Chapter 10
Prospective Mathematics Teacher Argumentation While Interpreting Classroom Incidents

Despina Potari and Giorgos Psycharis

Abstract This paper aims to analyze the structure and quality of prospective mathematics teachers' (PMTs)' argumentation when identifying and interpreting critical incidents from their initial field experiences. We use Toulmin's model and recent elaborations of it to analyze the discussions that took place at the university where PMTs reflected on their recent classroom experiences. Our aim is to identify the structure of the argumentation and characterize the emerging warrants, backings, and rebuttals. Results indicate different argumentation structures and types of warrants, backings, and rebuttals in the process of PMTs' interpretations of students' mathematical activity. We discuss these findings from the perspective of noticing to identify shifts at the level of PMTs' interpretations.

Keywords Teacher argumentation · Argumentation structures · Warrants
Noticing · Critical incidents

10.1 Introduction

Current approaches in research in mathematics teacher education acknowledge the importance of noticing as a construct to study what and how prospective mathematics teachers (PMTs) attend to when observing, analyzing, and interpreting teaching (Scherer & Steinbring, 2006). Noticing has been considered as a complex action that involves teachers in identifying what is significant in a classroom interaction, interpreting this noteworthy incident on the basis of their knowledge and experiences, and linking these with broader principles of teaching and learning (van Es & Sherin, 2010). Moreover, at the level of teacher education and in

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collaborative contexts, interpreting teaching phenomena is a joint action that involves the development of claims, conjectures, and arguments. Studying teachers' argumentation is a means of understanding the resources upon which teachers base their interpretations. As regards PMTs, a challenge is to gain insight into the nature and structure of argumentation in relation to their multiple experiences from school, teacher education courses, and field experiences. Steele (2005) points out that pedagogical argumentation has a diverse and fragile knowledge base and thus, "It is difficult to access and agree upon fundamental elements of a pedagogical argument" (p. 296). He also considers teaching as an interpretive act that is highly contextualized and dependent on teachers' experiences in the educational culture. Therefore, we consider pedagogical argumentation as a lens to have access to PMTs' interpretive acts of teaching and to the sources upon which they base their interpretations.

In this paper, we focus on PMTs' argumentation in the process of selecting and interpreting critical classroom incidents as part of their fieldwork activities. The study took place in the context of a 14-week undergraduate mathematics education course with the philosophy of linking theory-driven instruction on the teaching and learning of secondary school math with actual mathematics teaching in classroom settings. In a recent paper based on this context (Potari & Psycharis, Submitted), we used critical incidents as a structure to facilitate PMTs' reflection and study their conceptualizations of mathematics teaching and learning. The analysis showed PMTs' shifts from observing teaching to questioning aspects of it and conceiving it in a relational way. It also brought to the fore a richness of arguments developed by PMTs as they supported their claims or challenged their peers' interpretations. In this paper, we use Toulmin's model of argumentation and recent elaborations of it to analyze the quality of PMTs' argumentation and its development while identifying and interpreting critical incidents. The research questions are:

- What is the structure of PMTs' argumentation while interpreting classroom incidents?
- What are the sources upon which PMTs base their interpretations of critical incidents?
- How do the PMTs' interpretations evolve in the context of the course?

10.2 Theoretical Framework

10.2.1 Teacher Noticing and Critical Incidents

Noticing has been introduced to mathematics teacher education to study shifts in the structure of teachers' attention and, through this, address different levels of awareness in mathematics and in mathematics teaching (Mason, 2002). In resonance with a number of current research approaches (c.f., Jansen & Spitzer, 2009;

Scherer & Steinbring, 2006), noticing is an activity involving description, analysis, and interpretation of teaching practice. According to van Es and Sherin (2002), noticing is a more complicated action than just observing teaching. Rather, it requires teachers to identify significant teaching and learning incidents, to interpret them on the basis of their knowledge and experiences, and to link these with broader principles of teaching and learning. Further, van Es and Sherin also highlighted the importance of interpreting classroom interactions as a way of informing teachers' pedagogical decisions. Therefore, promoting teachers' noticing includes, "to first notice what is significant in a classroom interaction, then interpret that event, and then use those interpretations to inform pedagogical decisions" (p. 575).

Researchers have been concerned about the introduction of sufficient structures for making the act of teacher noticing more concrete. Critical incidents are an example of a structured framework for reflection on classroom episodes. A critical incident can be considered as an everyday classroom event that has significance for the teachers, makes them question their practice, and seems to provide an entry for their better understanding of teaching-learning situations (Hole & McEntee, 1999). Critical incidents have been used in mathematics education for analytical and developmental purposes (Goodell, 2006; Skott, 2001). For example, Skott (2001) used the term "critical incidents of practice" to describe moments of a teacher's decision-making in which multiple and possibly conflicting motives of their activity evolved that challenged the teacher's own school mathematics images and provided learning opportunities for students. Goodell (2006) used critical incidents to promote PMTs' noticing, as well as to address her own development as a mathematics teacher educator. The issues raised by PMTs in her study included: teaching and classroom management; student factors; relationships with colleagues, parents and students; and school policies and procedures. She also identified that PMTs fruitfully addressed important aspects of mathematics teaching that supported students' understanding. Similar to Goodell's study, our research has a developmental character as it aims to address the quality of PMTs' argumentation and its development while identifying and interpreting critical incidents. While Goodell investigates PMTs' learning through the analysis of their written reports of critical incidents, we focus on PMTs' discussions in the context of a teacher education course while sharing their reflections with their peers and the teacher educator. In this paper, our focus is on PMTs' interpretation of their selected classroom incidents and in particular on their argumentation when analyzing and interpreting them.

10.2.2 Teacher Argumentation and Toulmin's Model

In the mathematics education field, teacher argumentation has been studied in the context of the classroom (Knipping & Reid, 2015), in teacher education programs (Metaxas, Potari, & Zachariades, 2016), as well as in teachers' responses in hypothetical scenarios (Nardi, Biza, & Zachariades, 2012). Toulmin's theory has

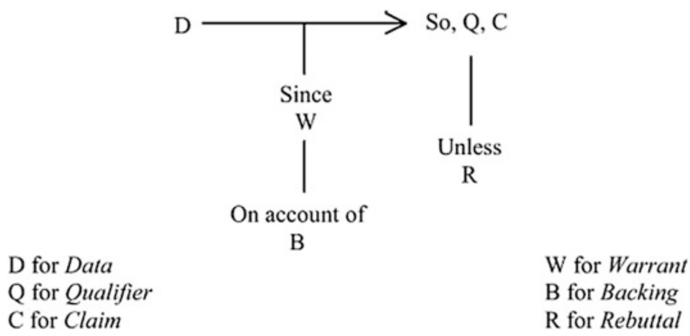

Fig. 10.1 A representation of Toulmin's model

been the basis of most studies in the analysis of teacher argumentation and in particular its model for the layout of arguments (Toulmin, 1958). His model (see Fig. 10.1) consists of six basic elements: the claim (C) is the position or claim being argued for; the data (D) are the foundation or supporting evidence on which the argument is based; the warrant (W) is a general rule of inference that authorizes the step from the data to the claim; the backing (B) supports the legitimacy of the warrant; the modal qualifier (Q) represents the degree of force or strength that the data confer on a claim in virtue of the warrant; and the rebuttal (R) consists of exceptions to the applicability of the warrant.

Toulmin's model has often been combined with other frameworks so as to address not only the structure, but also the quality of argumentation. For example, Metaxas et al. (2016) used argumentation schemes to analyze the internal coherence of mathematics teachers' arguments in the context of a master's course, while Nardi et al. (2012) adopted Freeman's framework to identify the different types of warrants mathematics teachers use to support their claims when they interpret hypothetical classroom scenarios. Nardi et al. (2012) address the quality of teachers' argumentation by placing their arguments in relation to teacher considerations and priorities—pedagogical, curricular, professional, and personal. In particular, they distinguish seven types of warrants in teachers' arguments: a priori-epistemological and a priori-pedagogical (based on mathematical or pedagogical principles); empirical-professional and empirical-personal (based on their teaching or learning experiences); institutional-epistemological and institutional-curricular (based on practices for the mathematics community or on curriculum and textbook recommendations/requirements); and evaluative (based on a personally held view/value/belief). This categorization helped Nardi et al. (2012) to analyze teachers' warrants and to identify the sources on which they based their arguments. In our study, we adopted the same categorization to analyze PMTs' warrants, backings, and rebuttals as we considered all indicators of the sources of PMTs' argumentation. Since our study refers to PMTs, we considered their professional experiences stemming from their fieldwork practices and other personal teaching experiences they had.

Moreover, Knipping and Reid (2015) distinguished local from global arguments to study proving processes in the mathematics classroom. Local arguments represent a step of an argument that can be analyzed by Toulmin's model. Global arguments lay out the structure of interconnected local arguments indicating the structure of an argumentation process as a whole. Knipping and Reid also identified different types of global argumentation structures (e.g., source-structure, spiral-structure) as constructs to address differences in the argumentative process. To explain these differences, they considered the nature of local arguments that make up global structures. While both source and spiral structures have several similar characteristic features (e.g., parallel arguments, argumentation steps with more than one datum, refutations), they differ in the way these features appear in the global structure. Spiral structures involve parallel arguments leading directly to the final conclusion, whereas in the source structure the parallel arguments lead to different data that provide a basis for another argumentation step supporting the conclusion. Furthermore, in the spiral structure more refutations are used to oppose the main claim. These structures emerged from the study of proving processes in upper secondary mathematical classrooms. We adopt this framework to study PMTs' argumentation in the teacher education context.

Taking into account the theoretical constructs discussed above, in our study, we use Toulmin's model, the classification of the warrants proposed by Nardi et al. (2012) and the structures developed by Knipping and Reid (2015) to analyze PMTs' interpretations of critical incidents they identified when reflecting on lessons observed and/or taught. Toulmin's model provides a structure to analyze PMTs' local arguments and relates to our first research question. Knipping and Reid's elaborations allow us to compare different argumentative processes to address potential shifts in PMTs' interpretations of classroom phenomena. This helps us to answer our first and third research questions. Nardi et al.'s approach helps us to characterize the sources of warrants, backings, and rebuttals and address our second and third research questions. The combined use of these approaches offers us a tool to address the quality of PMTs' argumentation.

10.3 Methodology

10.3.1 Context of the Study and Participants

Prior to enrolling in the course which provided the context of the present study, PMTs had a background of undertaking at least four other mathematics education courses as a part of their teacher education program at the university. In parallel to their university studies, most PMTs were helping school students on a private basis with their math homework. The course aim was to engage PMTs in critical consideration of aspects of mathematics teaching as these emerged from the complexity of teaching practice in schools. Every second week (for the entire semester) PMTs were asked to participate in a number of field activities in secondary schools (over six field activity-weeks) while each week following the activities in schools included a 3-h meeting at the university. We note that in Greece students enter secondary education (Grades 7–12) at the age of 13 after six years of primary education. PMTs' field activities consisted of observing other teachers' mathematics teaching for 6 h in total (first three field activity-weeks) and designing and teaching lessons for the whole class for three teaching hours (fourth, fifth, and sixth field activity-weeks).

During their fieldwork in schools, the 22 PMTs (9 males, 13 females) who served as participants in this study, were asked to: (a) identify the specific content of a lesson in the curriculum and to trace it throughout the different grades; (b) look for possible research evidence related to potential students' difficulties; (c) make a lesson plan describing the main tasks and their rationale; (d) keep systematic notes from and/or record the lessons; (e) select critical incidents and provide a reflective account on the basis of justifying their selection, interpreting them, and proposing potential teaching actions. The PMTs were divided into pairs and carried out collaboratively the field activities under the supervision of eight mentors (postgraduate students of mathematics education). The mentors accompanied the PMTs to schools and supported them in their fieldwork activities by providing feedback on the PMTs' designs and discussing with them events from the lessons. Before the beginning of the course, the group of mentors met twice with the teacher educators to discuss the course philosophy and the PMTs' responsibilities. The mentors had access to the course materials and participated in the university meetings.

Instructional practice in the university sessions aimed to support PMTs' reflection on their recent field experiences and link emergent issues with existing mathematics education research in order to develop deeper levels of awareness. Typical activities in which the PMTs were engaged in the university meetings were: (a) to present critical events they had identified in their observations and in the analysis of their own teaching; (b) to discuss and question emerging issues; (c) to present their analyses of transcriptions of events with the aim of interpreting their criticality and linking them to their research readings; and finally, (d) to propose alternative teaching actions.

The teacher educator (first author) facilitated the discussion, but also challenged the PMTs to justify their selection of critical incidents, provide evidence of their claims, make interpretations, and describe their potential teaching decisions. The teacher educator also enriched the discussion by offering research—informed input. The second author participated in the university meetings as a participant observer and provided input during the discussions with the aim of promoting PMTs' reflections. The researchers took into account ethical issues related to PMTs' consent to participate in the research study. To avoid biases related to the dual role of teacher educator (as teacher and researcher), the second author had the main role in the data collection and discussed with her conflicting issues emerging in data analysis.

10.3.2 Research Design and Data Collection

Noticing critical incidents from mathematics classrooms and questioning aspects of mathematics teaching was a rather new practice for PMTs. It was supported through the discussions in the university meetings and the field activities. We considered critical incidents as a methodological tool for triggering PMTs' reflection on teaching practice. In the first two university meetings, the teacher educator introduced PMTs to the idea of critical incidents through (a) a brief presentation of Goodell's (2006) study (including the meaning of critical incidents, the classification of them, and examples from PMTs' written reports) and (b) analysis of lesson transcripts to identify critical incidents and discuss/justify in the class their criticality. In all subsequent meetings, the pairs of PMTs were asked to select and present in the next session a critical incident that they had experienced during their fieldwork activities.

We collected the data for this study over the entire semester, which consisted of: (a) PMTs' personal portfolios, including their written accounts of critical incidents and material related to the design, implementation, and presentation of the field activities in the classroom (e.g., worksheets, lesson plans, presentation files); (b) video recordings of all meetings at the university (eight in total); and (c) researchers' field notes. We base the present paper on the analysis of the transcripts of the university meetings.

10.3.3 Data Analysis

Under a grounded theory perspective and open coding (Strauss & Corbin, 1998), we identified four themes of critical incidents discussed in the meetings (i.e. students' activity, epistemological issues, lesson planning and classroom management, and wider contextual and social factors). Within each theme, we conducted a fine-grained analysis of the data in terms of the three dimensions of van Es and Sherin's (2002) description of teachers' noticing (i.e. what they observed, how they interpreted it, and how this informed their pedagogical decisions). Next, we used our combined theoretical approach described above to analyze the PMTs' argumentations developed when interpreting critical incidents related to each theme. In particular, for each theme we identified the claims that the PMTs made while reflecting on classroom observations and their own teaching, the data on which they based their claims, the warrants and backings they used to support them, and the rebuttals they provided when the validity of the conclusions was under question. Then, we focused on the interrelationships among the arguments related to the specific theme throughout the university meetings. Our purpose in this part of the analysis was to describe argumentation structures and to trace their progressive development in order to identify shifts in PMTs' interpretations. Finally, we analyzed the warrants, backings, and rebuttals to identify the sources on which PMTs based their arguments following Nardi et al.'s (2012) classification. In this paper, our focus is on PMTs' argumentation concerning the theme "students' activity" and in particular the construction of mathematical meaning that appeared to be dominant in what PMTs' noticed.

10.4 Results

From the initial sessions, PMTs' noticing of students' activity focused primarily on students' difficulties. However, our grounded analysis of the discussions in the meetings revealed that towards the end of the course, PMTs' selection and interpretation of critical incidents regarding students' mathematical learning was progressively enriched. In particular, the PMTs' interpretations were justified by relating students' activity to a multiplicity of factors acting as warrants for their claims. For example, PMTs interrelated teachers' actions, the nature of tasks, the classroom communication, and the use of language with students' mathematical learning (e.g., classroom interaction, norms). They also offered backings based on research in mathematics education, as this was targeted in the course. For example, PMTs appeared to: identify different forms of mathematical thinking and understanding (e.g., formal versus informal, procedural versus conceptual, understanding versus memorization); value students' mathematical ideas; consider epistemological aspects underlying students' learning; and appreciate the role of affective issues in the process of learning mathematics.

To illustrate the above findings, we analyzed transcripts of discussions of nearly the same length related to the theme of students' activity in two university meetings. The first meeting took place at the beginning of the course, after PMTs' initial experiences with classroom observations, and the second towards the end of it, after the completion of PMTs' own teaching. Our focus is on the structure of PMTs' argumentation and the resources upon which they based their arguments to highlight and trace the quality of their interpretations throughout the course.

10.4.1 Argumentation in the Third University Meeting

The structure of argumentation

The teacher educator encouraged PMTs to report on critical incidents they had identified during their first classroom observation. A main issue discussed in this meeting was the construction of mathematical meaning. Initially, the focus was on students' mistakes emerging primarily from their difficulties with connecting algorithmic procedures to the underlying concepts and properties. For example, PMT's discussed different arithmetic and algebraic mistakes in the meeting. The teacher educator challenged the PMTs to interpret why these mistakes appeared and what they revealed about the students' conceptual understanding. At that point, the PMTs identified a number of factors they considered relevant for explaining students' difficulties, such as the language and the use of symbols, the classroom norms, and the students' lack of motivation.

The main claim (C1) developed in the discussion was that students face difficulties in moving beyond a surface understanding to a deeper conceptualization of the underlying concepts and properties. PMTs used a number of different data sources upon which they based their claim. These sources came from their classroom observations and concerned students' difficulty in transforming a fraction to an equivalent one, using the algebraic properties to solve a first-degree equation, or simplifying an arithmetic or algebraic expression.

For example, the data (D1) a PMT reported was about a classroom interaction between two students concerning the transformation of the fraction $\frac{7}{5}$ to its equivalent with 30 as the denominator. The first student completed the transformation by multiplying both terms of the fraction by 6. Then, the second student wondered why their classmate had not used a technique taught traditionally in the Greek primary schools. According to this technique, the quotient of the division of the least common multiple of the denominators by the denominator of each fraction is placed over each numerator and then it is multiplied by both nominator and denominator for each fraction. The students usually follow the procedure without understanding why they do this. The prospective teacher interpreted the phenomenon by considering this technique as a "picture" in the student's mind that might provide a barrier to conceptual understanding: "The second student seems to have clear in their mind a picture without knowing why this method works, the essence of the method" (Kostas) (W1).

Another PMT (Petros) brought data (D2) from his fieldwork observation regarding how the teacher managed a similar situation in the context of algebra. Instead of stressing the rule "change side, change sign" commonly used in solving algebraic equations, the schoolteacher emphasized the properties involved in the solution process. Petros found this approach fascinating as it was beyond his own experiences.

A third PMT, Orestis, brought the data (D3) related to students' difficulties with simplifying arithmetic or algebraic expressions and referred to his observation in an 8th grade classroom:

> While solving an equation of two fractional expressions with numerical denominators, the students transformed them into equivalent fractions but they did not change the nominators. Then, they equated the two nominators without understanding their mistake. (Orestis)

After encouragement from the teacher educator ("What does it mean for you that the students use techniques without understanding? How do you explain this?"), the PMTs started to provide justifications for their arguments. Orestis interpreted students' difficulty with conceptualizing mathematical ideas with the warrant that in school textbooks mathematics loses its meaning:

> We use terms or expressions that have nothing to do with mathematics. For instance, the Rule of Three,[1] central in school textbooks at primary level, is a technique, rather than a mathematical method. (W2)

Another PMT, Leonidas, offered a new warrant by referring to the different meaning of symbols in arithmetic and algebraic expressions in the school textbooks. He mentioned that:

> In some cases, 3½ is a mixed number, while $3x$ where x is $\frac{1}{2}$ is a product. These two different meanings of similar representations come one after the other in the textbook. (W3)

In a subsequent phase, PMTs' attention moved to what students bring into the lesson and how this influences teaching. The new claim (C2) they were formulating concerned the important role of students' contributions to the lesson. The argumentation was enriched by other data coming from classroom observations.

For example, the PMT Irene described a critical incident as it emerged in a 9th grade classroom during a geometry lesson on congruency of triangles concerning the fact that some students brought pieces of knowledge that had not yet been taught in the classroom:

> One student mentioned the term 'adjacent angles' that had not been taught. The classroom teacher responded by saying, "We have not said anything yet about adjacent angles here," and she continued the lesson. I initially thought that the student might have read it in the textbook. But the word 'adjacent' is rather difficult for students to remember even at the upper secondary school level. Finally, I think that this knowledge came from private lessons. Sometimes this knowledge does not empower the students as I had expected from giving private lessons myself. In contrast, it constrains the classroom interaction as I see it now from the classroom teacher's perspective.

Other PMTs brought similar examples from their classroom observations. For instance, Marina mentioned a case where the teacher introduced the concept of angles in the 7th grade, but the students referred to its measure that they had encountered in primary school: "When the teacher asked, 'What is an angle?,' one student said, 'degrees.' The teacher commented that, 'We have not discussed that yet.'"

In the above two extracts, Irene and Marina bring new data (D4 and D5) to support the claim. Irene offers also as a warrant that the students have private lessons (W4) while the discussion follows Marina's observation that, "The pupils have already met the same concepts in primary school" (W5). In this way, PMTs started to identify factors related to curriculum and to wider cultural context that interfere with teachers' attempts to promote conceptual understanding. In their attempts to support the warrant W4, PMTs offered the following backings: "They [school students] take private tuition because parents do not have the knowledge or the time to help their children with their homework" (B1); "The requirements of

[1]The Rule of Three is a mechanical method for solving proportions. Briefly, it says that if we know three numbers a, b, and c, and want to find d such that a/b = c/d then d = cb/a. Algebraically, one can multiply the equation (proportion) by bd, giving ad = bc and then divide by a.

school mathematics are increasing so students need help in order to be successful" (B2); "The national examinations are rather demanding" (B3); and "The students need more individualized teaching" (B4).

In the realm of this discussion, Orestis expressed a rebuttal by questioning the tendency to support students to become good at mathematics through continuous guidance:

> It is not necessary for every child to be successful in mathematics. So, close guidance does not allow students to take decisions for their future according to their interests. (R1)

Anta referred to her own experience with her parents who had always helped her at home "although they had worked all day" (R2).

Towards the end of the meeting, the PMTs brought new data from their classroom observation about students' algebraic mistakes coming back to the initial claim (C1):

> In the 7th grade classroom that I observed, the teacher assigned the task to explore if $2^2 \cdot 3 \cdot 5^2$ is a multiple of 90 expressed in the form $2 \cdot 3^2 \cdot 5$. The students wanted to make the calculations first and then check if $2^2 \cdot 3 \cdot 5^2$ is a multiple of 90. When the teacher asked them if $547 \cdot 90$ is a multiple of 90, they did not respond. They wanted to do the calculations again. (Thenia) (D6)

> The teacher gave the task of simplifying the expression $2 + 4(2x + 1)$ and one student wrote $4 \cdot 3x$. Although the teacher reminded the students about the distributive law, some of them provided a wrong answer. (Thenia, D7)

Here, the PMTs started to identify elements of students' mathematical thinking by offering as a warrant that, "The students conceive the distributive rule visually as a picture in their minds and use it without understanding its meaning" (Anta, W6). They also started to provide justifications with reference to the classroom norms established by the teacher. For instance, Irene interpreted D7 by arguing that:

> The student might be embarrassed to ask again although she had not understood. The teacher's authority could possibly be an obstacle. Thus, the student pretended that she had understood. This is what we used to do as students. (W7)

Orestis offered a new rebuttal (R3) to Irene's warrant by interpreting the incident through taking into account students' motives, "The students might prefer to be out of the classroom and playing, but they are obliged to respect the rules and pretend that they understand."

Focusing on the interrelationships of arguments, we recognized a number of argumentation steps based on different data sources (D1, D2, D3, D6 and D7) that appeared at the beginning and at the end of the discussion. These steps indicated the existence of parallel arguments supporting the claim C1 concerning students' difficulty in developing mathematical meaning in algebra. For the claim C2, new data sources appeared (D4, D5) that provided the basis of new argumentation steps. Warrants and backings (W1–W7, B1–B4) were used to support both claims, while emerging rebuttals (R1, R2, and R3) challenged claims and warrants. We could possibly argue that the argumentation structure that emerged in PMTs' initial attempt to address students' construction of mathematical meaning followed

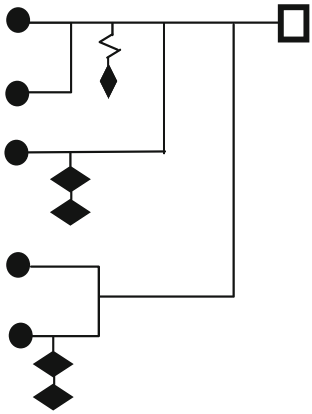

Fig. 10.2 Argumentation structure of C1 (3rd university meeting)

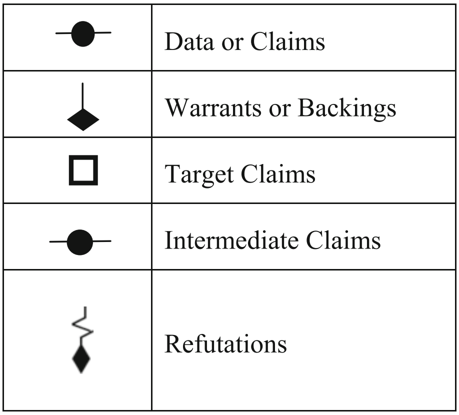

Fig. 10.3 Symbols used in the argumentation structure diagrams

characteristic features of what Knipping and Reid (2015) named as source-structure, where the emphasis is more on collecting information (data and conclusions) than on the connections between the different argumentation steps (see Fig. 10.2 the argumentation structure of the claim C1, and Fig. 10.3 for an explanation of the symbols used in the diagrams).

The sources of PMTs' interpretations

In this part of the discussion, the PMTs grounded their warrants, backings, and rebuttals mainly on their broader views about teaching and learning (evaluative), as well as on recommendations of the curriculum and the textbooks (institutional-curricular). PMTs' personal experiences as learners (empirical-personal), held pedagogical principles (a priori-pedagogical), and practices of the mathematics community (institutional-epistemological) emerged as sources in PMTs argumentation. In Table 10.1, we summarize our classification according to the framework of Nardi et al. (2012).

Table 10.1 Classification of warrants, backings, and rebuttals in the third meeting

A priori-pedagogical	B4
Institutional-epistemological	W2
Institutional-curricular	W3, W4, W5
Empirical-personal	W7, R2
Evaluative	W1, B1, B2, B3, R1, W6

10.4.2 Argumentation in the Eighth University Meeting

The structure of argumentation

At this meeting, the PMTs presented critical incidents selected from their own teaching and provided transcripts from the classroom interaction related to the incidents. This process had started in the sixth meeting. Here, the focus of the discussion was on a critical incident reported by two PMTs, Anna, and Marina. The incident that constituted the data (D1) for the subsequent argumentative process concerned the difficulty that a student had with linking the algebraic identity $(a+b)^2 = a^2 + 2ab + b^2$ and its geometrical representation. The students had been given a model representing the design of a square shaped house with side $(a + b)$ divided into rooms with areas a^2, ab, ab, and b^2. Both Anna and Marina referred to a case of a student that although he was encouraged by the teacher to work with the geometrical model and then to recognize the algebraic identity, he only recalled the algebraic formula that he had already known. The main claim (C1) in the discussion was the students' difficulty with connecting different representations of mathematical knowledge, such as algebraic and geometrical.

Different PMTs expressed their interpretations about this incident. Marina reflected on her own experience as a school student to interpret the student's reaction:

> Actually, the student offered a safe answer! I also used to do the same as a student at school. When the teacher asked me something that I did not know, I provided a formula I could relate to the question. This is what the student did here.

In her comment, Marina refuted the initial claim (R1) and provided as a warrant that, "the student offered a safe answer" (W1). She further supported her warrant by implicitly referring to existing norms in the classroom where a student feels obliged to give an answer to any question (B1). She brought data from her own experience as a learner (D2).

Sofia offered as a warrant the students' difficulty with applying their prior formal knowledge to an open task (W2) and brought new data from similar cases that she had met in her classroom teaching to back it up, "We met similar incidents many times in our teaching (D3). Often students' prior knowledge was an obstacle to engaging them in an activity" (B2). Then, Irene and Anta refuted the initial claim by arguing that the student might have been able to make the connections very fast (R2, R3). Anna provided further evidence from her interaction with the student and offered data from another incident, "In a similar task of connecting the identity of

$(a+b)^3$ with the volume of a solid, the student made the same mistake" (D4). Similarly, Marina brought more data from the student's attempts to link the areas of the decomposed rectangles and squares to the algebraic identity (D5).

In the following excerpt, Leonidas supported the main claim by referring to the student's use of language in the excerpt provided by Anna and Marina, "The student used the word 'solution' to refer to the algebraic identity" (D6). He offered as a warrant that the student, "cannot see the equivalence of the two parts of the identity, but he considers it as a procedure that needs to be followed" (W3). In this phase of the discussion, the PMTs provided multiple warrants. These warrants referred to classroom norms (Efi and Leonidas offered as a warrant that the student provided an algebraic answer because the lesson was on algebra and not on geometry, W4 and W5), the curriculum (four PMTs offered as a warrant that the curriculum did not connect algebra and geometry, W6–W9), and students' attitudes (preference to algebra over geometry, W10). Irene brought another warrant, which recognized that linking algebra and geometry was not a simple task even for PMTs: "This connection is too difficult for the students" (W11), "It is difficult even for us to see how a geometrical situation can be expressed by algebraic symbols and operations" (B3). Here, the warrant was backed by PMTs' similar difficulties as learners at the university. In this case, Irene's experience at the university operated as a new source of data (D7). Later in the discussion, after the challenge from the teacher educator ("Can you interpret this incident based on what you have learned in mathematics education courses?"), PMTs enriched their interpretation of this specific incident by bringing data from research and theory of mathematics education (D8). Irene offered as a warrant that, "The students are used to applying the mathematical content to exercises" (W12). She supported it further by offering a backing that included the qualifier "I think" (Q1) and it was influenced by her research readings in the course. She said, "I think that this has to do with the didactic contract and the social norms of the classroom" (B4). Alexandros refuted W12, saying that, "The children are more creative than adults" (R4) and offered a warrant for this: "For the kids to use models to form algebraic relations is like playing a game, so they are successful" (W13). He referred to his experience at the university (D9) and used this as data to back W13: "We [as university students] tend to follow complicated solutions. Our minds are not used to seeing the simple solution" (B5). Irene and Anta referred to the critical role of representations in the construction of mathematical meaning in algebra (W14, W15), while Kostas referred to geometry and the dominant role of prototypical figures to further support Irene and Anta's argument (B6). Kostas also offered another warrant for students' difficulty with conceptualizing the meaning of the equal sign in algebra (W16).

A second critical incident that Anna and Marina brought concerned once again a student's difficulty with conceptualizing the identity $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$ in a similar geometrical context involving the decomposition of the volume of a cube with side $(a + b)$ into other solids. The main problem that the PMTs addressed was that the student could not relate the concept of volume with the space occupied by the solid. The PMTs offered new data from their teaching (D10) and the formulated claim was that conceptual understanding is a complex process (C2). The discussion initially centered on the teacher's role and classroom management issues. Then the students' activity was once more the main focus. Anna and Marina brought data from the transcript of their lessons about two unexpected incidents. In the first one, Dina, a student who had appeared not to participate in the interaction between Dimitris (a competent student) and the teacher (D11), provided the correct answer. Anna offered as a warrant with a qualifier, "probably" (Q2), that "Dina was thinking differently from Dimitris. Probably, she was not constrained by the existing formal knowledge, thus she had an open mind without reproducing mechanically taught methods" (W17). Irene further supported this warrant: "Her thinking does not follow a specific channel. Students like this can approach mathematical concepts in a more global way and see the meaning behind them" (B7). Irene also added an affective dimension to explain Dina's response, "As she was not interacting with the teacher at that time, she was not as anxious to provide a correct answer as Dimitris was, so she felt free to express her thinking" (W18).

In the second incident, the students easily recognized the algebraic identity (square difference) by transforming area manipulatives provided by the PMTs (D12). Concerning this incident, the PMTs offered warrants to support their expectation that the task would be difficult for the students and indicated the validity of the claim C2. Therefore, Anna argued that, "Even my friends could not visualize the identity" (W19), while Marina admitted that, "I had no idea how to rearrange the tiles to form a rectangle" (W20). Backings to these warrants were based on PMTs learning experiences at school and at university. As Anta put it, "Our thinking is highly constrained by the formal knowledge at school and at university so that we cannot think in a simpler way" (B8). Irene offered another backing by arguing that, "The children engage easily in playing with bricks, puzzles, and constructions and use their imagination. But we are far away from this" (B9).

In terms of the argumentation structure, PMTs made two main claims (C1, C2), as well as a number of warrants and backings in parallel argumentation steps that led to the support of the main claims based on different sources of data (D1–D12). In this process, we also observed the presence of refutations (R1, R2, R3, R4) in the argumentation structure, claims that were supported by a multiplicity of warrants (W1–W20) and backings (B1–B9), as well as the use of qualifiers (Q1, Q2) in PMTs' attempts to consider the claims from different viewpoints. This structure has similar features as the spiral argumentation structure of Knipping and Reid (2015). This is because it involves parallel arguments that could stand alone leading to the final claims, warrants, and backings that adequately justify the claims, and refutations of the main claims (See the argumentation structure of C1 in Fig. 10.4).

The sources of PMTs' interpretations

In this part of the discussion, the PMTs grounded their warrants, backings, and rebuttals mainly on the theory and research on mathematics education (a priori-pedagogical) their personal experiences as learners at school and at the university (empirical-personal), and their broader views about teaching and learning (evaluative). They also based their interpretations on their current teaching

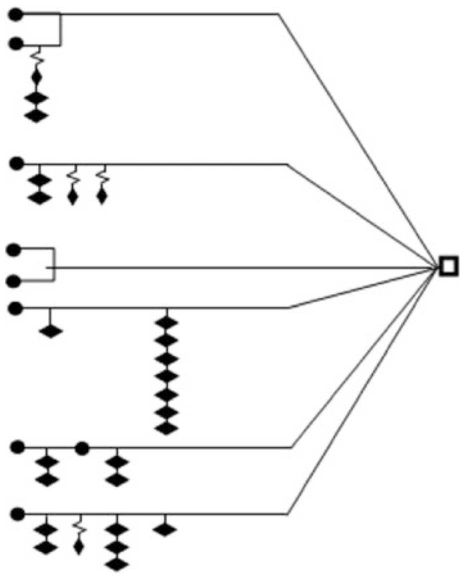

Fig. 10.4 Argumentation structure of C1 (8th university meeting)

Table 10.2 Classification of warrants, backings, and rebuttals in the eighth meeting

A priori-pedagogical	W3, W11, B4, W13, W14, W15, W16, B6
Institutional-curricular	W6, W7, W8, W9
Empirical-professional	W2, B2, W4, W5
Empirical-personal	W1, B1, B3, B5, W19, W20, B8, B9, R1
Evaluative	R2, R3, W10, W12, R4, W17, B7, W18

experiences (empirical-professional), as well as on curriculum and textbook recommendations (institutional-curricular). In Table 10.2, we summarize our classification according to the framework of Nardi et al. (2012).

Warrants, backings, and rebuttals are based on a multiplicity of sources. The co-existence of a priori-pedagogical, empirical-personal, and evaluative types indicated that PMTs had started to interpret students' activity in different ways. For example, they combined research findings and theories they had encountered at the university with their personal experiences as learners and with their broad views about teaching and learning. These connections indicated a reflective stance towards classroom phenomena and a development of awareness of the complex interrelationships that underlie these phenomena.

10.4.3 Comparing Argumentation in the Two Meetings

The analysis of the discussions in the two university meetings showed shifts in the process of interpreting students' activity through different argumentation structures and types of warrants, backings, and rebuttals. While observing other teachers' teaching PMTs' argumentation was based on different data they brought from their observations with a small number of warrants, backings, and rebuttals. The emphasis then was on collecting data and conclusions without making connections between them and warrants or backings. In contrast, the PMTs' argumentation based on their actual teaching was enriched by a large number of warrants, backings, and rebuttals, as well as by qualifiers. In this case, the emphasis was on the connections between data and conclusions based on justified arguments that could support the final claims in different independent ways.

Through the analysis of the types of warrants, backings, and rebuttals, we identified a balanced distribution of them in the categories of Nardi et al. (2012). Evaluative arguments, evident in both university meetings, indicated the role of personal views and beliefs about learning and teaching mathematics in the process of interpreting classroom incidents and noticing in general. However, the presence of a priori pedagogical arguments in the second case revealed the research-based character of arguments that strongly related to the aims of the course and the PMTs' experiences in their university studies. At the same time, the reflective stance promoted in the course seemed to play a unifying role between the experiences that PMTs brought from research, personal learning histories and views, and current teaching practices.

10.5 Conclusions

For interpreting students' activity, the PMTs used different sources of data based on their prior school experiences, current university studies, and fieldwork. In particular, they made links between students' conceptualizations and their own experiences as learners at school and university and they looked for evidence in their classroom observations and teaching. The analysis of the discussions about students' activity in the two university meetings showed different argumentation structures in terms of the use of warrants, backings, and rebuttals and their interrelations. The structure that emerged from the analysis of PMTs' reflections on their classroom observations (third university meeting) involved parallel arguments, warrants, and backings without rich connections between them. Conversely, towards the end of the course while PMTs reflected on their own teaching (eighth university meeting) their argumentation was based on argumentation steps consisting of a large number of warrants, backings, and rebuttals that targeted the final claim. Using Knipping and Reid's (2015) framework in a teacher education context, we identified similar argumentation structures in the pedagogical discourse.

However, in both structures that we identified, the argumentation steps seemed to lead directly to the main claim. Thus, in our case the difference between the two structures related to the number of warrants, backings, and rebuttals that were richer in the second case. This indicated that the PMTs had developed deeper and justified interpretations.

Using Nardi et al.'s (2012) approach in analyzing PMTs' warrants, backings, and rebuttals, we identified a multiplicity of sources on which PMTs grounded their interpretations. Towards the end of the course, it was more evident that PMTs had developed justifications balancing sources from theory and research in mathematics education, from personal views about learning and teaching mathematics, and from their experiences as learners and teachers.

The differences between PMTs' interpretations of incidents selected when observing other teachers' teaching and their own teaching can be explained from different points of view. First, existing research (e.g., Stockero, 2008) shows that PMTs' experiences in analyzing other teachers' lessons can enhance deeper levels of reflection on their own teaching. Another explanation could be that towards the end of the course, PMTs started to have examples from their own teaching as a basis for reflection. This experience seemed to have facilitated their progress in realizing interrelationships between teaching and learning. Also, the use of critical incidents as a teacher education strategy in the course seemed to have supported PMTs' in reconstructing prior experiences about teaching and learning mathematics in the light of new experiences in the teacher education context.

Our study offers an analytical framework (argumentation structures and classification of warrants and backings) that can contribute to the field of research in teachers' noticing. First, by analyzing the argumentation structures, we relate noticing to PMTs' justification of their claims. According to Mason (2002), this is an indication of their awareness of mathematics teaching that constitutes an important aspect of noticing. Second, our approach allows researchers to address the sources upon which PMTs base their interpretations. Although existing research has emphasized the important role that PMT's prior learning experience, beliefs and orientations play in noticing (Ding & Dominguez, 2016), our study provides a lens to analyze how PMTs' experiences (personal learning, fieldwork, university courses) influence the process of noticing. Finally, our analysis of the types of warrants and backings in PMTs' argumentation makes it possible to address the developmental trajectory of noticing. According to van Es (2011), teachers' transition to higher levels of "how they notice" is characterized by their ability to make connections between events and principles of teaching and learning and to propose alternative pedagogical solutions based on their interpretations. Our findings show that PMTs reached higher levels of noticing indicated by their making of connections between events and research on teaching and learning. Therefore, the analysis of the warrants and backings allows us to identify the shifts in PMTs' interpretations. Further research is needed in order to address the potential of this approach to mathematics teachers' noticing.

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