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ORIGINAL ARTICLE

Comparing theoretical frameworks enacted in experimental research: TELMA experience

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Abstract In the context of the Kaleidoscope Network of Excellence, six European research teams developed a methodology for integrating their research approaches. In this paper, we present the methodology based on a cross-experimentation, showing how it gave insight to the understanding of each team's research and on the relationship between theoretical frameworks and experimental research.

1 TELMA project and its objectives

This contribution is about a research activity that is jointly carried out by six teams belonging to Kaleidoscope, a European Network of Excellence[1] that brings together many research teams in technology-enhanced learning. The aims are, on the one hand, to develop a rich and coherent theoretical and practical research foundation, and on the other hand, to develop new tools and methodologies for an interdisciplinary approach to research on learning with digital technologies at a European level (TELMA ERT 2006).

Within the activities of Kaleidoscope, a European Research Team (ERT) TELMA—Technology Enhanced Learning in Mathematics—has been established to focus on the improvements and changes that technology can bring to teaching and learning activities in mathematics. TELMA ERT includes six teams[2] with a strong tradition in the field, most of which have also been engaged in designing, developing, testing and integrating interactive learning environments (ILE)[3] for use in mathematics learning.

As a Kaleidoskope's ERT, TELMA first aim is by contract *to promote integration among its constituting teams* and to favour (a) the construction of a shared

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[1] Kaleidoscope is an initiative founded by the European Community (IST-507838) under the VI Framework Programme. See http://www.noe-kaleidoscope.org.

[2] The teams (whose acronym is indicated in brackets) belong to the following Institutions: Consiglio Nazionale delle Ricerche—Istituto Tecnologie Didattiche—Italy (ITD); Università di Siena—Dipartimento di Scienze Matematiche ed Informatiche—Italy (UNISI); University of Paris VII—France (DIDIREM); Grenoble University and CNRS—LIG Laboratory—France (MeTAH); University of London—Institute of Education—UK (UNILON); National Kapodistrian University of Athens—Educational Technology laboratory—Greece (ETL-NKUA).

[3] The term ILE is used instead of the more general term of ICT tool in order to better qualify the kind of technology developed or used by TELMA partners.

scientific vision, (b) the development of common projects and (c) the building of complementarities and common priorities in the area of digital technologies and mathematics education.

TELMA teams have brought with them different research questions, theoretical frameworks, work methodologies, cultural perspectives and views of the use of digital technologies for the teaching and learning of mathematics. Thus, in order to achieve the project objectives, the teams started sharing knowledge, developing a common language and common topics of interest. This demanding task was addressed by analysing documents and some of the most significant papers provided by each team, focusing on topics considered as important for mutual knowledge and comparison among teams, such as digital technologies developed and used by the teams, theoretical frameworks and work methodologies, and contexts of digital technologies use. This comparative study proved useful for improving mutual understanding and allowed identifying TELMA teams' common concerns (e.g. contextual, social and cultural dimensions of learning, instrumental issues, etc.), but it also put forward a diversity of ways to deal with these concerns, which is due mainly to the variety of theoretical frameworks used by the teams (ibid.). Furthermore, from a methodological point of view, it had evident limits: the exact role played by theories remains largely implicit in most published papers, and the data one would like to access to in order to understand this role better are rarely provided. For the sake of developing an integrated approach to the research on technology enhanced learning of mathematics, there was a need to get a deeper insight on the role played by the theoretical frameworks each team uses in its own research.

In order to overcome the reported limits, TELMA teams decided to complement this analysis by *developing a specific strategy to gain more intimate insight into each team's respective research and design practices*. This strategy consisted in the simultaneous development of a set of teaching experiments based on the use of a piece of technology and of a *methodological tool for systematic exploration of the role played by theoretical frames in the design and analysis of these teaching experiments*. A joint short-term empirical study was designed and implemented based on the developed methodology.

Although this methodology was not intended to provide a tool for comparing theoretical frames in general, but rather a tool for studying how theoretical frameworks are "enacted in the researchers' practice" when they design and analyse teaching experiments, it may contribute to the development of tools for comparing, combining, networking and complementing different theoretical approaches.

With this respect it is important to stress that the aim was not to study "in general" how research teams (and theoretical frameworks) can be integrated or networked. TELMA teams needed to achieve such integration for practical reasons, in order to be able to work together and to understand each other. Thus, our aim was not to study how in principle the integration could be achieved, even if it was perceived that, as a side (and valuable) effect, an exportable methodology could be developed, but the addressed task was rather very concrete and with well defined time and resources constrains.

This paper focuses on *this methodology* (Sect. 2) showing how it provides a tool for investigating the roles played by theoretical frameworks in the design of teaching experiments (Sect. 3), but also how it allows comparing different theoretical frameworks used by different TELMA teams (Sect. 4). Within the paper potentialities of this methodology are also discussed.

2 The developed methodology

The methodology developed within the TELMA project consists of two main methodological tools: (1) the *construct of didactical functionality* (*DF*); (2) an experimental methodology, called *cross-experimentation*, framed by and developed together with collaboratively produced *guidelines*.

2.1 The construct of didactical functionality

The construct of didactical functionality has been described in detail in Cerulli et al. (2006) and here we only recall its main aspects. It was built with the aim of providing a common perspective, independent from specific theoretical frameworks, to address the variety of approaches (possibly depending on theoretical references) to the use of ILEs (as ICT tools) in mathematics education, and to link theoretical reflections with actual uses of ILEs in given contexts.

> "With didactical functionalities we mean those properties (or characteristics) of a given ICT, and/or its (or their) modalities of employment, which may favour or enhance teaching/learning processes according to a specific educational goal.

The three key elements of the definition of the didactical functionalities of an ICT tool are: (1) a set of features/characteristics of the tool; (2) a specific educational goal; (3) a set of modalities of employing the tool in a teaching/learning process referred to the chosen educational goal" (ibid. p. 1390).

These three dimensions are inter-related: although characteristics and features of the ILE itself can be identified through an a priori inspection, these features only become functionally meaningful when understood in relation to the educational goal for which the ILE is being used and to the modalities of its use. Note that, when designing an ILE, designers have to have in mind some specific DFs, but these are not necessarily those emerging when the tool is used. This may be especially the case when an ILE is used outside the control of its designers, according to different epistemological or educational perspectives, or in contexts different from those envisaged by the designers.

The notion of DF took a central and unifying role in the design and development of the cross-experimentation:

On the one hand, the cross-experimentation aimed at exploring the DFs that the different teams would associate with the ILEs they have not designed;
On the other hand, this notion was also used to structure the methodology for exploring the role played by theoretical frames in designing empirical research.

In fact, the three dimensions constituting the notion of DF are supposed to be always addressable when designing or analysing empirical research studies based on the use of ILEs, no matter what the theoretical assumptions of the research are taken into consideration.

2.2 Cross-experimentation

As mentioned above, the *cross-experimentation* consisted in the simultaneous development of a set of teaching experiments and of a methodological tool for systematic exploration of the role played by theoretical frames in the design and analysis of these teaching experiments. The main characteristics of the developed experimental methodology relevant to the scope of this paper are the following:

- Design and implementation by each research team of a teaching experiment making use of an ILE developed by another team. Each team was asked to select an ILE among those developed by other teams and design and implement a teaching experiment making use of that ILE. This decision was expected to induce deeper exchanges between the teams, and to make the influence of theoretical frames more visible through comparison of the DFs envisaged by the ILEs designers and those identified by the experimenting teams. Table 1 summarises the ILEs chosen, the teams who developed the ILEs and the teams conducting experiments with these ILEs.
- Joint construction of a common set of *guidelines* expressing questions to be answered by each designing and experimenting team in order to frame the process of cross-team communication. The cross-experimentation was intended to enhance integration among the teams by addressing a shared set of research questions derived from the three key themes of interest of the project: contexts, representations and theoretical frameworks. On the one hand, the investigation of these themes constitutes a first level of integration among TELMA teams, at least in terms of addressing shared issues. On the other hand, such themes are wide and open the space for a huge number of possible research questions: there was a need to restrict to a feasible smaller number of questions. Generally speaking, the choice of specific questions to address may depend on one's interests, on possible theoretical frameworks of reference, or on other constrains. This potentially constituted a sort of centrifugal force among the teams, which could contrast with the aims of the cross-experimentation itself. Thus, common questions were chosen according to a specific methodology, as detailed in the next paragraph.

Table 1 The tools employed by TELMA teams in the cross-experimentation

ILE	Developer's team	Experimenting team(s)
Aplusix	MeTAH	ITD, UNISI
E-Slate	ETL-NKUA	UNILON
AriLab2	ITD	MeTAH, DIDIREM, ETL-NKUA

Finally, in order to allow as much comparability as possible between the research settings, it was also agreed to address common mathematical knowledge domains (fractions and algebra), with students between years 7 and 11 of schooling in experiments lasting approximately one month.

2.2.1 *The guidelines*

The guidelines is a document collaboratively produced during the cross-experimentation which includes the research questions to be answered by each designing and experimenting team in order to frame the process of cross-team communication, as well as the answers provided by the teams before, during and after the experiments (Cerulli et al. 2007). This document was meant to draw a framework of common questions providing a methodological tool for comparing the theoretical basis of the individual studies, their methodologies and outcomes. Thus the questions had to reflect, on the one hand, the shared objectives of the cross-experimentation and its constraints, and, on the other hand, the specificities of each research team. Thus the guidelines were jointly built according to the following procedure:

- Three researchers of the TELMA group, experts in the subjects, developed three documents (one for each of the three key themes addressed by TELMA) each consisting of a set of possible research questions to focus on.

- The teams reviewed such documents and jointly chose a small set of questions to be addressed. The choice followed the criteria of (a) relevance to teams' interests and (b) feasibility within the constraints of the cross-experimentation.
- A priori, a posteriori and a priori/a posteriori sets of questions were developed to be answered by the experimenting teams respectively before, after and both before and after the experiments.
- In addition, each team that produced a tool employed in the experiment was required to provide a description of the educational principles underlying the design of the tool and to indicate possible DFs of the tool.

Two examples of questions concerning theoretical frameworks are the following:

Example 1 (*theoretical frameworks*: a priori):

What theoretical frame(s) do you use and what motivated your choice? How do you see their potential and eventually limitations for this project?

Example 2 (*theoretical frameworks*: a posteriori):

In your opinion, in which ways have your theoretical choices influenced:

- The analysis of the software and the identification of its didactical functionalities?
- The design of the experiment?
- The choices of the data and their analysis?
- The results you obtain and the conclusions you draw from these?

2.2.2 The cross-experimentation and the guidelines

After the production of the first version of the guidelines document containing the set of key questions to be addressed and identifying basic information to be provided by each team, the guidelines became the key element around which the main phases of the cross-experimentation were developed:

1. Production of a pre-classroom experiment version, containing plans for each experiment and answers to some questions (a-priori questions).
2. Implementation of the classroom experiments.
3. Analysis of the experiments.
4. Production of the final version of the guidelines containing answers to all of the addressed questions (including the a-posteriori questions).

The guidelines may be considered both as a product and as a tool supporting TELMA collaborative work. A product in the sense that the final version contains questions and answers to questions as well as plans, descriptions of the experiments and results. A tool in the sense that the guidelines structured each team's work by:

- Providing research questions concerning contexts, representations, and theoretical frameworks;
- Establishing the time when to address each question (ex. before, or after the classroom experiment, etc.);
- Establishing common concerns to focus on when describing classroom experiments, on the basis of the definition of DF;
- Gathering, under the same document, the answers provided by each team to the chosen questions, in a format that could possibly help comparisons.

Thus, the guidelines became also a tool for analysing the role played by theoretical frameworks in the design, implementation and analysis of experiments themselves and for comparing and possibly integrating the different research approaches of the teams. In fact, the process of building the guidelines and using them simultaneously as a reference for comparing the teams' researches, contributed to:

- Investigate the relationships between each team's assumed theoretical frameworks, and the employed/defined DFs (and questioning the effectiveness of such DFs).
- Analyse the teams' approaches to the design of their classroom experiments, and explaining the key choices characterising them, could they be depending on theoretical assumptions, institutional/cultural constrains, or any other reason.

Such objectives were addressed, on the one hand, by comparing and questioning the teams' answers to the questions contained in the guidelines and, on the other hand, by addressing extra questions, like the following one (*Example 3*), aiming at preparing the terrain for answering the a posteriori question of the guidelines reported in *Example 2*:

Example 3 (*DF*: *extra question*):

If you were to design a new experiment aiming at the same mathematical educational goal and employing the same ICT tool, which characteristics of the experiment would you keep unchanged? Which of these characteristics do you think, according to the theoretical framework you chose, are necessary conditions for the experiment to be successful?

This kind of questions aimed at making explicit the links between the DFs employed/defined by the teams for their experiments and the theoretical frameworks they assumed.

In what follows, we illustrate the potentialities of the methodological tools developed within the TELMA project for investigating the roles played by theoretical frameworks in the design and analysis of the teaching experiments and for comparing and possibly starting networking "theoretical frameworks in action" as well.

3 Outcomes of the cross-experimentation

In this section, we outline some of the most striking results of a comparative analysis of the local experiments in terms of the impact of theoretical frameworks chosen by TELMA teams on the design, implementation and analysis of the experiments. For more details, see Artigue et al. (2006a, b).

3.1 Theoretical frames and observed priorities

Contextual and representational issues were central aspects of the study developed within the TELMA project together with issues related to the role of the teacher, the social interaction and so on; consequently, these were central issues of the cross-experimentation as well. Nevertheless, the research teams did not address such aspects in the same way: rather, the cross-experimentation shows that though addressing the same main issues, *different teams had different priorities when designing their experiments*.

Such priorities (and differences among teams' approaches) may be determined by cultural backgrounds, theoretical frameworks and ways of approaching and conceiving research in mathematics education. For instance, in the experiment carried out by the DIDIREM team, the main theoretical frameworks were the *Theory of Didactic Situations* (TDS) (Brousseau 1997) and the *Anthropological Theory of Didactics* (ATD) (Chevallard 1992). As a result, major attention was paid to (a) a detailed organization of a (potentially) cognitively rich "a-didactic milieu" and (b) a distance between the experimental and the usual institutional contexts, as well as the necessity to keep this distance manageable by the teacher. Consequently, other aspects, even if considered interesting, were less emphasized (e.g. students' collaborative work, teacher's role beyond the management of the devolution and institutionalization processes).

On the contrary, the ITD team mainly referring to *Socio-constructivism* and *Activity Theory* (Cole and Engeström 1993; Engeström 1987; Vygotsky 1978) assigned a high priority to social construction of knowledge and to the role of the teacher. Therefore, their experiment was mainly focused on these issues and minor attention was paid to other aspects (e.g. detailed organization of the milieu), and many choices were not set up by the experimenting team but left to teachers (e.g. specific tasks and orchestration of the work).

Finally, let us mention ETL-NKUA team's theory-driven choice of not defining a "strictu sensu" didactical goal for its experiment. Referring mainly to theories concerning "the generation of mathematical meanings" such as *Constructionism* (Harel & Papert 1991) and *Situated Abstraction* (Noss & Hoyles 1996), ETL-NKUA researchers paid emphasis not on "closed didactical goals" but rather on pupils' active construction of meanings as they operationalize the use of the available tools while making judgments and taking decisions in the process of solving a problem.

We hypothesize that such priorities may remain implicit and act as hidden variables—out of ones control—when designing experiments. The request of making clear and communicating allows/makes these variables revealed.

3.2 Theoretical frames and adapting ILEs

The analysis of the teaching experiments reveals some difficulties the teams met in using or adapting a given ILE to contexts different from those within and for which it was designed. In general, such difficulties might be related to a pile of different aspects: a team could meet difficulties with designing the use of an ILE conceived according to different educational hypotheses, or with designing the use of an ILE conceived to be used in a different school context, but there might be other sources of difficulty not envisaged a priori. In the absence of specific methodologies, this variety of aspects might conceal the exact source of the difficulties hindering the design of the use of a given ILE.

Our analysis confirms that the developed methodology can be an efficient tool for revealing the sources of the possible experienced difficulties.

For example, the software Aplusix was designed (by the French team MeTAH) to be a constitutive element of an autonomous milieu for an a-didactic situation. The software allows students to build and transform algebraic expressions; for each step, the system gives an indication of correctness as feedback. Aplusix was designed to support the standard activity of algebraic manipulation (referring to the French mathematics curriculum), based on solving calculation tasks like *expand and simplify*, *factor*, *solve equation or inequality*, etc. and it was not designed for supporting activity based on solving open-ended tasks. Thus, when ITD team tried to design its experiment based on the use of Aplusix, consistently with a socio-constructivist approach, they met a problem of planning open-ended tasks within the ILE. According to this theoretical framework, such tasks favour pupils' construction of meanings through exploratory activities. In the experiment, this was achieved through a radical change of perspective on the use of Aplusix within the class. In fact, Aplusix was no longer used autonomously by students; rather the teacher orchestrated the whole activity by asking the students to make their strategies explicit, to justify them and to discuss them with their classmates.

MeTAH, DIDIREM and ETL-NKUA teams in their respective experiments also met difficulties in the design of use of an ILE. They all explored the possibility of using the Fraction microworld of AriLab2 software designed by ITD team (Italy), taking a socio-constructivist approach. Both French teams, MeTAH and DIDIREM, were inspired by the TDS and ATD theoretical frameworks, while the Greek team ETL-NKUA followed a constructionist theoretical perspective. It is noticeable that all three teams reported the same problem due to the difference between the school context of the experiment and the school context for which this ILE was designed. In fact, the Fraction Microworld provides a geometric construction of fractions based on Thales theorem (i.e., the projection principle), which is usually introduced later than fractions themselves in the French and Greek curricula.

However, the teams chose different ways to cope with this inconsistency. The French teams, in spite of the fact that they shared the same theoretical background, reacted in radically different ways. On the one hand, MeTAH team tried to use the "construction of a fraction" functionality as a "black box" but found this caused problems when pupils needed to make sense of feedback provided by the tool. On the other hand, DIDIREM team preferred to switch to other AriLab2 microworlds because they judged it was not realistic to ask the teacher to change the mathematics organisation of the school year. As far as the Greek team, it chose the "black box" approach, like MeTAH team, but only because of the time limits of the cross-experimentation. Consistent with their constructionist theoretical framework, the ETL-NKUA researchers considered this specific representation of fractions as offering a novel way to introduce primary students to the geometrical aspects of fractions before—and independent of—the learning of it in the traditional classroom in a future study and under specially designed tasks.

One may argue that the difficulties encountered by TELMA teams are due in part to the limitations of the theoretical frameworks adopted by the experimenting teams. Also one may question how much the success of an experiment is dependent on the theoretical framework adopted by the involved researchers. Exportability to other contexts, especially non-research contexts or different didactic/school contexts, of ILEs might by strongly affected by institutional restrictions coming from researcher's own educational system exceeding the narrow frame of the classroom. Thus it becomes relevant what is (and what is not) the actual impact of theoretical frameworks on teaching experiments. This also brings in the foreground the necessity of the mathematics education research to take into account explicitly such contextual aspects in order to provide more efficient tools for studying the teaching and learning of mathematics.

3.3 What theoretical frameworks do not say

In the previous paragraph, we cited a few examples of how theoretical frameworks may—implicitly or explicitly—drive the design of a teaching experiment. This is but a part of the story; in fact the cross-experimentation revealed that though a theoretical framework may influence/inspire an experiment at a global level, it may not address/determine many specific relevant aspects for the actual set up of the experiment itself. There seems to be a sort of a gap between what a theoretical framework offers and what is needed to put into practice within a classroom teaching experiment. Such a gap is at the core of the relationship between theoretical reflections and cases of practice, and it remains often implicit. Indeed, since theoretical frameworks seem to be strongly linked to research communities (e.g. French mathematics education community refers the most often to TDS and to ATD), it is often rather difficult distinguishing between what researchers exactly do when referring to these frameworks and the frameworks themselves, with their potentials and limitations. In the case of the TELMA cross-experimentation, this gap was made clear through comparisons among the different teams' experiments, in particular UNISI and ITD experiments on the one hand, and between MeTAH and DIDIREM ones on the other hand.

UNISI and ITD teams referred to compatible theoretical frameworks—respectively the *Vygotskian theory* (as for the construction of higher psychological functions) and the *Activity theory*—and centered their experiments on the use of the same ILE, namely Aplusix. Nevertheless, from the ILE analysis they identified different educational aims for their experiments, which resulted in two teaching experiments, both consistent with the respective theoretical frames, but deeply contrasting as far as the role of the teacher, the kind of tasks given to pupils, the validation of pupils' work, the use and set up of the tool are concerned.

Similarly, MeTAH and DIDIREM teams shared the same theoretical background: TDS and ATD and experimented with the same ILE: AriLab2. However, their experiments still differed, though less dramatically than UNISI and ITD experiments, in important aspects such as who/what is responsible for validating pupils' work, does validation emerge as a social product, does it rest with the teacher or does it rest with the ILE, are pupils allowed/obliged/forbidden to use systems of representations other than those provided by AriLab2 (e.g. paper and pencil).

The comparative analysis of the local experiments designs and implementations shows that the theoretical frameworks underlying the research work influence these in different ways. They have impact first on the analysis of the ILE used in the experiment and of the didactical functionalities assigned to it by the experimenting teams following the hierarchy of priority concerns. Once educational goals and modalities of use of the ILE have been globally decided, theoretical frameworks impact the design of the experiment at another level, by determining up to what point the details of the design have to be planned in advance and what is left to be decided on the spot in the design enactment, what respective roles and responsibilities will be given to the teacher and to the students. Theoretical frameworks also impact the design by the influence they have on the vision researchers develop of "distances" and the way they cope with them: "distances" between the representations of mathematical objects and actions on these objects in usual contexts and those provided by the ILE, "distances" between the educational cultures that have supported the work of the designers of an ILE and those of its users in the cross-experimentation. As was expected, the characteristics of this cross-experimentation made visible these influences and the researchers made clear that many influences which became visible through a comparative analysis were not visible from the inside of the teams because theory lived there in some sense "naturalized". At the same time, the reports also show that theoretical frameworks determine only partially the identification of DFs and the design of the experiments. Teams using the same ILE and the same theoretical frameworks built, coherently with these, very different designs, and as stressed in the reports, there exists a real gap between most macro-level theories and the decisions to be taken in the design phase. This is an important issue that directly tackles the difficulty of connecting theoretical approaches on learning mathematics with digital media. It concerns the fact that the respective theoretical frameworks have been fragmented and involve assumptions bound to the specific contexts from which they emerged and, thus, making difficult to distinguish the theoretical frameworks used in the design and analysis of a teaching experiment from the research practice developed by the respective teams.

These interesting results and the necessity of investigating more precisely the role played by theoretical frames in the a posteriori analysis led us to focus on the relationship between theoretical assumptions and actual research experiments.

4 Investigating theoretical frameworks enacted in actual practice

Our methodology for studying how theoretical frameworks are "enacted in actual practice" uses the *cross-experimentation* as a means for making explicit the relationship between the theoretical assumptions and the actual enactment of experimental research. A peculiarity of our methodology is the fact that, during all the phases of the cross-experimentation, thanks to the collective elaboration in itinere of the *Guidelines*, each team's experiment was analysed by the external eyes of researchers from other teams who could ask for clarifications and explanations. We remark that TELMA researchers belonging to different teams are confident of different theoretical frameworks. Thus a given team's experiment was analysed by researchers taking perspectives of their own theoretical frameworks different from that assumed by the experimenting team. This kind of analysis can lead to interesting insights on how different theoretical frameworks can shape the design and implementation of an experiment, but also to gain a deeper understanding of the theoretical frameworks themselves, applicability, usefulness and efficiency of theoretical constructs they offer to support empirical research, as shown by the example presented in the following section.

4.1 A case study: ITD experiment through the lens of the TDS framework

In this section, we attempt an analysis from the point of view of the TDS framework of the experiment carried out by ITD research team whose design and implementation has been driven by the AT theory. We will try to show that such a "cross-analysis" can contribute to shed a new light upon the decisions made in the design and the implementation of a teaching sequence, to explain some unexpected events and perhaps also to get a deeper insight into the two theoretical frameworks themselves.

4.1.1 The ITD experiment

Recall that ITD team (Italy) used Aplusix developed by the French team MeTAH in the experiment designed for working with the notion of fraction and implemented in two Grade 7 classes (11–12 years old students). The general aim of the experiment was to study "*how new technologies, if inserted in suited contexts, can contribute to the construction of innovative environments that can enhance learning processes and can also change traditional approach to school teaching*" (Cerulli et al. 2007, p. 7).

4.1.1.1 Theoretical framework underpinning the ITD local experiment The researchers refer to the Activity theory (AT), more specifically to the model proposed by Cole and Engeström (1993) describing the relationships between elements in an activity (Fig. 1), as the main theoretical framework. Briefly, AT is a philosophical and cross-disciplinary theory bringing the idea of artefact-mediated and

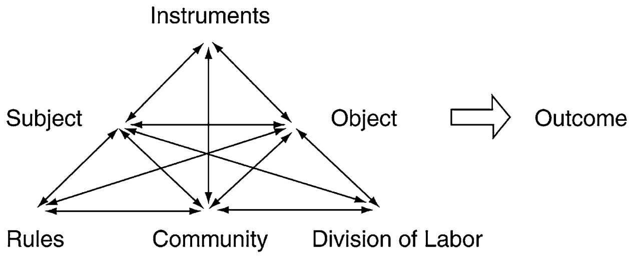


Fig. 1 Cole and Engeström (1993) model of an activity

object-oriented action. In this theory, an activity is a form of acting whose aim is transforming an object into an outcome. Considering activity theory applied to the educational field, the object of an activity is the learning of a given piece of knowledge or the development of a given skill; the outcome of this activity is students' acquisition of this piece of knowledge or this skill (Bellamy, 1996). The model proposed by Cole and Engeström (1993) highlights three mutual relationships involved in every activity, namely the relationships between subject and object, between subject and community, and between community and object. Each of these relationships is mediated by the third entity. The relationship between subject and object is mediated by artefacts (instruments) that both enable and constrain the subject's action. The relationship between subject and community is mediated by rules (explicit or implicit norms, conventions and social interactions). The relationship between community and object is mediated by the division of labour (different roles characterizing labour organization).

4.1.1.2 Main choices in the design of the experiment The key idea of the experiment was to exploit "*feedback provided by Aplusix as a means for supporting pupils in remedial activities concerning numerical fractions*" (Cerulli et al. 2007, p. 10). Educational goals were specified as follows:

Reinforcing pupils operational skills with fractions;
Reinforcing relationships concepts such as equivalence of fractions and ordering of fractions;
Reinforcing pupils' self-control systems concerning the handling of fractions (ibid.).

When identifying didactical functionalities of Aplusix, the researchers focused on the feedback provided by the tool. They noticed that Aplusix checks the equivalence between a statement or an expression entered by the user and a statement or an expression produced by the user in the next step, and returns one of the following values: correct when there is equivalence, incorrect when there is no equivalence and unknown when the produced statement or expression is not well-formed. Based on the analysis of the tool, the researchers hypothesized that

> "This kind of feedback may enable the student to accomplish a task and validate his/her solution of a problem with the aid of the computer, without the intervention of the teacher. Moreover, because this feedback is given constantly, at any moment of the interaction, we hypothesize that the user may be constantly stimulated to reflect on each single step. Moreover, we believe that ad hoc designed activities with Aplusix may help the pupil to foster/develop his/her own control systems" (ibid. p. 8).

Figure 2 shows an example of activity proposed to students who were working in pairs with the computer (ibid. p. 19).

In terms of the Cole and Engeström's model, the elements of the activity can be described as follows: Fig. 3.

The Table 2 below summarizes the main didactical functionalities of Aplusix identified by the ITD research team at the beginning of the experiment:

4.1.1.3 A change of activity With respect to the above-mentioned hypothesis, the researchers claim that the experiment suggests that "Aplusix can be of help in fostering pupils' control systems, and can be suitable for supporting pupils with difficulties." (Cerulli et al. 2007, p. 12). However, one part of the hypothesis turned out to be false since "*the considered feedback **did not** stimulate constantly pupils to reflect on each single step*; on the contrary, it resulted to be an incentive for pupils to "random alike" or "trial and error" strategies, simply because it is easy to try out many solutions, and sooner or later one will guess the right one" (ibid. p. 12).

The students did not spontaneously go beyond a result that they guessed right and were not searching for better, more efficient strategies. To provoke an evolution of the students' initial, trial and error strategies, the teacher needed to intervene and ask them to formulate the strategies, either using comments in Aplusix or just paper and pencil.

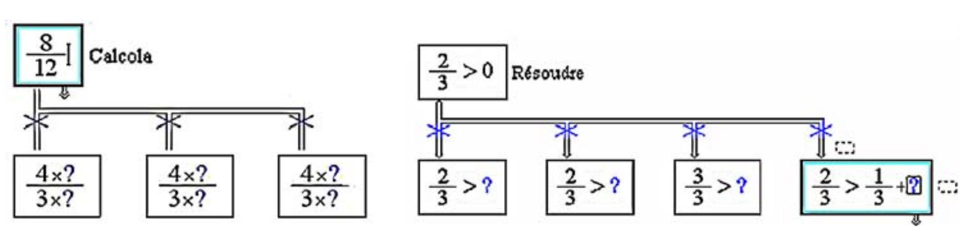


Fig. 2 Examples of trees with empty placeholders. Students were asked to replace them in a way to obtain expressions (*left*) or statements (*right*) equivalent to the given expression (fraction 8/12) or statement (2/3 > 0)

Fig. 3 ITD experiment: a model of the planned activity

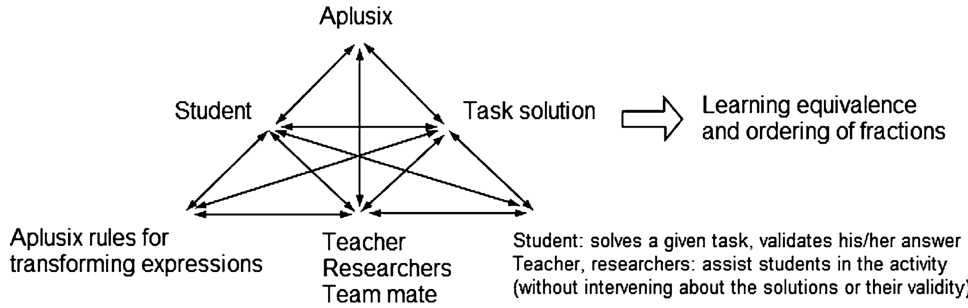


Table 2 Didactic functionalities of Aplusix

Educational goals	Reinforcing pupils operational skills with fractions
	Reinforcing relationships concepts such as equivalence of fractions and ordering of fractions
	Reinforcing pupils' self-control systems concerning the handling of fractions
Characteristics/features of Aplusix	Feedback: verifying equivalence between two consecutive statements of expressions
Modalities of employment	Open-ended activities proposed to pupils working in pairs with Aplusix software

However, the tool not being able to validate the described strategies, the teacher had to take the validation in charge and organize a classroom discussion aimed at discussing and validating or not students' strategies. For the researchers, such a change of the activity was not a problem, as they assumed the teacher to be a constitutive element of the activity, thus influencing it. The planned activity could be refined in itinere, and it was actually changed into the following one: Fig. 4.

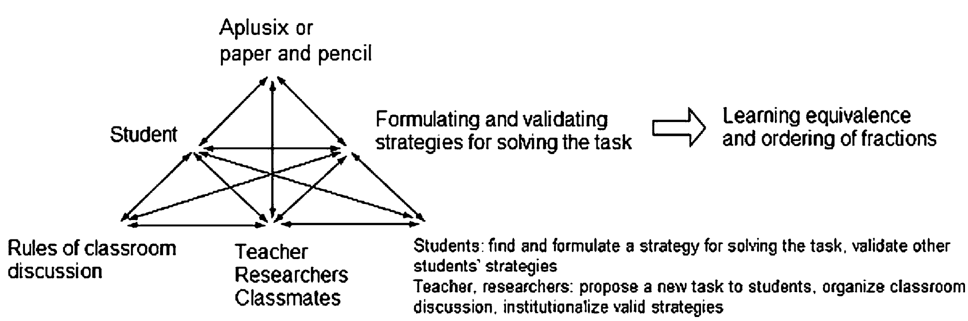
At the end of the experiment ITD researchers were thus able to reformulate the adopted (and hopefully re-adoptable) didactical functionality of Aplusix defining more elaborated *modalities of employment*: "open-ended activities proposed to pupils working in pairs with Aplusix software, and to be developed with a constant interaction with the teacher providing feedback on the strategies employed by pupils to bring forward the activities".

As we can see from the model, although the outcome of the activity remains the same, its object has changed: the focus has shifted from the task solution to searching for strategies leading to the task solution. The role of the teacher has changed as well: it became crucial for making the initial students' strategies evolve. It is interesting to mention that in the design of the experiment, such issues have not been anticipated and the teacher has taken this decision on the spot. There seems to be a gap between what the theoretical framework offers and the needs of the implementation of the experiment (see Sect. 3.3).

4.1.2 Analysing ITD experiment through the TDS lens

4.1.2.1 Theory of didactical situations (TDS) TDS (Brousseau 1997) refers to the work of Piaget and to a model of learning by adaptation. This theory models the interactions between the three components of a didactical system: teacher, student and "milieu" with respect to a certain piece of knowledge. The main interactions are those between the student and the "milieu". Student's acting on the "milieu" provokes feedback provided to the student by the "milieu" calling for modifying or adjusting the student's action. The piece of knowledge at stake is the optimal and stable solution to a set of constraints. Learning

Fig. 4 ITD experiment: a model of the new activity refined in itinere



thus results from the student's adaptation to an antagonist "milieu". Teaching consists in organising these constraints and keeping optimal the conditions of the interaction.

An important issue in the theory is the distinction between didactical and a-didactical situations. A didactical situation is an activity in which the student has to mobilize or construct knowledge in order to achieve a goal set up by the teacher. An a-didactical situation is a situation designed in a way that the desired outcome can be obtained only by applying the knowledge aimed at in the situation, but the student cannot "read" the teacher's intentions as regards this knowledge to take his/her decisions.

4.1.2.2 Analysis of the ITD experiment As we mentioned above, studying the feedback provided by Aplusix was the first step the researchers made towards the definition of the didactical functionalities of the tool. This approach is similar to the TDS framework where characterizing the "milieu" the students are going to interact with in order to solve proposed tasks and the provided feedback is crucial. The task ITD team firstly designed could be considered as an a-didactical situation: the "milieu" provided feedback that was judged as sufficient to help the students to solve the given problem (replace the question marks by numbers in a way to obtain equivalent expressions or statements) and the teacher intervention was not necessary. It was expected that the students would adopt reflective position with respect to the feedback coming from the "milieu", especially to the error message following an incorrect action, and would try to avoid errors. But most of the students did not behave as expected and the teacher was led to modify the task by asking the students to formulate and discuss the validity of their strategies, thus revealing his/her intention as regards the aim of the activity.

TDS offers an explanation for this event which was unexpected by the experimenting team: by carefully examining the milieu, the feedback it provides allows validating the students' solutions, but it gives no information about the reason why a given solution is erroneous, thus it does not help the students in reflecting about their strategies. Moreover, there is no constraint in the milieu inciting the students to analyse their errors and change the initial strategies which are mostly trial-and-error ones. In other words, the milieu as it is organised enables "the student to accomplish a task and validate his/her solution of a problem with the aid of the computer", which was the first ITD team research hypothesis, but is not rich enough to guarantee the evolution of these strategies (second research hypothesis).

The TDS offers a possibility to avoid such a breakdown caused by the intervention of the teacher by carefully examining the "milieu" and anticipating students' behaviours in the a priori analysis. In this case, one would certainly expect that the students would start solving the given problem by applying trial and error strategies that are quite efficient in the tool. Therefore, they do not need to abandon the initial strategies and search for other, more efficient strategies. The teacher's intervention was intended to fill this gap, and was legitimated by the fact that the situation was not assumed to be an a-didactical one. Within the TDS, another choice can be envisaged: the one consisting in adding a constraint to the "milieu" obliging the students to search for better, more efficient strategies. One could for example modify slightly the initial problem by adding a constraint consisting in the requirement to replace the question marks by numbers in a way that the equivalence between expressions or statements is obtained at the first attempt. Clearly, trial and error strategies would not be efficient anymore. Searching for better strategies would be motivated by the need to adapt to the "milieu" rather than to respond to a teacher's explicit demand, this resulting in a new *modality of employment* for Aplusix.

4.2 Comparing AT and TDS

The analysis presented in the previous section can certainly be discussed, and perhaps even deepened, for example by considering the three types of a-didactical situations (action, formulation and validation), but we find that this level of analysis highlights interesting differences between the ways AT and TDS tend to frame the design and implementation of a teaching experiment. Moreover, it turns out that the TDS framework allows explaining some unexpected events observed in ITD experiment. In what follows, we discuss some of the most striking differences between these two theories, first from a general point of view and then those highlighted particularly by the above-mentioned case study of the ITD local experiment "cross-analysis".

4.2.1 AT versus TDS: general issues

In this section, we report briefly how AT and TDS theoretical frameworks view some of the general, but crucial issues in the teaching and learning mathematics with a technological tool. These notions are learning, teaching and learning environment.

4.2.1.1 Learning and teaching As we mentioned in the Sect. 4.1.1, AT provides a model of an activity whose object is the learning of a given piece of knowledge or the development of a given skill, and whose outcome is students' acquisition of this piece of knowledge or this skill (Bellamy, 1996). Learning is thus considered as the *outcome of the designed activity*. Consistently with this view of learning, AT considers teaching as *organizing and managing an activity towards its object and outcomes*.

According to TDS, the piece of knowledge a subject is supposed to be learning is the optimal and stable solution to a set of constraints of a milieu the subject interacts with (see Sect. 4.1.2). Learning is thus considered as the *result from the student's adaptation to a milieu*, and teaching *consists in organizing a milieu in a way to optimise student-milieu interactions*.

4.2.1.2 Learning environment Within AT, the learning environment is constituted by the *enactment of an activity* oriented towards an educational object, involving students, teacher and artefacts. Vygotskian frame, which is at the basis of AT, gives a *cooperative and social dimension* to the notion of learning environment. Thus, learning environment is negotiated, co-built by all the participants in the activity, students and teacher, and it evolves during the development of the activity.

As was mentioned previously, TDS views learning as student's adaptation to a milieu that is source of contradictions, difficulties and disequilibria (Brousseau 1997). The milieu opposes feedbacks to the students' answers or inadequate choices with respect to the a-didactical situation at stake. In order to learn, students have to understand the insufficiency of their control of the situation. The milieu is not a students' allied but rather a competitor. The milieu is thus an *antagonist system* for students.

4.2.2 AT versus TDS: differences highlighted by the case study

The case study presented in Sect. 4.1 highlights other interesting and insightful differences. In what follows, we discuss a few of them, namely the process of design of activities, the role of the teacher and the conditions for students' strategies evolution.

4.2.2.1 Design of activities It is worth to mention that the differences between theoretical approaches adopted by different TELMA teams appeared from the very beginning of the cross-experimentation, at the level of planning of the teaching experiments and designing experimental activities. In the case of the ITD experiment, many choices were deliberately not set up but rather left to teachers, in particular how to precisely enact the designed activities. This decision is consistent with the AT theoretical approach that considers the teacher as a co-actor of the activity and his/her role is thus crucial in the development of the activity. In the experiment, the teacher needed to negotiate a change of the activity in order to obtain the wished outcome. The activities were thus refined in itinere, according to their actual development.

TDS requires a detailed organization of an a-didactical milieu (setting up values of didactical variables) to guarantee the construction of the desired meaning of the piece of knowledge aimed at in the didactical situation. A priori analysis of the designed situations is a very important moment of the design since it is intended to determine how the chosen didactical variables allow controlling the students' behaviours and their meanings (Artigue 1988).

4.2.2.2 Teacher's role As we mentioned previously, AT assigns a crucial role to the teacher during the whole process of development of the activity: s/he may need to adapt to the actual development of the activity and can be obliged to negotiate a change of the initial activity if it turns out that it does not lead to a wished outcome. This was the case in the ITD experiment.

Within TDS, one of the teacher's roles is to build conditions under which the responsibility of the task solution is entirely submitted to the student (process of devolution). Between the moment the student accepts the task as her/his own problem (not as a school problem) and the moment when s/he produces a solution, the teacher has to step aside: the student has to construct her/his knowledge. The teacher's role is crucial also at the moment of institutionalising the constructed knowledge.

These are well known facts about the teacher's role in the AT and TDS theories, and are confirmed by experimental data, however to deepen such analysis is not in the scope of this paper.

4.2.2.3 Evolution of students' strategies The ITD experiment also shows how important are the teacher's decisions in order to make the students' initial strategies evolve. Indeed, in the experiment, the students used mostly trial and error strategies to solve the given problems that were appropriate and efficient in the Aplusix environment. However, the expected outcome of the activity was not to provide solutions for the problems, but rather find strategies allowing solving these problems. In order to make the development of the activity shift towards the achievement of this goal, the teacher was obliged to intervene and reveal the expected outcome to the students by asking them to formulate and discuss their strategies. On the basis of the DFs, the teacher's actions can be seen as related to the modalities of employing the tool and specifically to those regarding the social interaction between the different actors, their respective roles and responsibilities in order to achieve a specific educational goal.

From TDS perspective, one would anticipate the students' strategies and organize a milieu in a way to force them search for better, more efficient ones (see Sect. 4.1.2).

The Table 3 summarizes the reported differences between AT and TDS:

These differences have a consequence on the reproducibility of the designed activities: it is clear that from the AT perspective, the evolution of the activity and thus its outcome depends significantly on the teacher's decisions made on the spot, whereas TDS tends to determine a priori all the didactical variables necessary to make the situation evolve in a desired way.

They can be particularly relevant also when one questions the exportability of a given educational approach into new school settings: to what extent teachers used to work within a given theoretical perspective can be able to adopt an approach based on different theoretical assumptions? If a teacher is used to design activities mainly a priori, how confident will he/she be with an approach based on refinement in itinere? Vice versa, if a teacher is used to rely on refinements in itinere, will he/she be able to set up effective a-didactical situations? The same questions can be posed for researchers, and as we have shown, become particularly relevant when technology enhanced learning is considered, since any piece of technology may bring with itself the theoretical assumptions of its designers.

5 Summary and perspectives

In this paper, we presented and discussed the specific methodology developed by the TELMA teams to address the question of investigating how specific theories influence empirical research. We have reported on four main facets of the TELMA work: (a) the use of the construct of didactical functionality as a means to link theoretical reflections and actual uses of ILEs in given contexts; (b) the collaborative design and realisation of a cross-experimentation approach as a joint methodology to help different developing and experimenting teams to make explicit their assumptions and the set up of their experimental investigations; (c) the development of the Guidelines for comparing the theoretical basis of the individual studies, their methodologies and outcomes and (d) the analysis of the potentialities of the developed methodology for networking different theoretical frameworks by exploring the ways that these can infuence the actual enactment of a specific research experiment.

This analysis puts into evidence that *making the role played by theoretical frames visible and not just invoked needs specific methodologies*. The methodology developed by TELMA teams proved efficient in highlighting the influence of theoretical frameworks used by the teams on the choices made in the design and the implementation of empirical research. The cross-experimentation puts forward the fact that some choices are determined by cultural or institutional backgrounds and such contextual issues need to be taken into account especially when designing and/or using computer-based tools developed in a given research and educational context. In this sense, our research brought to light the existence of a gap between the needs required by the implementation of an empirical research and the constructs theoretical frameworks can offer.

Finally, we have shown that this methodology affords opportunities for starting networking theoretical frameworks. In fact it provides opportunities for exploring differences and complementarities between theoretical frameworks, as we showed in the analysis of the ITD case study by comparing Activity Theory and the Theory of Didactic Situations. Such analysis contributed to getting a deeper insight into both theoretical frameworks (e.g. about the role of the teacher and the "milieu" in the experiments) and the constructs they offer to deal with issues related to the enactment of experimental research.

Table 3 Comparing AT and TDS perspectives

	Activity theory	Theory of didactical situations
General issues		
Learning	Outcome of an activity	Adaptation to a milieu
Teaching	Organizing and conducting the activity towards its object and outcomes	Organizing a milieu in a way to optimise student-milieu interactions
Learning environment	Cooperative activity-oriented system	Antagonist system
Issues highlighted by the case study		
Design of activities	Refinement in itinere	A priori
Role of the teacher	Crucial with respect to the evolution of the activity	Devolution of a-didactical situations; institutionalisation of the constructed knowledge
Evolution of students' strategies	May depend on teacher's decisions	Guaranteed by the evolution of the milieu

With this respect, the results sketched above corroborate the efficiency of the methodology developed within the TELMA project as a tool for analysing and comparing how theoretical frameworks are "enacted in actual practice"; but the exportability of the presented methodology cannot be taken for granted. Is it applicable to other research projects? What are the conditions for its applicability? Moreover, mantained that different forms of rationality are implicitly engaged in the design and implementation of teaching experiences: to what extent may such implicit factors be accessible to an explicit study?

We conclude this paper just mentioning two different recent directions originated from the experience reported in this paper.

- Within TELMA project, delayed reflective interviews have been carried out for investigating still more in depth the actual role played by theoretical frames in the design of teaching experiments ("interviews for making explicit" (Vermersch & Maurel, 1997). Researchers involved in the experiment were not questioned about the theoretical frames they have used and why, but about the decisions taken in the design, the implementation, the collection of data and their analysis. The analysis of such interviews is still in progress.
- Within TELMA project, the cross-experimentation methodology was conceived as a methodological tool aiming at fostering communication and integration among different teams per se. Such methodological tool is currently used within the European Project ReMath[4] (originated from TELMA Project) to achieve integration among the results of different teaching experiments (carried out by six different research teams) in order to produce common results on specific research goals.

We believe that the kind of research reported in the paper is of particular importance in the European context where more and more teams are involved in cross-country projects. With this respect, TELMA experience brings forth a methodological tool for comparing and possibly networking different theoretical frames enacted in actual practice without loosing the richness of diversity of approaches.

[4] "Representing Mathematics with Digital Media", http://www.remath.cti.gr, European Community, 6th Framework Programme, Information Society Technologies (IST), IST-4-26751-STP, 2005–2008.

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