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Teachers' designs with the use of digital tools as a means of redefining their relationship with the mathematics curriculum

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The present article reports a study concerning the analysis of 19 activity plans (we call them 'scenarios') developed by mathematics teacher educators-in-training for the pedagogical use of digital tools. The development of these scenarios took place during their training program and was designed as an activity for increasing reflection, for expressing creative pedagogical ideas and for an active engagement in the design of curricula enriched with the use of technology. Our analysis showed that the trainee teacher educators deconstructed and reconstructed respective parts of the formal curriculum regarding the mathematical concepts they chose to embody in their scenarios.

1. Theoretical framework

This article focuses on the relationship between teachers and curriculum, approached through the study of the teaching designs—in the form of scenarios—developed by mathematics teacher educators-in-training. The study took place in the context of an in-service educational program at the University of Athens. The program was part of a large-scale nationwide initiative of the Ministry of Education concerning teachers' familiarization with the use of digital technologies for teaching and learning in their respective subjects. As for mathematics, the aim of the program was to provide the participants who were selected experienced mathematics teachers with methods, knowledge and experience in in-service teacher education and to educate them in the pedagogical uses of expressive digital media and communication technologies for the teaching and learning of mathematics. The Educational Technology Lab (ETL) had the responsibility to develop curricula for this program and the completion of training four educator groups in the University of Athens.

The Greek educational system is characterized by a central national curriculum in which goals, concepts and activities are explicitly described. Despite existing efforts to implement an interdisciplinary curriculum, the teaching of extended material continues to be linear. The concepts are sometimes introduced with activities. However, there are few provided representations and, still, with limited connections inside and outside mathematics. Even though there are abundant concepts, the conceptual fields (Vergnaud, 1996) finally constructed are particularly fragmented and rigid. The teacher has the role of the technical implementer of this curriculum, as she/he strictly follows the textbook, along with the guidelines of the special book addressed to the teacher. The use of technology, in spite of the fact that in some cases is proposed, it is rarely put into practice and when that happens takes the form of a simple demonstration. The integration of technology in the teaching and learning process implies new kinds of learning activity, which focuses on the generation of meanings, problem solving, students' mathematizations, use of representations, creation of socio-cultural norms in the classroom (Kynigos *et al*., 2009). These aspects of the teaching and learning process, require the development of new curricula and change of the teacher's role, not only as a facilitator to this kind of learning, but as a decision maker with an active role in designing innovative curricula (Budin, 1991) and this was one of the program's purposes.

The teacher–curriculum relationship has been studied less than other fields in the domain of teacher education (Remillard, 2005). When curriculum is referred to, we have to distinguish between the intended curriculum (that which resides in state frameworks, guides, textbooks and in teachers' minds as they plan what they will do) and what it appears to be, which is curriculum as enacted by teachers in the classroom and curriculum as experienced by students (Gehrke *et al*., 1992). This distinction recognizes the significant role of the teacher as mediator on what is planned and what really takes place in the classroom. Very important factors that influence teachers' mediation are their knowledge, beliefs and experiences. We consider that this mediation phase is of great importance, as it is not the practice itself, but it is directly interlinked with practice. According to Carlgren (1999) teachers should not be considered as reflective practitioners (in the sense of Schön) only for actions internal to the classroom, but for external ones as well. As such, she proposes the development of local curricula by the teachers, through reflective processes incorporating knowledge, beliefs and classroom experiences that are transmitted to their colleagues as a kind of apprenticeship. The specific program's emphasis was on the teachers' involvement with design in the form of activity plans, we call 'scenarios', aiming to think up investigative activities involving the productive use of digital media by the students. These scenarios were written by the trainee teacher educators and were addressed to teachers. They were thus perceived by them as material which they would subsequently use in teacher education courses in which they would be employed to teach by the Ministry. The ETL in collaboration with research colleagues and teachers developed a structure for addressing those aspects of a learning environment based on using digital media which were important for the learning process. This structure includes the description of specific characteristics related to the teaching and learning process, such as the subject area and topic, the added value by the use of technology, theoretical framework, students' learning problems addressed in the specific topic, time and space parameters, resources, social orchestration, goals, phases of enactment (anticipated teaching and learning process: the kind of meanings the students are expected to construct through the use of the specific software tools, teaching management, roles), evaluation after implementation, possible extensions and references. In that sense, a scenario is a starting point for the teacher to reflect upon the teaching process as a whole. Simultaneously, the possibility of adapting scenarios, according to different teachers' epistemologies and classroom conditions, constitutes them malleable informal curriculum units that may strengthen teacher's knowledge, autonomy and reflection alongside the provision of ideas for using digital media for added value in learning

During the course, the trainee educators got familiarized with the pedagogical use of five kinds of digital media: Data Handling, Computer Algebra Systems (CAS), Dynamic Geometry Systems (DGS), Simulations and Programmable microworlds software. A central construct for scenario design with a medium from any of these kinds, was the design of a specific focused microworld

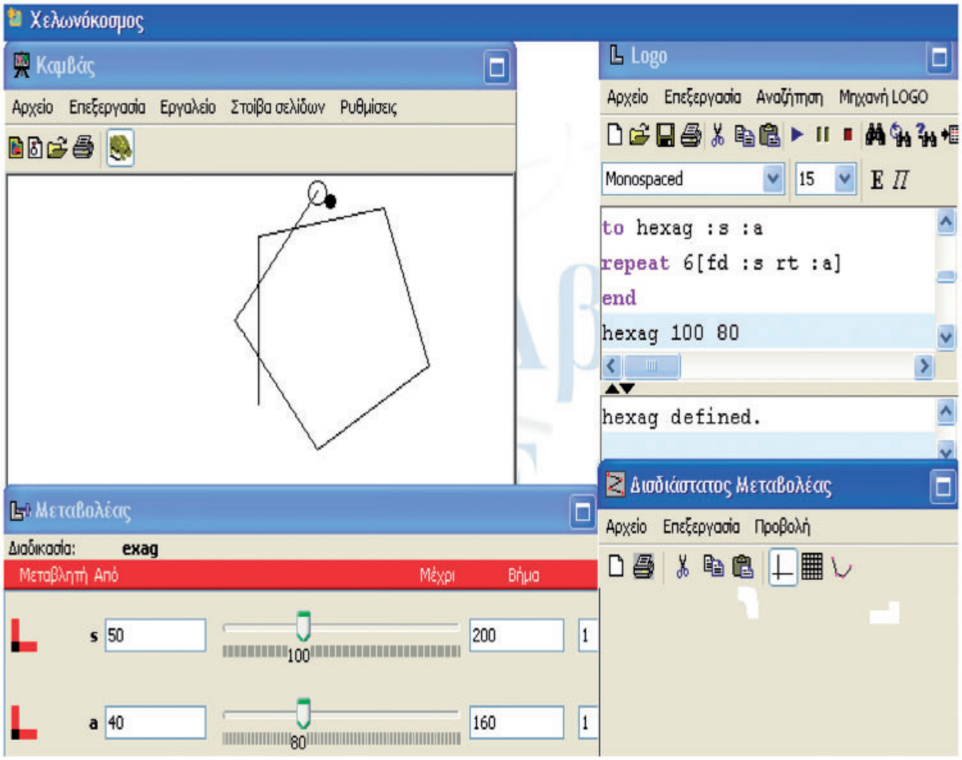

FIG. 1. The half-baked microworld 'hexagon'.

that we introduced and mediated to the teachers, as a 'half-baked microworld' (Fig. 1). Half-baked microworlds are models or constructs that are designed to be incomplete in a way which is visible to their intended users. They are given to these users who are then asked to study what's missing or buggy and try to fix it. The design is such that these activities bring the users to generate meanings for the embodied mathematics to the extent to which they can express these by putting them to use to fix the microworld. We employed the idea of half-baked microworlds at two levels, the teacher as designer level and the student level. At the first level our aim was for our trainees to build on them, change them or de-compose parts of them in order to carry out some mathematics for themselves. At the second level, to consider them as half-baked microworlds for their students and to make changes to fine-tune them to implement their own scenarios (Kynigos, 2007). After offering a small set of such microworlds we then challenged our trainees to design and develop their own. A very important decision regarding the design of a half-baked microworld is which mathematics should be embodied and which should be removed in order to give the students the opportunity to investigate and fix the proposed model. The decisions regarding design may vary according to different teachers' pedagogies and their classrooms' specific contexts. In this sense, scenarios and microworlds constituted *improvable artefacts* and objects-to-think with. Their design was meant to motivate communication amongst colleagues, and provoke community building. Thus, their development can be considered as a constructionist activity, integral to teachers' professional development (Kynigos, 2007). According to the requirements of this specific professional development program, the teachers had to design five scenarios, each involving a half-baked microworld for each kind of digital media, respectively.

2. The computational environment

The scenarios analysed here involve the use of 'Turtleworlds' a programmable Turtle Geometry medium designed to integrate formal mathematical notation with dynamic manipulation of variable values (Kynigos *et al.*, 1997). In Turtleworlds, the elements of a geometrical construction can be expressed in a Logo procedure. After a variable procedure is defined and executed with a specific value for each variable, clicking the mouse on the turtle trace activates the 'variation tool' that provides a slider for each variable (see at the bottom of Fig. 1). Dragging a slider has the effect of the figure dynamically changing DGS-style, as the value of the variable changes sequentially. The novel character of dynamic manipulation in Turtle Geometry is that what is manipulated is not the figure itself as in DGS, but the value of a variable of a procedure. The variation tool provides an alternative way to control changes that affect both the graphics and the symbolic expression through which it has been defined, combining these two kinds of representations that appear rather static in most other geometrical construction computational settings. Thus, employment of the variation tool provides opportunities for the students to take a reflective view on the geometrical construction process by confirming or dismissing conjectures derived from the use of the tool in conjunction with the other representations (e.g. by adding/erasing commands or variables, by expressing or modifying magnitudes with variables, etc.).

3. Method and research questions

The research presented in this article part of a wider one, concerning the trainees' choices in teaching design in different educational contexts. It is an intervention research, aiming to study the features of human artefacts produced in a specific educational context designed by us (Cobb *et al.*, 2003). Our aim was first to gain insight into the nature of the teaching design of technology enhanced mathematics learning developed by the trainees in the form of scenarios based on the use of specially designed computational tools. Secondly, we were interested in investigating if and how these tools shaped the mathematical content of the respective scenarios. In particular, we were interested in identifying: (a) The main characteristics of the teaching design (concepts involved, situations/contexts of tasks, connections within or outside mathematics, use of representations) and their relation to the characteristics of the formal curriculum, (b) The way the trainees used the available tools as teachers to orchestrate learning situations that promote conceptual understanding and generation of mathematical meanings. Our emphasis was on the ways in which they integrated the dynamic manipulation feature of Turtleworlds in their design of tasks and if and how this integration favoured the possibility of introducing new components in the formal curriculum. To analyse the scenarios we organized their content into tables. Every line of the table corresponded to a scenario and every column to the specific characteristics according to the scenario structure we described earlier. This process led to the categorization of the scenarios according to their topics. For each topic we traced the sequence of the chosen concepts through diagrams in order to clarify the connections between them. Afterwards, the teaching sequence was studied in each scenario and new tables related to the use of technology were created. Some of the parameters that we studied were the software tools that the teachers used for designing their microworlds and the potential contribution of these tools to the students' meaningful engagement with the underlying mathematical concepts. In particular, the analysis of the anticipated students' use of dynamic manipulation lead to the definition and refinement of new codes characterizing a number of qualitatively different dragging categories that are presented in the second part of the analysis.

4. Results

4.1 Domains, topics and mathematical concepts

Geometry almost monopolized teachers' interest, since 18/19 scenarios had Geometry as the starting point of their activities and only one had Trigonometry. Two kinds of connections were seeked: (a) Connections inside mathematics: the majority of scenarios (eight) were developed on a specific topic of Planar Geometry, with no further connections. Six of the scenarios connected Geometry with Trigonometry, three of them Geometry with Algebra and one Planar Geometry with Stereometry. (b) Connections outside mathematics: there were no connections with other disciplines. We also examined the different situations and contexts upon which the teachers based their scenarios: four scenarios involved Arts as a context, eight scenarios were developed under specific situations of every-day life (jewels, wheel of the amusement park, houses, kites, building plots, sailing-boats and stairs) and seven scenarios were developed into the mathematics world. The 19 scenarios were developed around six topics: regular polygons (6), triangles (6), quadrilaterals (4), slope of a straight line (1), similarity (1) and areas (1). A notable finding stemming from the examination of the 19 scenarios is that the trainees followed different paths for approaching the same topic not only regarding the targeted concepts but also the technological tools used. The same mathematical concept could be included in different topics through the exploitation of its different properties. For example, the equilateral triangle appeared: (a) in the topic of regular polygons, both as the polygon with the minimum number of sides and as the structural element for the construction of a regular hexagon; (b) in the topic of triangles as a special case; (c) in the topic of areas as a unit for covering plane shapes; and (d) in the domain of Algebra as a unit for Pascal's triangle.

4.2 Phases of anticipated learning activities and students' tool use

Two representative scenarios from the topic of regular polygons are analysed in the next paragraph. Our aim is to indicate the variety of the teaching designs for the same topic involving also distinct differences in the exploitation of the available tools. In the Greek formal curriculum, this topic is approached through a formal definition of regular polygons that is followed by the construction of a regular polygon with n sides (i.e. through the division of the cycle's periphery into n equal arcs) and the proof of the correctness of this construction. The description of the teaching sequence included in the scenarios S12 and S13 that we chose to present here highlights in each case the teaching process as *anticipated* by the designer according to his/her own choice of activities and tool use.

4.2.1 (S12) «Let's create regular polygons»

1st phase: (A) Process—activities: three half-baked microworlds are given to the students, providing open jagged lines of 3, 4, 6 equal sides, respectively (Fig. 1: jagged line of 6 sides). The students work in groups and try to modify the code so that the lines close and create an equilateral triangle, a square and a regular hexagon, respectively. (B) The role of the tools: Students are expected to correspond the sliders of the variation tool to the variables of the code, by dragging them and observing the graphical outcome. The students are expected to recognize that the slider determining the line's closure corresponds to the turtle's turn. After moving the side's slider on a certain value, the students are expected to experiment by dragging the turn's slider to different values until they discover that the regular polygon is created when the turtle's turn becomes 120°, 90°, 60°, respectively. The students are expected to relate the turn to the polygon's exterior angle and conjecture its relationship with the number of sides. Then they can modify the code by taking off the variable that corresponds to the turn and replace it by the functional relation 360/:n. These changes can be further tested by executing the code and observing the graphical outcome.

```
to line :n :b :c
repeat :n [fd :b rt :c]
end
```

2nd phase: (A) Process—activities: the students have to modify a given half-baked microworld, called 'line' which constructs an open jagged line of n equal sides with length :b. (B) The role of the tools: students correspond the sliders of the variation tool to the variables of the code and ascertain that the slider which does not influence the closure of the line is the one of length, which is moved on a certain value. By dragging the slider (:c) of the exterior angle, they find out that for every value of n the line closes (the regular polygon whose exterior angle equals :c). By dragging the slider (:n) of number of sides, they find out the value of the angle that closes the line (the exterior angle of a regular polygon of n sides). They conjecture that the exterior angle which creates a regular polygon of :n sides, equals 360/:n. They take off the variable :c from the code, replacing it by 360/:n. Their conjecture can be verified by executing the code and taking the graphical feedback.

3rd phase: the scenario is extended to the circle, and then, to the circumscribed circle of a given regular polygon (the description cannot be given here due to the space limits).

4.2.2 (S13) «The wheel of the amusement park»

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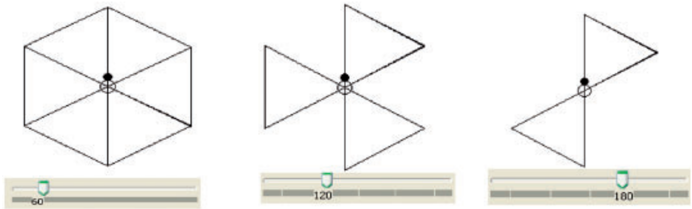
to scheme :a :b :c :d
fd :a rt :b fd :a rt :c fd :a rt :d
end
```

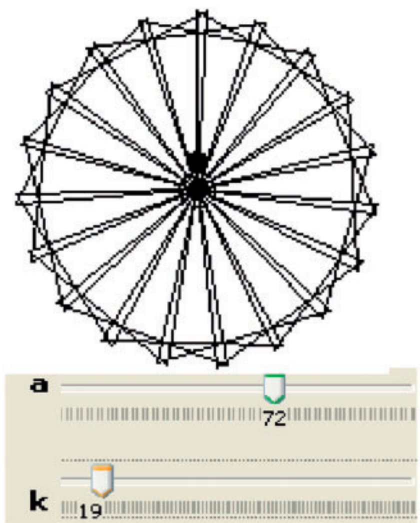
1st phase: (A) Process—activities: The students have to modify a given half-baked microworld, called 'scheme', which constructs an open jagged line of three equal sides, length's :a, in order to construct an equilateral triangle. (B) The role of the tools: the students correspond the sliders to the variables and ascertain that the one which does not influence the line's closure is the slider of length :a. They move it in a certain value. By dragging the sliders of angles :b and :c, they find out when the line closes. Then the students put forward the conjecture «each triangle with equal sides, has also equal angles». They are expected to recognize that the slider :d controls the turtle's orientation for the construction of the equilateral triangles in succession. They modify the code, by eliminating the variables to one and use the command repeat 3 [fd :a rt 120] for the construction of the equilateral triangle. The new code is called 'triangle'.

```
to hexagon :a :b
repeat 6 [triangle :a rt :b]
end
```

2nd phase: (A) Process—activities: a class discussion is scheduled to be conducted related to the way a regular hexagon derives from an equilateral triangle and the students are expected to create a microworld of six successive equilateral triangles with common vertex. (B) The role of the tools: in order for the students to create the code 'hexagon', they initially correspond sliders to variables. The slider of the side's length is moved to a constant value. Then, they drag the slider :b that 'turns' the equilateral triangle. If, for example, :b = 10°, then six equilateral triangles will be created (each one of them derives from its previous by being rotated 10°). Here, the concept of rotational symmetry is embodied. In the instances shown besides six equilateral triangles are positioned in the plane after their rotation of 60°, 120° (two by two coincide) and 180° (three by three coincide), respectively.



3rd phase: (A) Process—activities: for the construction of 'the wheel of the amusement park', the students have to create an equilateral triangle of length 'a' which has to be repeated for 'k' times after being rotated by 'b' degrees each time. The code 'wheel' is expected to be created by the students through modifications of the code 'hexagon' (i.e. insertion of a variable :k for the number of repetitions of the equilateral triangle). (B) The role of the tools: students are expected to experiment with the slider :k in conjunction with the turtle's turn :b and replace the turn by 360/:k. An instance of :a = 72 and :k = 19 is shown in the figure besides. In this phase, the students generalize their understandings and connect them to a specific situation of everyday life.



Through the analysis of the activities' phases it can be noted that:

(I) The concept of regular hexagon is approached in two different ways: (a) Differentially in S12 through the intrinsic turtle's movement (every position and turn of the turtle is defined according to its previous one) in a situation oriented within the mathematics world. In this case, the focus of the designer is on the polygonal line rather than the polygonal surface. This approach favours the extension of the scenario to the concept of circle. (b) Using a simulation of a real-world object, the designer of S13 attributes emphasis on the polygonal surface and this approach favours the connection of the scenario with the topic of Areas. In S13, the regular hexagon derives from transformations of an equilateral triangle and particularly through rotational symmetry of 6th order. Two other scenarios for the construction of regular hexagons used the concept of rotational symmetry in a different way than the S13. According to the formal

Greek mathematics curriculum only few teaching hours are devoted to the concepts of central and axial symmetry during the first grade of the secondary level. The concept of rotational symmetry is not taught at any grade of secondary education despite the fact that central symmetry is a specific case of rotational symmetry (i.e. rotation of 180°). We suggest that the use of the concept of rotational symmetry in the design of these particular scenarios was favoured by the potential of the available tools. Specifically, the use of the 'repeat' command in conjunction with the VT facilitates the generalization of the concept of rotational symmetry of different orders and seems to have challenged teachers to adopt an alternative didactic approach by-passing the formal curriculum.

(II) As far as the role of the tools is concerned, the vast majority of the activities revolved around the use of the variation tool. From the body of the 19 scenarios we distinguished four expected dynamic manipulation schemes (DMS) taking into account findings of previous researches focusing on the use of the variation tool (Psycharis & Kynigos, 2009). These schemes are present in the anticipated students' use of the variation tool in the Scenarios S12 and S13 mentioned before. Next, we describe briefly these schemes with particular references to the corresponding stages indicating the evolution of the anticipated learning process.

Reconnaissance DMS: the students are expected to begin with random manipulation of sliders and finally to correspond sliders to particular magnitudes of the geometrical construction and recognize the interdependence between the variables used.

Correlation–orientation DMS (two levels can be described): after distinguishing which sliders influence the construction of the geometrical figure and which do not, the students are expected to experiment with the varying magnitudes so as to complete the construction of the figure. If there is only one slider that influences the construction of the figure, the students determine the value or the values for which the figure is completed. If there are two or more sliders that influence the construction of the figure, the students move them in order to find the 'combination' of particular values that leads to the completion of the figure. In this level dynamic manipulation is oriented towards the completion of the figure through the use of specific techniques involving the change of the initial and end value of specific variables, the change of the variation step and the identification of the marginal values. The students are expected to develop conjectures for possible relations between variables and modify the code.

Verification DMS: after having corrected the code, the students are expected to verify the correctness of their conjectures by dragging the sliders for enlarging and shrinking the geometrical figure. According to the provided feedback the students confirm or dismiss their own conjectures.

Design DMS: in order to compose a new figure from the initial one, the students can use the variation tool to control the number of iterations. This kind of dragging signals students' ability to use a particular geometrical figure as a unit for the creation of more complicated ones.

5. Conclusion

Our concern was to shed light on the nature of the teaching design based on the use of digital tools and on how these tools might shape the mathematical content in comparison with the formal mathematical curriculum. According to our findings, the development of scenarios based on the use of specially designed digital tools can create contexts for teachers to build meaningful situations for their students. Our analysis reveals that these scenarios might constitute curriculum units that take into consideration different aspects of mathematical concepts and introduce new components in the formal curriculum and create new widening conceptual fields out of the limitations of the formal curriculum. In that sense, designing scenarios based on the use of digital tools can be a professional activity that redefines the relationship between teachers and curriculum.

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