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ENACTMENT OF INQUIRY-BASED MATHEMATICS TEACHING AND LEARNING: THE CASE OF STATISTICAL ESTIMATION

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This paper investigates the enactment of inquiry-based mathematics teaching (IBMT) and inquiry-based mathematics learning (IBML). The focus is on how two teachers enacted the same task into their classrooms and on how this enactment framed students' mathematical activity concerning the notion of statistical estimation. Through the use of the sociodidactical tetrachedron, our study shows that IBMT and IBML are framed by different factors such as the selection and use of artefacts, the newly established and existing classroom norms and the meanings that teachers and students attribute to statistical estimation and to inquiry-based teaching and learning.

INTRODUCTION

Inquiry in mathematics – described through terms IBML and IBMT - can be defined loosely "as a way of teaching in which students are invited to work in ways similar to how mathematicians and scientists work" (Artigue & Blomhøj, 2013, p. 797). As regards its classroom implementation, inquiry is considered an opposing approach to teacher-centred ones integrating a combination of at least two of the following three characteristics: opportunities for students to generate several options and solutions; discuss together; and make justified decisions (Chan, 2006). Existing empirical studies indicate the benefits of inquiry in the teaching and learning of mathematics and science in terms of students' outcomes and teaching quality at all educational levels and systems (Bruder & Pescott, 2013). However, integrating inquiry in the real classrooms is a rather complicated task bringing to the fore a number of issues such as the nature of the designed tasks, the provided resources, the teaching management and the students' learning (Artigue & Blomjoi, 2013). Thus, teachers aiming to enact inquiry in their teaching face a number of concerns at the level of design and implementation and according to their decisions and actions inquiry is likely to take different forms with respect to the surrounding (e.g., institutional) conditions. To address this issue, we adopt a sociocultural perspective to study the teaching activity of mathematics teachers in relation to students' activity while enacting inquiry in their classroom. The teachers in our study worked collaboratively on an open-ended task engaging students in developing strategies to count the number of people in a demonstration based on an air photograph. We focus on the notion of statistical estimation that appeared to be central in the classroom activity. We address the following research question: How are IBMT and IBML enacted into the mathematics classroom?

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THEORETICAL FRAMEWORK

Elaborating on the conceptualization of IBML and IBMT, Artigue and Blomjoi (2013) indicated a number of concerns that need to be addressed as a basis for integrating inquiry in classroom teaching. These include: the 'authenticity' of tasks and students' activity in terms of connection with real life activities; the epistemological relevance of the tasks from a mathematical point of view; the modelling dimension of the inquiry process and the extra-mathematical sources of rationality; the experimental dimension of mathematics; the students' autonomy and responsibility for producing and validating answers; the guiding role of the teacher and teacher–student(s) communication in the classroom. In our attempt to study IBMT focusing on the teacher's decisions and actions and IBML focusing on students' actions and behaviours, we use the sociodidactical tetrahedron (SDT) (Rezat & Strässer, 2012).

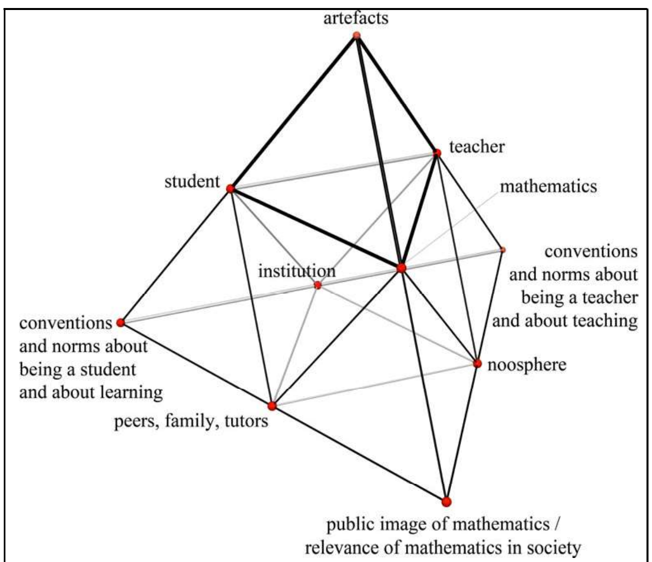

Fig. 1: The sociodidactical tetrahedron.

SDT allows us to focus on the classroom interaction and explain how IBMT and IBML are enacted in the classroom by taking into account the sociocultural setting. Rezat and Strässer (2012) used the Engeström's triangle and distinguished two activity systems: (a) the teaching of mathematics taking as subject the teacher and (b) the learning of mathematics taking as subject the students. So, they extended the classical didactical triangle (teacher – student – mathematics), first to the didactical tetrahedron (teacher – student – mathematics – artefacts), and finally based on EMT to the *Sociodidactical Tetrahedron (SDT)* (Fig. 1). The community of teachers is made up of teachers and mathematics educators, the *noosphere* that has created certain images about what is mathematics teaching and learning. The community of students belongs to *peers, family* and possibly *tutors.* In the base of the STD, the two vertices represent *conventions and norms about being a student and about learning* and *conventions and norms about being a teacher and about teaching* while in the third vertex the role of mathematics in relation to the division of labour in the society and the public image of it (*public image of mathematics*) are considered. The communities include both teachers' and students' communities as well as the *institution* that is represented as another point in the STD. In our study, we focus on the relations between the teacher and the students with the mathematics content (the statistical estimation), the artefacts (the materials provided by the teacher and those used by the students), the public image of mathematics in the society (the authentic workplace setting), the noosphere (the meaning of IBML and IBMT as they are discussed in the community of mathematics education researchers and teachers), the classroom norms and conventions (how teachers and students conceive mathematics teaching and learning in the classroom with regard to different approaches to teaching and learning) and the institution (curriculum, school schedule, school rules).

The notion of statistical estimation in our study refers to the informal inferential reasoning process in which students make arguments to support inferences about unknown populations based on observed samples (Zieffler et al., 2008). A statistical inference is formed by (a) a statement of generalization "beyond the data," (b) use of data as evidence to support this generalization, and (c) probabilistic (non-deterministic) language that expresses some uncertainty about the generalization (Makar & Rubin, 2009). Here, we explore IBMT and IBML when the main mathematical idea is the statistical estimation.

METHODOLOGY

The reported study took place in the Mascil context (www.mascil-project.eu) that targeted mathematics and science teachers' professional development through the integration of inquiry-based learning and workplace into their teaching. To this end, professional development (PD) groups of in-service mathematics and science secondary teachers have been established. Each group, supported by a teacher educator, participated in cycles of designing, implementing and reflecting during a period of a school year. Before and after each implementation of the designed lessons professional development (PD) meetings took place. In this paper we focus on two mathematics teachers (Vangelis and Eirini) who worked in lower secondary schools in Athens and were members of the same PD group. The teachers collaborated in the transformation and implementation of the mascil task *Counting People* that was proposed to the group by the teacher educator in the 3rd PD meeting. Both implemented the task after adapting it to the Greek context in their 9th Grade classes (14 years old) for two teaching hours each. In both cases, the students were separated into groups (4-5 students) and all groups worked collaboratively for the solution of the problem. We chose to focus on the cases of Vangelis and Eirini because they expressed different perspectives in relation to IBMT and this allowed us to address issues related to our research question.

The *Counting People* task, as was transformed by Vangelis and Eirini, engages students to devise their own plan for counting the number of people in a particular antiracist demonstration that took place in Athens in front of the Parliament House. In the beginning of the two lessons both teachers provided the students with one photo (see Fig. 2) showing people demonstrating in three streets in front of the Parliament House (the Vas. Georgiou str., Vas. Amalias str., and Othonos str.) where the crowd density varies. Students are asked to adopt the role of a journalist and to provide information about the number of demonstrators in the photo. Key steps for a possible solution can be: (i) relating the counting of people with calculating the area of the specific streets; (ii) devising a plan to estimate the area of the three streets (the teachers did not provide a scale); (iii) estimating the total number of demonstrators in the three streets. From the above, it seems that statistical estimation becomes a central inquiry issue in this task.

Fig. 2: The photo.

Fig. 3: The map of the area.

The data consists of (a) the audio recordings of three 2- hour PD meetings (one for designing teaching (3rdmeeting) and two (4th and 5th meeting) for reflecting)); (b) a video-taped lesson for each teacher; (c) audio taped students' group discussions in each classroom; (d) teachers' reflective interviews. Concerning the data analysis, first from the transcribed lessons and the group discussions we identified key actions that each teacher and the students had undertaken in relation to IBMT and IBML and the statistical estimation. Then through the analysis of the PD meetings and the reflection interviews we looked for the teachers' perspectives of IBMT and IBML and statistical estimation. At the next stage of analysis, we focused on the interaction between the teacher's and students' actions in relation to the elements of the STD. Finally, the two cases were contrasting to identify key differences on how IBMT and IBML were enacted in their classes.

RESULTS

Teachers' perspectives

In the 3rd PD meeting the teacher educator introduced the *Counting People* task. Vangelis (V) realized at once that statistical estimation was a central mathematical notion in this task: *"You can approach it through statistics [...] estimating the number of a group of animals for example ... you take a sample and consider how many they are in areas of high or low density respectively."* Vangelis had a strong background in statistics something that helped him to connect the task directly with central statistical methods and concepts such as statistical estimation and sample representativeness. Eirini acknowledged statistical estimation as a central learning objective but she related it with the idea of approximation *"It is reasonable to have great deviation in the results since everything is a matter of approximation"* (5th PD meeting).

Both teachers realized the task's potential to promote inquiry. However, the two teachers seem to have different approaches of what inquiry-based teaching is about. Eirini characterizes her teaching as 'guided inquiry' and she adds *"I leave my students to negotiate up to a point, when I realise that they go beyond the problem, I intervene. I always hear them trying to understand their difficulties and then I try to guide them in finding the solution"* (Eirini's interview). Vangelis, on the other hand, characterizes his teaching as 'open inquiry' by arguing that he minimizes his interventions and allows students to develop their own strategies. In the 4rth reflective PD meeting, Vangelis explained why he chose to leave students work without strict guidance: *"I chose not to guide them because every student has his own pace, others work quicker and others slower"*.

Setting up the task

Eirini and Vangelis set up the task in different ways in relation the selected artefacts. Eirini gave students only printed materials, a photo of the demonstration (Fig. 2) and a printed Google map of the area under investigation (Fig. 3). Vangelis provided the students only the photo (Fig. 2) and he said to them *"You can search in the internet for the appropriate resources to handle the task"*. All groups in Vangelis' class tried to locate the area under consideration in the Google earth maps.

Teachers' and students' enactment

Below we compare teachers' and students' enactment in the two classrooms in relation to two main mathematical ideas related to statistical estimation: The scale selection and the density anticipation.

The scale selection was one of the main students' objectives in both classrooms implementations. Students in Vangelis' class developed various strategies to address the given problem (e.g., they developed a plan to find the scale in a map representation of the area by using the length of a car as a unit measurement or used the scale that appeared automatically in the bottom of the screen in a specific map representation in Google maps). This scale showed the length in meters of a particular length in the map. In a subsequent PD meeting, Vangelis mentioned *"I could not manage to hide this representation"* expressing this way his intention to make the activity more exploratory for his students. In Eirini's class all groups developed the same plan for the scale selection. Eirini observed very closely her students' group work. The following is a typical example of the discussion between Eirini and her students while they were trying to define a unit measurement and estimating the scale in the printed map shown in Fig. 2.

1. E: What mathematical notion is relevant to this activity?
2. St1: Find the area of the road […] the scale.
3. E: How do we find the scale?
4. St1: I need to have a real object.
5. E: Can you identify a real object in the map representation [Fig. 2]?
6. St2: Let's see … The length of a car or a bus.
7. E: Better a car, not a bus.
8. St2: We have to know the dimensions of a real car and the dimensions of it in the photo ... But which car? … It could be a Volvo or a Smart; it could be 3 meters long or 4 meters long.
9. E: Discuss it with your group and decide on that.

As we can see in the above extract, Eirini guides her students to the desired mathematical object (line 1) and she provides hints to facilitate the process (lines 3, 5, 7). At the same time, we see the emergence of some informal indications of statistical inference in St2' attempts to identify an appropriate object for modelling the situation (line 8). The expressions of uncertainty highlight St2's encounter with the early steps of this statistical notion.

The density anticipation of demonstrators per one square meter is another central issue that it came up in students' discussions in both classrooms. Vangelis interfered very little in students' discussions while his students exploited many statistical ideas even though in an informal way. We present the following extract as a typical one showing students' collaboration in Vangelis' classroom and how Vangelis handled this discussion.

10. St3: If we take the half [area of the three streets] with 5 persons per square and the other half with 6 persons per square so we will be more close to…
11. V: Why?
12. St3: Because some people may be fatter and other thinner… (laughs!)
13. St2: Why don't we estimate how many people can fit in a square meter [he means to actually define a square meter and see how many fit]?
14. St5: Why don't you try first with 5 and then with 6 so we consider something in between.
15. St4: We don't care so much for who [could be the representing sample]. We can say that all persons are like St3. We just want an approximation.

    [The group stops for a little to talk and observes another group of students who simulated the problem by forming 1 square meter with a measuring tape on the floor, standing inside to find out the number of people that fit in this area. This group estimated 6 persons per 1 square meter. Then Vangelis asked]
16. V: Every square meter in the photo can have 6 persons?
17. St3: No, there are some empty spaces [spaces with no demonstrators in Fig. 2].

As we can see in the above extract, the students negotiated a lot about their choice for the number of people in a square meter (lines 12-15). After Vangelis inquiry question (line 16) students located spaces of low density. Statistical ideas that came up in students' discussions where the features of the persons (fat/thin) implying the representativeness of a sample or suggestions to take elsewhere 5 and elsewhere 6, implying a mean value of a high and a low density.

The extract below comes from one of Eirini's group of students who were asked to describe what they did while estimating the number of demonstrators in Fig. 2: *"E: How did you estimate the population? St: We first found the area in the streets where there were demonstrators. Then we agreed that two persons fit well in a square meter. [...] This is our sense, without [doing] measurements."* Eirini asked her students to report on how they calculated the density anticipation without involving them in a systematic exploration of the problem as in the case of Vangelis class. As we can see in the above extract the students in Eirini's class made their estimation based on intuitive approximations.

Tetrahedron	Vangelis' implementation	Eirini's implementation
Institution	Statistical estimation is not included in the official curriculum; authentic realistic tasks are not usually part of the Greek curriculum and textbooks.	
Teachers and mathematics	Strong background in statistics; related statistical estimation with selection of the best sample representativeness; appreciation of the multiple approaches in a statistical investigation.	Related statistical estimation with the idea of approximations; appreciation the multiple approaches in a statistical investigation.
Teachers and noosphere	Encouragement of the students to follow their own paths; limited intervention.	Guiding instruction on the basis of students' responses.
Teachers and artefacts	On line resources (photo, maps).	Printed resources (photo, map).
Teachers and conventions	Students explore and share their ideas; students have their own learning pace; emphasis is given on the mathematical processes.	Students express their ideas; emphasis is given on the mathematical concepts and properties.
Teachers, mathematics and society	Encouraging students to link their strategies with methods used by professionals.	Encouraging students to link the task question to other professions.
Students and mathematics	Extended use of probabilistic language; development of early steps of stochastic thinking through systematic experimentation; enactment of simulations.	Limited use of probabilistic language; informal indications of statistical inference in an intuitive way; use of data-request processes in a deterministic context.
Students and artefacts	Selecting artefacts beyond those proposed by the teacher.	Using the artefacts proposed by the teacher.
Students and conventions	A mathematical problem has one correct answer; the teacher verifies the correctness of a solution.	

Table 1: The analysis of the two implementations through SDT.

In the end of the task implementation both teachers asked the groups to present their results. Eirini addressed the question: "*Why do we have different answers?*" and then she referred to the particularity of situations with estimations. Vangelis did the same and even though students insisted on asking him *"What is the correct answer?"* Vangelis refused to answer which value was close to the actual situation. Later on, Eirini asked her students to discuss which professions may have to deal with the problem of estimating a population. Vangelis encouraged his students to inquire further the estimations given by news sites and suggested them to explore the methods used by the journalists and comparing to their own.

Table 1 summarizes the results from the analysis of the two different implementations by using the SDT.

CONCLUSION

In this paper we studied how IBMT and IBML were enacted in two classrooms. We see similarities between the two implementations such as the use of an open-ended task and the existence of norms allowing students to generate their solutions. The above characteristics are crucial as reported by Chan (2006). However, our analysis based on SDT brought to the fore differences in the two classrooms concerning the teachers' perspectives and actions and the students' mathematical activity. In particular, the different meanings teachers attributed to IBMT (open or guided inquiry) or to statistical estimation (e.g., connected to sample representativeness or the idea of approximation) and the selection of different artefacts (printed or on-line) promoted diverse possibilities for students' mathematical exploration. For example, Vangelis' students selected artefacts beyond those proposed by him and developed early steps of stochastic way of thinking while Eirini's students developed indications of statistical inferences but in a fragmental way and they were working mostly in a more deterministic context. Additionally, in both classrooms a number of concerns about the nature of mathematical solutions and the role of extra-mathematical sources of rationality are raised (Artigue & Blomhøj, 2013).

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