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Editors: Ewa Bergqvist, Magnus Österholm,
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CROSSING THE BOUNDARIES BETWEEN SCHOOL MATHEMATICS AND WORKPLACE THROUGH AUTHENTIC TASKS

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This paper reports a case study research aiming to connect the teaching and learning of mathematics in upper secondary education with the workplace. Four 10th grade students engaged in authentic tasks from the merchant navy context concerning the navigation of a ship through the use of original tools (e.g., the nautical map). We use the notions of activity system and boundary crossing to study students' construction of meanings for geometrical concepts. Results indicate that the students took a new look at the school taught geometry, by adopting the workplace perspective (perspective taking) and addressing authentic workplace problems through the lens of school mathematics (perspective making).

INTRODUCTION

Students' preparation for their professional life constitutes one of the aims of mathematics education (FitzSimons, 2014). However, the relationship between mathematics and workplace is complex and has been studied in many research studies over the last 30 years. Almost all these studies converge to the conclusion that mathematics used by professionals has some unique characteristics emerging from the cultural nature of the workplace which puts an indelible mark on the mathematical ideas developed in it (Millroy, 1992). These unique elements make workplace mathematics different from those in typical education enabling them to be identified as a distinct genre within the workplace discourse practice (Williams & Wake, 2007). Besides, several studies in the workplace reveal that apprentices or novices face a lot of difficulties in understanding the mathematical practices involved in authentic work situations. These difficulties indicate the limitations of school and academic knowledge as well as the complexity of linking this knowledge and workplace *(skill gap)* (Fitzsimons, 2014). Even though, studies have revealed how professionals engaged in mathematics when handling breakdown situations (e.g., Pozzi, Noss and Hoyles, 1998) we do not know if these situations could provide a context for students' mathematics meaning making at schools. While the teaching exploitation of the workplace in vocational education has received considerable attention over the past years (Bakker et al., 2014), only recently has the connection between general education and workplace been an area of research focus (e.g., Psyharis & Potari, 2017). The reported study aims to contribute in this direction by connecting the merchant navy context (ship navigation) and the teaching and learning of mathematics in upper secondary education. It was carried out through

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the close collaboration between a mathematics teacher (who acted also as a researcher) and a ship captain.

THEORETICAL FRAMEWORK

By perceiving the sociocultural nature of learning process, we have adopted the third generation of Activity Theory, as the theoretical framework for our study, considering two interacting activity systems (Fig. 1) as unit of analysis (Engeström et al., 1995). In the present research, we study the interaction between teaching and learning mathematics in school (students, teacher) and merchant navy workplace (captain, ship navigation). We acknowledge Activity Theory an appropriate theoretical background as it treats the learning process ingrained in a system of *object*-oriented, *tool*-mediated and *rule*-defined actions without neglecting its individual and communal dimension. The latter elements are also highlighted by the *subject*; the *community* and the *division of labor* which include the structural elements of the activity systems (see Fig. 1). The categories of *objects, goals, tools/artifacts* and *rules* are shaping mathematics within workplace activity into a recognizable kind of mathematic practice, in direct connection with the workplace context. Among two distinct activity systems emerge sociocultural differences as subjects engage in new practices unfamiliar to them. From this perspective, difficulties in using the typical school knowledge in realistic workplace situations can be considered as an example of discontinuity (Bakker & Akkerman, 2014).

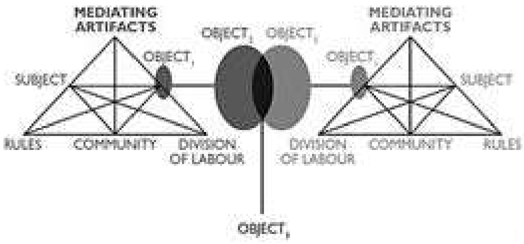

Fig. 1: Interacting activity systems.

The aforementioned discontinuities are defined by Bakker and Akkerman (2014) as *boundaries*. These authors use the term *boundary crossing* to refer to the interactions between the subjects in order to establish or restore communication among the activity systems. Within the bidirectional boundary crossing the tools/artifacts utilized by the subjects to bridge the two activity systems are defined by Star και Griesemer (1989) as *boundary objects*, i.e. objects that both inhabit several intersecting words and satisfy the informational requirements of each of them. Bakker and Akkerman (2014) consider boundary crossing as a cognitive process that can be described through the possible activation of four learning mechanisms: *identification* of the intersecting practices; *coordination* of practices by developing tools/objects to establish effective communication between them; *reflection* while subjects become aware of their own perspectives by redefining them in relation to the perspectives of others (perspective making) as well as by taking a new look to their own perspectives through the eyes of others (perspective taking); and *transformation* leading to changes in the existing practices, even the emerging of a new hybrid practice. We recognize boundary crossing perspective to be a fruitful alternative to understanding knowledge transfer between contexts, as we consider reasonable the assumption that students' engagement into workplace situations can highlight the discontinuities between typical mathematical knowledge and its use in work (Wake & Williams, 2001).

In the present study, we consider two activity systems: teaching and learning of mathematics in upper secondary schools (geometry) and merchant navy (ship navigation). We focus on the interaction of the two systems and the students' learning as they engage in authentic workplace activities. Boundary crossing offers us the tools to describe learning as meaning generation in terms of the students' bidirectional moves from one system to the other. Our research questions are: "How do students construct meanings for mathematical concepts when working on authentic ship navigation tasks?" "What is role of the workplace context - including authentic tools, rules and practices - in this process?"

METHODOLOGY

The research constitutes the pilot study of the first author's PhD aiming to explore the potential of authentic workplace situations as a context for mathematics learning in upper secondary schools [1]. It took the form of a case study with four 10th grade students and took place in a secondary school in Athens where the first author (who is also a teacher) works. A professional captain was invited to participate in all phases of the study. The practitioner contributed to the familiarization of teacher and students with the workplace, being bearer of nautical knowledge. He judged the correctness and compatibility of the activities with the workplace (designing phase) and also the solutions proposed by students in terms of the professional practice (implementation phase). The teacher acted as an agent of mathematical knowledge bringing back to the forefront the typical mathematics, supporting students' inquiry of mathematical concepts, encouraging them to express their ideas and strategies, and asking for refinement and revision when appropriate.

The implementation part of the research was divided into three phases. The students have been engaged in authentic activities from the workplace of the Merchant Navy (ship captain). In the first introductory phase, after the captain presented the main features of the workplace (e.g., nautical map, professional's tools); the students were asked to design the ship's course on the map so as to get familiar with the workplace context. In the second phase, the professional familiarized the students with the measures and tools he uses to find the position of the ship on the map. In particular, he showed them how to take *bearing* (a straight line of sight connecting the ship and visual prominent landmark on the shore), *range* (distance from the ship to an object represented as a point through radar) and *horizontal angle* (angle which has vertex the ship and sides two straight lines linking the ship with two landmarks) with the use of ruler, divider, protractor and parallel rulers (i.e. two rulers moving in parallel lines). Thus, the aforementioned measures correspond to the geometrical notions of straight line, circle, knowing radius, and inscribed angle, respectively. Then, the students were engaged in solving a series of realistic problems (e.g., *Avoid Obstacle*, *Safe Passage*). In the third phase, the captain gave to the students, six measures (two bearings, two ranges and two horizontal angles) and asked them to find the ship's position on the nautical map using as many as possible ways. In the results section, we present how the students propose a solution to the task *Safe Passage* concerning the navigation of a boat through a hazardous area.

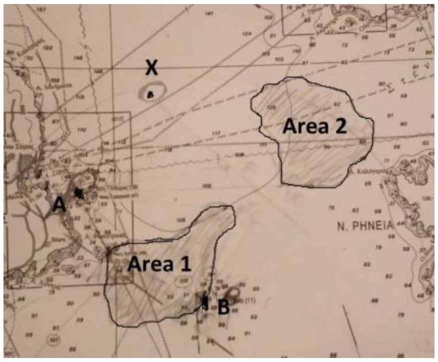

Fig. 2: The problem.

Safe Passage, that constitutes a typical duty/process in the navy context, was given to the students as an open geometrical problem. Starting from a point X the ship must pass through a dangerous (hatched) area with underwater obstacles (that are not visible). On the map the captain had marked landmarks A and B as reference points (Fig. 2). The students were asked, to use landmarks A and B and the aforementioned measures (bearing, range and horizontal angle) and tools in order to find a way to keep track of the ship's course to ensure its safe passage. This is a kind of problem not typical of those that students encounter in school. However, the students could make sense of it in the context of school mathematics.

The collected data consisted of: transcriptions of videos recording the main phase of task implementation; teachers' personal notes; teachers' resources and materials (lesson plan, worksheets, ppts, digital files); outcomes of the students' activities on the nautical map. The analysis was carried out in two levels. After carefully watching the videos and examining the corresponding transcripts, critical episodes were selected and coded under a grounded theory approach (Charmaz, 2006). The episodes were related to the construction of meanings that arose as students tried to "decode" workplace practices, understand and use the captain's tools, or respond to the activities modelling the realistic problems faced by the professional during the course of the ship. In the second level, the analysis focused on the boundary crossing of students between the two activity systems (teaching and learning of school mathematics and merchant navy) in relation to the meaning generation identified at the first level. In this paper, the levels of analysis described above concern the notion of inscribed angles that appeared to be central during the students' activity in the implementation of *Safe Passage*. The analysed episode falls into the learning mechanism of *reflection*.

RESULTS

Initial solution

The task was given to the students by the professional. He marked points X, A and B on the map and asked the students to find a method to keep track of the ship's course through the hazardous area. Students started solving the problem by drawing the segment AB. After that, they drew a line from point X parallel to segment AB as a safe route of the ship (Fig. 3). One student proposed to measure the distance between the lines drawn from X and the segment AB so as to keep track of ship's course. This was a correct solution from a mathematical point of view.

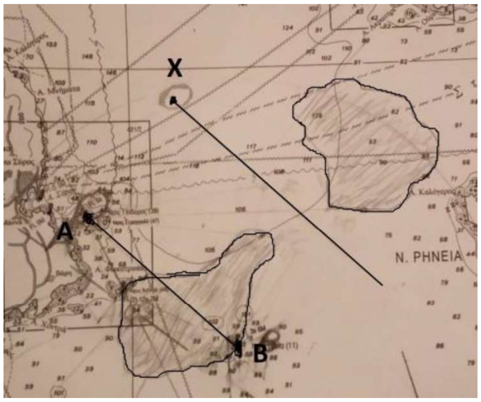

Fig. 3: Initial solution.

However, since AB in reality is an imaginary straight line (without visible landmarks), the students realized that it was not possible in the workplace context to measure the distance between the ship and AB with the use of the available measures/tools (i.e. from the ship). Student S2 communicated plainly his concern on this issue by wondering in which way he could be sure that the ship follows a safe course. Student S1 recognised that it was not possible to use straight lines (bearings) and distances (ranges) to keep track of the ship's course as there were no visible landmarks in the hazardous area. He proposed, for first time, to use inscribed (horizontal) angles.

1	S3:	Let's move like this. [He draws a straight line from X parallel to the segment AB]
2	S2:	How shall I know if I am far or near the hazardous area?
3	S3:	You are right. I have no landmarks to take measures. [Dangerous area is not visible]
4	S1:	[to S2] We must use horizontal angles… Bearings or ranges won't work.

The students' realization that they could not use their school taught geometrical knowledge to address the problem indicates a discontinuity between school mathematics and workplace. The constraints posed by the professional's measures/tools and the workplace rules (ship's safety) led the students to abandon their initial (mathematically accepted) solution discovering that it was ineffective in the new context. The fact that the students used terms from the professional's language during their exploration (lines 3 and 4) indicates that they had become familiar with the workplace tools.

Find a safe point

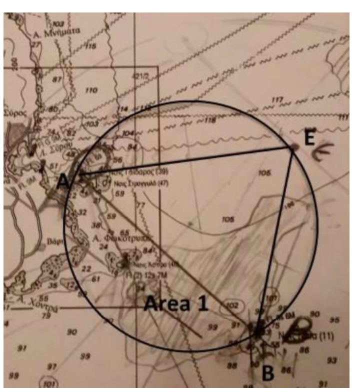

Fig. 4: Inscribed angle.

Later on, the teacher's intervention ("*How can I determine the dangerous waters?*") was crucial for the students to overcome their difficulties in finding a strategy to exploit the concept of inscribed angle they had recognised as relevant. He gave them the hint to mark a safe point on the chart and the students marked the point E (Fig. 4). That was not only a safe point between the two dangerous areas, but also a point on the line they drew in their initial solution. They still used as reference points A and B, though this time student S3 suggested to draw a circle passing through the points A, E and B and to measure the corresponding inscribed/horizontal angle (Fig. 4). The part of the dangerous area near the shore (area 1, see Fig. 4) was somehow contained in the designed circle. In that way students had come up with the idea to keep track of the ship's course through the use of an inscribed/horizontal angle. However, at this phase the students were experimenting with distinct safe points (e.g., E) without having formed a comprehensive strategy.

5	T:	Ok, why don't you mark on the chart a safe point for the ship?
6	S1:	[He marks point E] Wait, the circle should pass through all three points. [He refers to points A, E, B]
7	T:	So, how can I determine the dangerous waters? [Students had designed the circle and the inscribed angle AEB]
8	S4:	As the area surrounded by the arc of a circle. [He refers to arc AEB]

Through the design of the circle passing through points A, B and E the students achieved to orient accurately the dangerous area. Taking the role of a professional helped students to cross the boundaries between the two activity systems, using authentic tools (horizontal angle) as boundary objects, satisfying workplace rules (safety) and trying to fulfil a workplace demand (safe passage). Their choices were influenced jointly by the perspectives of both the school (accuracy in defining the dangerous area through a circle) and the workplace (tools, constraints, norms). Teacher's intervention helped them to connect the value of an inscribed/horizontal angle with the cyclical sector that defines the dangerous area.

Final solution

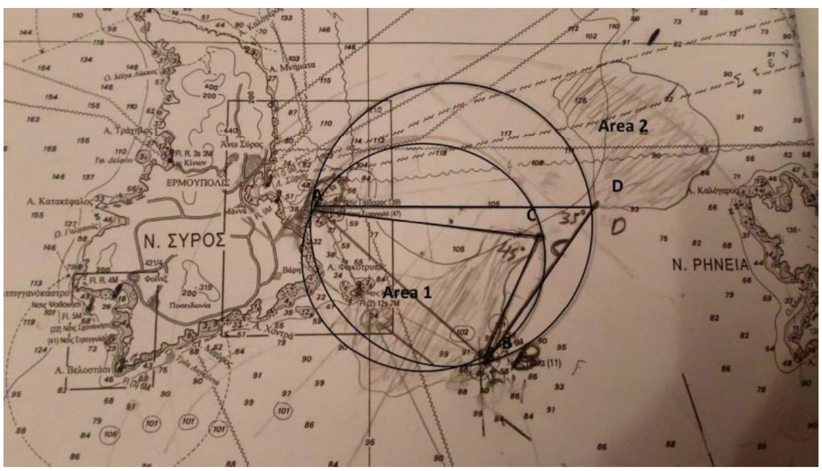

Fig. 5: Final solution.

In the final phase of their exploration, the students were engaged in finding a measure indicating that the ship sails in a safe area. For this they had the idea to design the point C (the ending point of the dangerous area near the shore, area 1, Fig. 5), the circle passing through A, C, and B and the inscribed/horizontal angle ACB. Their strategy was to use the value of the inscribed/horizontal angle (45°) to define the dangerous area oriented by the designed circle. With the intention to provoke students' mathematical reasoning, the teacher asked if the value of the inscribed/horizontal angle had to be bigger or smaller than the angle ACB to ensure the ship's safe passage. Student S1 suggested using the point D (the beginning point of the dangerous area away from the shore, area 2, Fig. 5) as they used the point C before. Thus, the students designed a new circle passing through the points A, D and B and the inscribed/horizontal angle ADB so as to have a visual representation of the 'safe' area for the ship's course. Measuring the inscribed/ horizontal angles (from the points C and D) and observing the difference in their values (45° and 35° respectively), the students accepted that as the radius of the circles (passing through A and B) increases the corresponding inscribed angle decreases (Fig. 5). In this way, they developed a strategy to check whether the ship is following a safe course or not based on the value of the inscribed/horizontal angle having as vertex the position of the ship and sides defined by the lines connecting the vertex with the points A and B respectively.

9	T:	Fine, the horizontal angle is 45°. To be safe, you need a wider or a narrower angle?
10	S3:	Wider. [Wrong answer]
11	S4:	No, for bigger radius, the angle becomes narrower. [After students make the second circle and measure the angle at D]
12	S1:	The second angle is 35°. It is narrower.
13	S2:	We must keep track of the horizontal angle.
14	S2:	The limits are from 35° to 45° for a safe passage.

Taking a global view of the above incidents, the students achieved to find a solution to the need to keep track of the ship's course based on the measure of inscribed/horizontal angles. In the end, the captain recognized that the students' final solution was identical to the one used in the workplace. To achieve this, the students reinvented the notion of the inscribed angle and constructed meaning for the alteration of it, a geometrical relationship that it was not taught at school. Thus, not only they used their existing knowledge in the new context but also they developed meanings for new geometrical concepts. Acting like professionals and through the teacher's and the captain's help, the students took a new look at the school taught geometry, by adopting the workplace perspective *(perspective taking)* and addressing authentic workplace problems through the lens of school mathematics *(perspective making)*.

CONCLUSIONS

In the current study we adopted the perspective that the integration of authentic workplace situations into mathematical teaching can enrich students' mathematical knowledge. Our analysis focused on how the merchant navy context motivated students to cross the boundaries between school mathematics and professional space. The realistic context (original workplace problem), the practitioner's tools (measures) and the workplace constraints (rules) acting as boundary objects revealed the insufficiency of school taught knowledge, highlighting a discontinuity between formal mathematics and the genre of mathematics developed in the workplace. Throughout their exploration for solving an authentic task the students were influenced by the school perspective (parallel lines), they moved to an intermediate model which jointed the school and the workplace perspective (horizontal/inscribed angle) and they gave the final solution taking the workplace perspective (official professional's practise). Through a reflection process they reinvented geometrical notions (inscribed angle), while they gave meaning to new geometrical relationships (alteration of inscribed angles). The professional acted as an agent of the workplace knowledge by bringing to the forefront the workplace context. The analysis indicates also the teacher's critical role in provoking students' mathematical reasoning and helping them overcome difficulties through inquiry-based questions and crucial interventions.

Notes

[1]. The study is inspired by the European project Mascil (http://www.Mascil-project.eu) that aims to promote the integration of inquiry-based learning and workplace in the teaching and learning of mathematics and science.

References

Bakker, A., & Akkerman, S. F. (2014). A boundary-crossing approach to support students' integration of statistical and work-related knowledge. *Educational Studies in Mathematics*, *86*(2), 223–237.

Bakker, A., Groenveld, D., Wijers, M., Akkerman, S. F., & Gravemeijer, K. P. E. (2014). Proportional reasoning in the laboratory: an intervention study in vocational education. *Educational Studies in Mathematics*, *86*(2), 211–221.

Charmaz, K. (2006). *Constructing grounded theory*. London ; Thousand Oaks, Calif: Sage Publications.

Engeström, Y., Engeström, R., & Kärkkäinen, M. (1995). Polycontextuality and boundary crossing in expert cognition: Learning and problem solving in complex work activities. *Learning and Instruction*, *5*(4), 319–336.

FitzSimons, G. E., (2014). Commentary on vocational mathematics education: where mathematics education confronts the realities of people's work. *Educational Studies in Mathematics*, *86*(2), 291–305.

Millroy, Wendy L. (1992). *An ethnographic study of the mathematical ideas of a group of carpenters*. Reston, Va: National Council of Teachers of Mathematics.

Pozzi, S., Noss, R., & Hoyles, C. (1998). Tools in practice, mathematics in use. *Educational Studies in Mathematics*, *36*(2), 105–122.

Psycharis, G. & Potari, D. (2017). Mathematics teachers' learning at the boundaries of teaching and workplace. In G. A. Stillman, W. Blum & G. Kaiser (Eds.) *Mathematical Modelling and Application: Crossing and Researching Boundaries in Mathematics Education* (pp. 301-311). ICTMA Series Springer.

Wake, G. D., & Williams, J. S. (2001). Using college mathematics in understanding workplace practice. *Summative Report of Research Project Funded by the Leverhulme Trust, Manchester University, Manchester.*

Williams, J. S., & Wake, G. D. (2007). Metaphors and models in translation between college and workplace mathematics. *Educational Studies in Mathematics*, *64*(3), 345– 371.