contradictions and convergences were selected within and across the common themes. Finally, we indentified interactions between the two activity systems (mathematics and science teaching). In this paper, the steps of the analysis described above concern the notion of function and its graphical representations that appeared to be central in mathematics and science teachers' interactions.

RESULTS

The meaning of contradictions is related to the elements of the AT triangles across the two activity systems. Convergences appear as common actions and goals that indicate an integration of the objects of these systems. Below, we address epistemological and didactical issues around the design and implementation of tasks integrating mathematics and science that emerged in different phases of teacher activity. A central theme of discussion throughout the PD meetings was the notion of function and the different ways by which it is approached in science and mathematics teaching.

Searching for tasks and concepts to integrate science and mathematics

In the first PD meeting, the teachers were introduced to the mascil philosophy through the analysis of existing mascil tasks and they were encouraged to collaborate in co-designing lessons based on these tasks or new ones developed by them. In the second PD meeting, the teachers brought their own ideas for tasks and started to discuss possible links between mathematics, physics and chemistry. The mathematics teacher (mtA) made explicit his willingness to work together with the science teachers by recognising that science teachers could provide ideas for contextual tasks where mathematics is embedded: "I see that you have the knowledge of the contexts that we can use to design lessons together". He also provided specific suggestions promoting their collaboration: "We can co-teach for four hours in the an 11th grade class". The science teachers provided different contexts for potential tasks (e.g., heat engines, biodiesel, elasticity of ropes) and with the encouragement of the teacher educator they suggested possible mathematical ideas related to these contexts. The physics teacher (sctA) suggested as a mathematical idea the concept of function appearing in the transformation of thermal energy in heat engines. He mentioned that graphs of functions such as straight lines, hyperbolas and exponentials used in this context can provide a bridge to mathematics. However, he recognized divergences between how mathematics and science approach functions:

> "In science, you first take measurements in an experiment and then you want to see what function is behind. Are you interested in this in mathematics? The function may be a

familiar one, a polynomial. What we actually do as physicists is to do the measurements and insert them in a software that gives us the corresponding function that can be a known one or not". (sctA, 2nd PD meeting)

In the realm of the discussion, mtA indicated that functions and graphs constitute objects of study in mathematics, but usually in context-free situations. The contradiction that appears here concerns epistemological issues on how function is considered in science and in pure mathematics. The view that sctA expresses is closer to how function is used in modelling, which is not emphasised in school mathematics. In the third PD meeting, the notion of function and its graph emerged when the two physics teachers proposed the *Elasticity of Ropes*. The discussion that followed was around didactical issues such as: students' tendency to consider all relations as linear; students' difficulty to connect graphs with physical phenomena; the meaning of inquiry in the task; the connection of the tasks to mathematics curriculum. Initially, mtA found the mathematics involved in the task rather trivial for high school students: "I cannot see how to contribute here. The law is too simple from a mathematical point of view … It is too experimental". Later on, the discussion moved in graphs for non-linear relations when the elasticity is destroyed and the students were asked to interpret the graph in relation to the behavior of materials. mtA at this point seemed to overcome his initial doubts and recognized the potential of the task to indicate the distance between real world phenomena and mathematical models: "A law models a situation under certain conditions. And this is important in mathematics as well".

Implementing the designed tasks

The students have already made the experiment with the springs and have collected their measurements. The two physics teachers had also performed the experiment for testing the elasticity of wires by using weights. At this phase, the mathematics teacher took over the management of the lesson by asking the students to draw a graph of the relation weight–displacement based on their measurements. The notion of function and its main properties again is the common tool pertaining mathematics and science teaching. In the classroom discussion, the teachers took the opportunity to make explicit to the students the different conventions and rules followed in mathematics and science as it appears in the following extract:

sctA: Let's see how we use graphs in science. In science, we are not allowed to put numbers in the two axes as well as on the graphs. Is this common in mathematics?

mtA: This is not a problem for us.

sctA: The criterion for selecting scale is to find the extreme measurements and their difference. I do not know what mathematics teachers do in the classroom.

mtA: We try to have the same scale in the two axes as we usually draw graph functions with a known formula.

sctA: This is very interesting. We never do this. And we do not have any problem with the origin of the axes.

mtA: And the way we treat the slope is different.

sctA: We discuss it because we realise that we say to our students different things. In science, the slope has always units of measurement, but not in mathematics.

The teachers also started to make connections between the function as a mathematical tool and the physical phenomenon. For instance, mtA working in the context of the physical phenomenon (the specific measurements of the spring displacements for different weights) used the notion of function as a tool for interpreting this phenomenon, an approach that is not common in mathematics teaching. In particular, he challenged students to make connections between the properties of function as a mathematical object with the experiment. Below, we list questions he posed to the students to illustrate his attempts: “I would like you to explain why two successive measurements as points in the graph can be connected only with a straight line”; “If the graph is a straight line what does it mean as regards the relation between the weight and the displacement?”; “Can you make predictions for different values of displacements and weights?; “What is the meaning of slope in this experiment?”; “How do you interpret the tangent of the angle in the Hook’s law?”. sctA extended the discussion by pointing out that in physics the function that describes a phenomenon is a dynamic object depending on the variability of the measurements: “Why some of your measurements are not on the straight line? It is not needed to connect all the points in one straight line; we draw the best line fit”.

Reflecting on the experience

Reflecting on the implementation in the fourth PD meeting, the teachers discussed about what the students gained from this lesson and what they themselves learned. They made explicit the epistemological divergences underlying the notion of function and they became aware of the fragmented way that this notion is approached in the teaching of mathematics and science in school. In the following two extracts we illustrate mtA’s and sctA’s development of awareness of these epistemological and didactical divergences:

> Through observing and interpreting weight-elongation graphs for different materials, the students recognized that the elasticity and the stiffness of the materials are related to the slope of the graph. (mtA’s reflection, 4th PD meeting)
>
> The students managed to connect a mathematical tool, the slope of a line, to the elasticity of materials. They had the opportunity to interpret the slope as we conceive it in physics, as a required force that can cause a unit of change in the length of a spring. They also realised that a graph in physics is beyond the formal way is taught in mathematics. It is a tool that helps them to interpret a physical phenomenon and also make predictions… Teaching mathematics and science together made us realise that we teach the same thing with completely different ways… (sctA’s reflection, 4th PD meeting)

In the interviews, teachers appreciated the collaboration and seemed to become aware of the different epistemological and didactical perspectives that mathematics and science teachers adopt in teaching. They also recognised that students’ learning was rather fragmented and their own difficulty to bridge the distance between mathematics and science in the actual classroom:

> In a school day, I teach my part and the mathematics teacher his own. However, the students can listen to many different things. It is a problem not to have an idea of other teachers' work. Through mascil we realised the diversity of our teaching approaches and the emerging problems. Before, we were not aware of it. (sctA's reflection, interview)

> Maybe more time was needed to integrate the actual teaching of the two subjects. In the reality, we distributed our responsibilities, I do this, you do that. Each of us remained in his own space. (mtA's reflection, interview)

The transformation of teaching

By analysing the mathematics and science teachers' collaborative activities we could trace developments and changes in teachers' perspectives as regards epistemological and didactical issues on functions and graphs. Particularly, mtA overcame his initial doubts and recognized the potential challenges of the *Elasticity of Ropes* task for his students. During their collaborative activity, they utilized common tools (e.g., the same worksheet) and transformed their initial goals (individual teaching goals) into shared goals and teaching practices. Their joint activity made them realize divergences in the meaning they attribute to the function concept and the representational conventions and rules they follow in their communities. They also developed awareness on the fragmental way of teaching the notion of function which could have an effect on students' understanding. Besides, we could identify mtA's shifts in his teaching practice of function when he posed context-specific questions to his students (e.g., what is the meaning of slope for this experiment?), or he used the notion of function as a prediction tool for the physical phenomenon under consideration. Finally, in their reflections all teachers appreciated the existed difficulties in achieving the fusion of mathematics and science teaching practices.

CONCLUNDING REMARKS

The short analysis supported also by other evidence emerged during the project reveals the strength of collaborative work between science and mathematics teachers and the value of sharing practices in actual science and mathematics teaching. The process of developing a shared understanding of common concepts and the meaning of their teaching for students appears to be rather demanding. However, it evolved through teachers' engagement in discussing connections, discerning epistemological aspects, finding complementary elements and sharing classroom experiences. As King et al. (2009) also argue, inquiry-oriented activities in the context of real world scenarios offered opportunities for science and mathematics teachers to integrate mathematical and scientific ideas and processes into their teaching.

Acknowledgements

Mascil has received funding from the European Union seventh Framework Programme (FP7/2007-2013) under grant agreement n° 320693. This paper reflects only the authors' views and the European Union is not liable for any use that may be made of the information contained herein.

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