Kink instability in relativistic magnetized jets

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Outline

- linear stability analysis
- unperturbed cylindrical, cold, magnetized jets
- resulting growth rates

Unperturbed flow

Unperturbed relativistic cylindrical jet

helical, axisymmetric, cylindrically symmetric and steady flow

$$\boldsymbol{V}_0 = V_{0z}(\varpi)\hat{z} + V_{0\phi}(\varpi)\hat{\phi}\,, \quad \gamma_0 = \gamma_0(\varpi) = (1 - V_{0z}^2 - V_{0\phi}^2)^{-1/2}\,,$$

$$\boldsymbol{B}_0 = B_{0z}(\varpi)\hat{z} + B_{0\phi}(\varpi)\hat{\phi}\,, \quad \boldsymbol{E}_0 = (V_{0z}B_{0\phi} - V_{0\phi}B_{0z})\hat{\varpi}\,,$$

$$\rho_{00} = \rho_{00}(\varpi)\,, \quad \xi_0 = \xi_0(\varpi)\,, \quad \Pi_0 = \frac{\Gamma - 1}{\Gamma}(\xi_0 - 1)\,\rho_{00} + \frac{B_0^2 - E_0^2}{2}\,.$$

Equilibrium condition $$\frac{B_{0\phi}^2 - E_0^2}{\varpi} - \xi_0\rho_{00}\frac{\gamma_0^2 V_{0\phi}^2}{\varpi} + \frac{d\Pi_0}{d\varpi} = 0\,.$$

The jet is expected to be unstable to current-driven instabilities (Kruskal-Shafranov) — role of inertia?

Linearized equations

$$Q(\varpi\,, z\,, \phi\,, t) = Q_0(\varpi) + Q_1(\varpi)\exp\left[i(m\phi + kz - \omega t)\right]$$

$$\begin{pmatrix} \\ \\ 10 \times 12 \text{ array} \\ \text{function of } \varpi\,, \omega\,, k \\ \\ \\ \end{pmatrix} \begin{pmatrix} \gamma_1 \\ \rho_{01} \\ B_{1z} \\ B_{1\phi} \\ iB_{1\varpi} \\ \xi_1 \\ V_{1z} \\ V_{1\phi} \\ d\left(i\varpi V_{1\varpi}\right)/d\varpi \\ d\Pi_1/d\varpi \\ i\varpi V_{1\varpi} \\ \Pi_1 \end{pmatrix} = 0$$

reduces to (4 equations in real space)

$$\frac{d}{d\varpi}\left(\begin{array}{c} y_1 \\ y_2 \end{array}\right) + \frac{1}{\mathcal{D}}\left(\begin{array}{cc} \mathcal{F}_{11} & \mathcal{F}_{12} \\ \mathcal{F}_{21} & \mathcal{F}_{22} \end{array}\right)\left(\begin{array}{c} y_1 \\ y_2 \end{array}\right) = 0\,,$$

where the (complex) unknowns are

$$y_1 = i\frac{\varpi V_{1\varpi}}{\omega_0}\,, \qquad y_2 = \Pi_1 + \frac{y_1}{\varpi}\frac{d\Pi_0}{d\varpi}$$

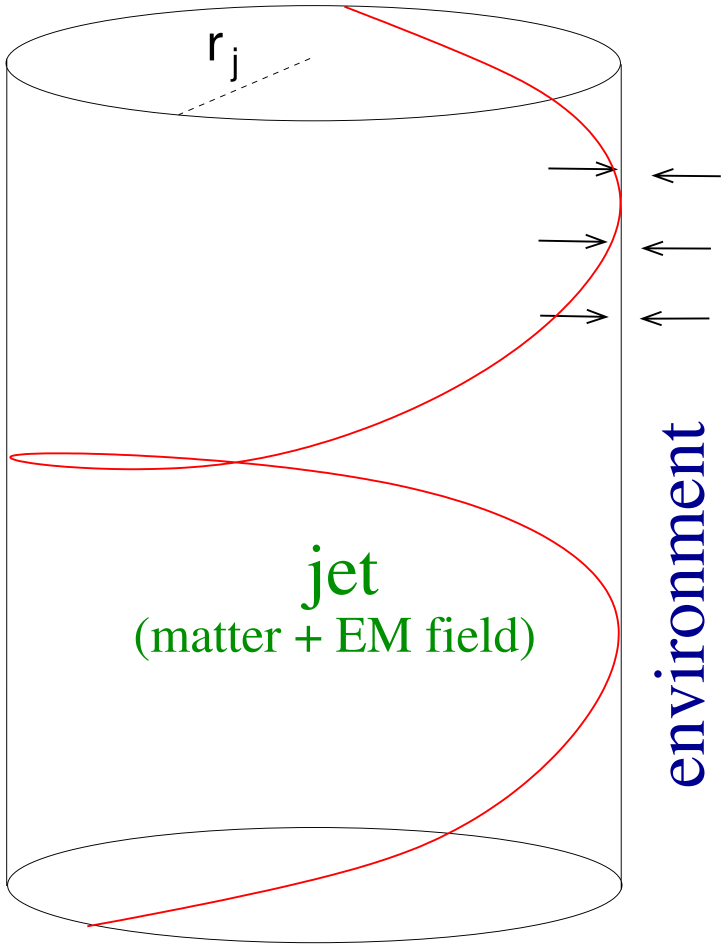
($\mathcal{D}\,, \mathcal{F}_{ij}$ are determinants of 10×10 arrays).

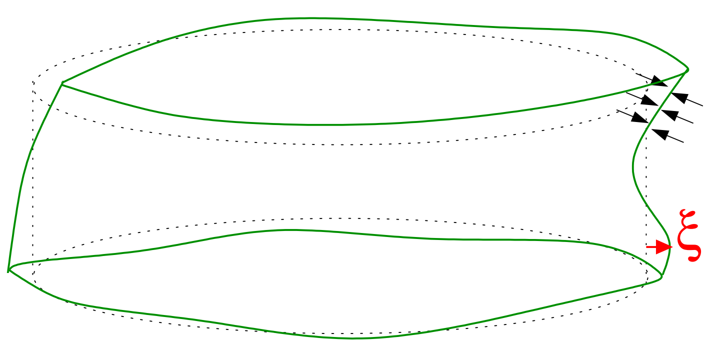
Equivalently

$$y_2'' + \left[\frac{\mathcal{F}_{11}+\mathcal{F}_{22}}{\mathcal{D}} + \frac{\mathcal{F}_{21}}{\mathcal{D}}\left(\frac{\mathcal{D}}{\mathcal{F}_{21}}\right)'\right] y_2' + \left[\frac{\mathcal{F}_{11}\mathcal{F}_{22}-\mathcal{F}_{12}\mathcal{F}_{21}}{\mathcal{D}^2} + \frac{\mathcal{F}_{21}}{\mathcal{D}}\left(\frac{\mathcal{F}_{22}}{\mathcal{F}_{21}}\right)'\right] y_2 = 0\,,$$

which for uniform flows with $V_{0\phi}=0$, $B_{0\phi}=0$, reduces to Bessel.

Eigenvalue problem





- solve the problem inside the jet (attention to regularity condition on the axis)

- similarly in the environment (solution vanishes at ∞)

- Match the solutions at r_j:
$[\![y_1]\!] = 0\,, [\![y_2]\!] = 0 \longrightarrow$
dispersion relation
$\star$ spatial approach: $\omega = \Re\omega$ and
$\Re k = \Re k(\omega), \Im k = \Im k(\omega)$
$Q = Q_0(\varpi) + Q_1(\varpi)e^{-\Im kz}e^{i(m\phi+\Re kz-\omega t)}$
$\star$ temporal approach: $k = \Re k$ and
$\Re\omega = \Re\omega(k), \Im\omega = \Im\omega(k)$
$Q = Q_0(\varpi) + Q_1(\varpi)e^{\Im\omega t}e^{i(m\phi+kz-\Re\omega t)}$

Unperturbed jet solutions

Try to mimic the Komissarov et al simulation results
(for AGN and GRB jets)

- cold, nonrotating jet

$$\boldsymbol{V}_0 = V_0(\varpi)\hat{z}\,, \quad \gamma_0 = \gamma_0(\varpi) = (1 - V_0^2)^{-1/2}\,,$$

$$\boldsymbol{B}_0 = B_{0z}(\varpi)\hat{z} + B_{0\phi}(\varpi)\hat{\phi}\,, \quad \boldsymbol{E}_0 = V_0 B_{0\phi}\hat{\varpi}\,,$$
$$\rho_{00} = \rho_{00}(\varpi)\,, \quad \xi_0 = 1\,.$$

- Equilibrium condition ("force-free")

$$\frac{B_{0\phi}^2/\gamma_0^2}{\varpi} + \frac{d}{d\varpi}\left(\frac{B_{0z}^2 + B_{0\phi}^2/\gamma_0^2}{2}\right) = 0\,,$$

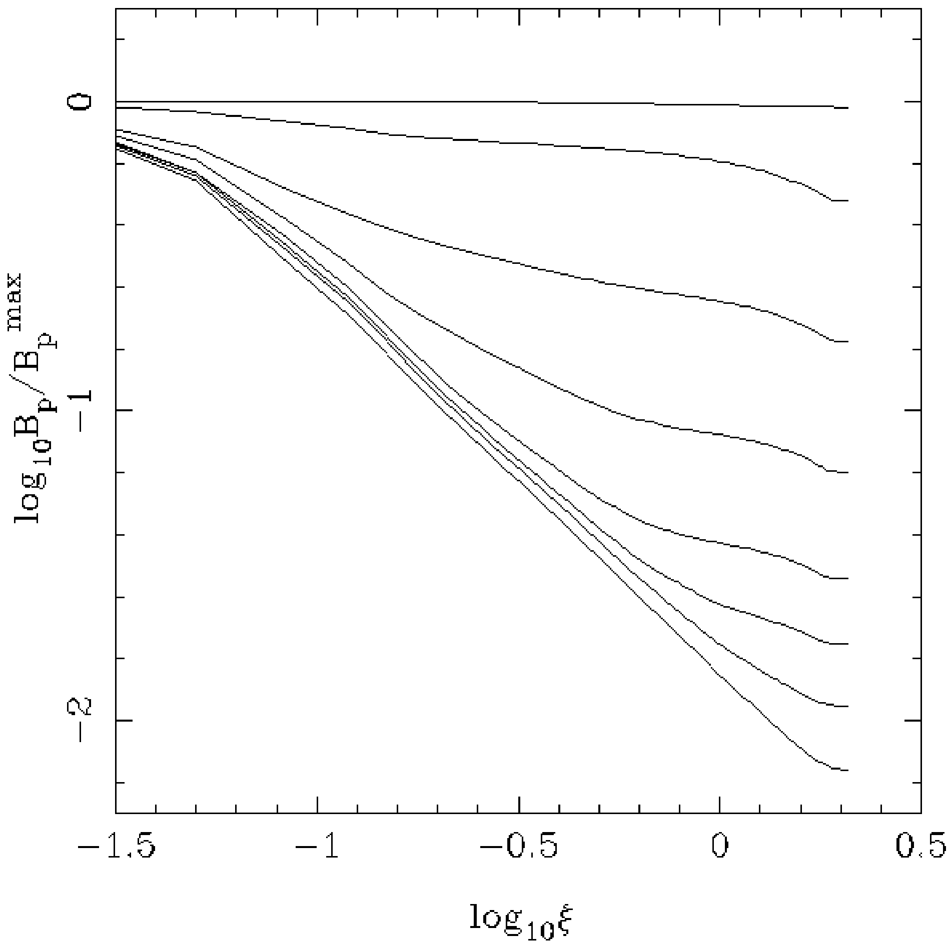
relates B_{0z} with $B_{0\phi}/\gamma_0$.

A cold, nonrotating solution:

$$B_{0z}=\frac{B_j}{\left[1+(\varpi/\varpi_0)^2\right]^{\zeta}}\,,\qquad B_{0\phi}=-\gamma_0 B_{0z}\sqrt{\frac{\left[1+(\varpi/\varpi_0)^2\right]^{2\zeta}-1-2\zeta(\varpi/\varpi_0)^2}{(2\zeta-1)(\varpi/\varpi_0)^2}}\,.$$

ϖ_0 , ζ free parameters, γ_0 , ρ_{00} free functions.

- choice of ζ:



$$B_{0z}\propto\varpi^{-1.2}$$

$$\zeta=0.6$$

Formation of core crucial for the acceleration.

The bunching function $\mathcal{S} \equiv \dfrac{\overbrace{\pi\varpi^2}^{S} B_{0z}}{\int_0^{\varpi} B_{0z} \underbrace{2\pi\varpi d\varpi}_{dS}}$ is related to the acceleration efficiency $\sigma = \dfrac{1}{\frac{\mathcal{S}_f}{\mathcal{S}} - 1}$, where $\mathcal{S}_f$ integral of motion ~ 0.9.

Since $\mathcal{S} \approx 1 - \zeta$ we get $\sigma = \dfrac{1-\zeta}{\zeta - 0.1} = 0.8$.

- choice of $\gamma_0(\varpi)$:

From Ferraro's law $V_{0\phi} = \varpi\Omega + V_{0z}B_{0\phi}/B_{0z}$, where Ω integral of motion, we get $-B_{0\phi}/B_{0z} \approx \varpi\Omega/V_{0z}$, or, $-B_{0\phi}/B_{0z} \approx \varpi/\varpi_{\rm LC}$.

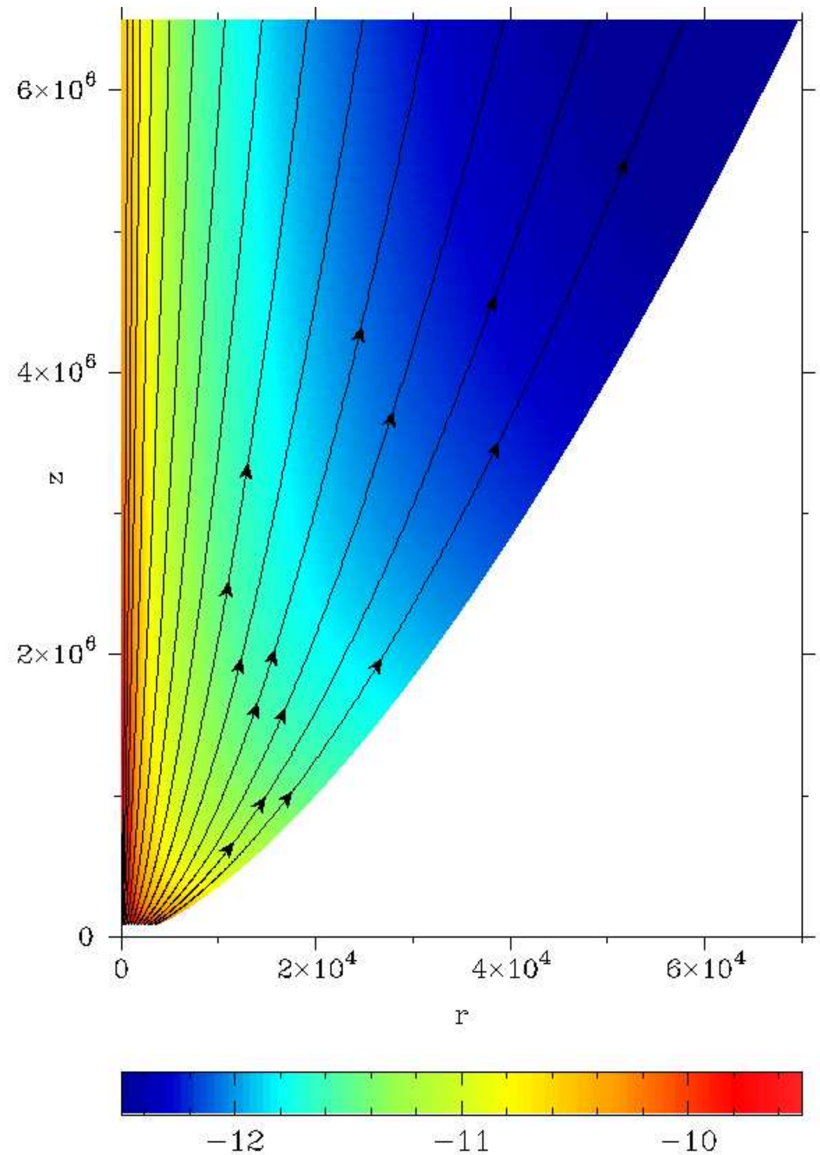
For a BH-jet $-\dfrac{B_{0\phi}}{B_{0z}} \approx 150 \left(\dfrac{r_j}{10^{16}{\rm cm}}\right)\left(\dfrac{M}{10^8 M_\odot}\right)^{-1}$

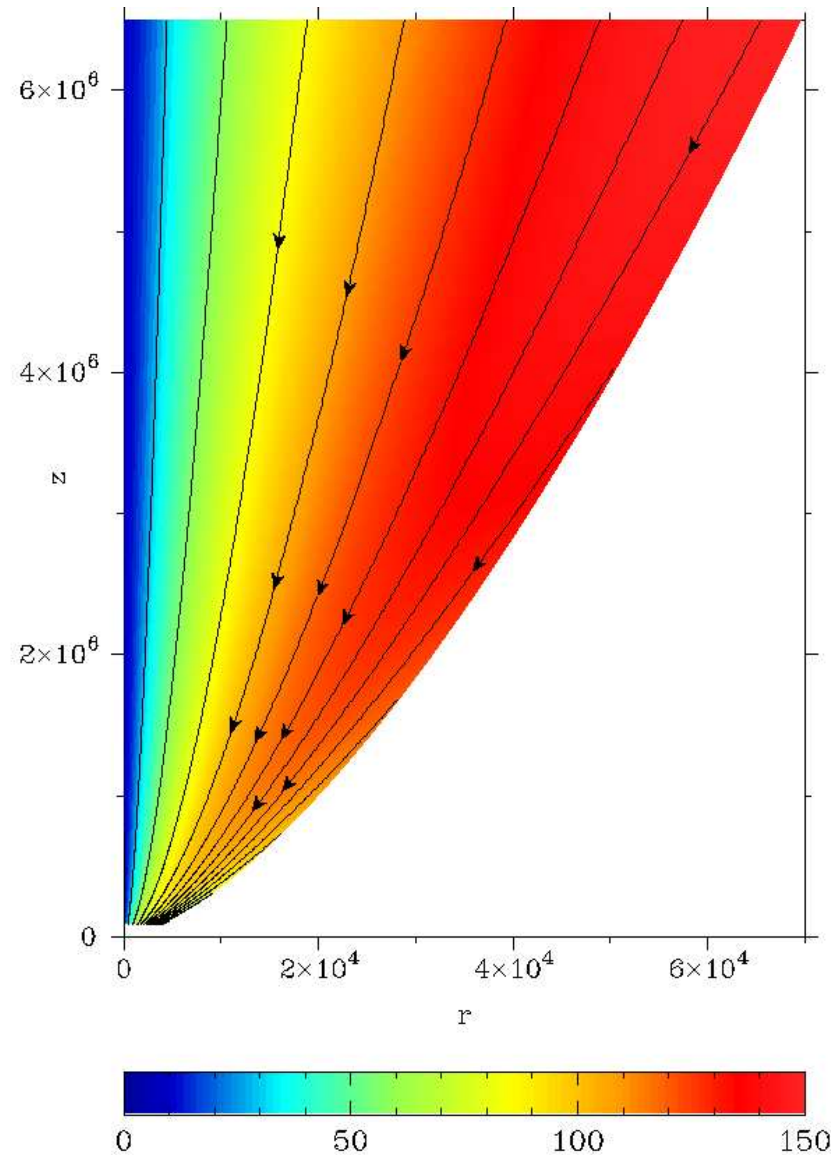
For a disk-jet $\dfrac{|B_{0\phi}|}{B_{0z}} \approx 20 \left(\dfrac{r_j}{10^{16}{\rm cm}}\right)\left(\dfrac{r_0}{10GM/c^2}\right)^{-3/2}\left(\dfrac{M}{10^8 M_\odot}\right)^{-1}$

For the given expressions of $B_{0\phi}/\gamma_0$, B_{0z},
$\gamma_0 = \sqrt{1 + \varpi_0^2\Omega^2 \frac{(2\zeta-1)(\varpi/\varpi_0)^4}{\left[1+(\varpi/\varpi_0)^2\right]^{2\zeta}-1-2\zeta(\varpi/\varpi_0)^2}}$.

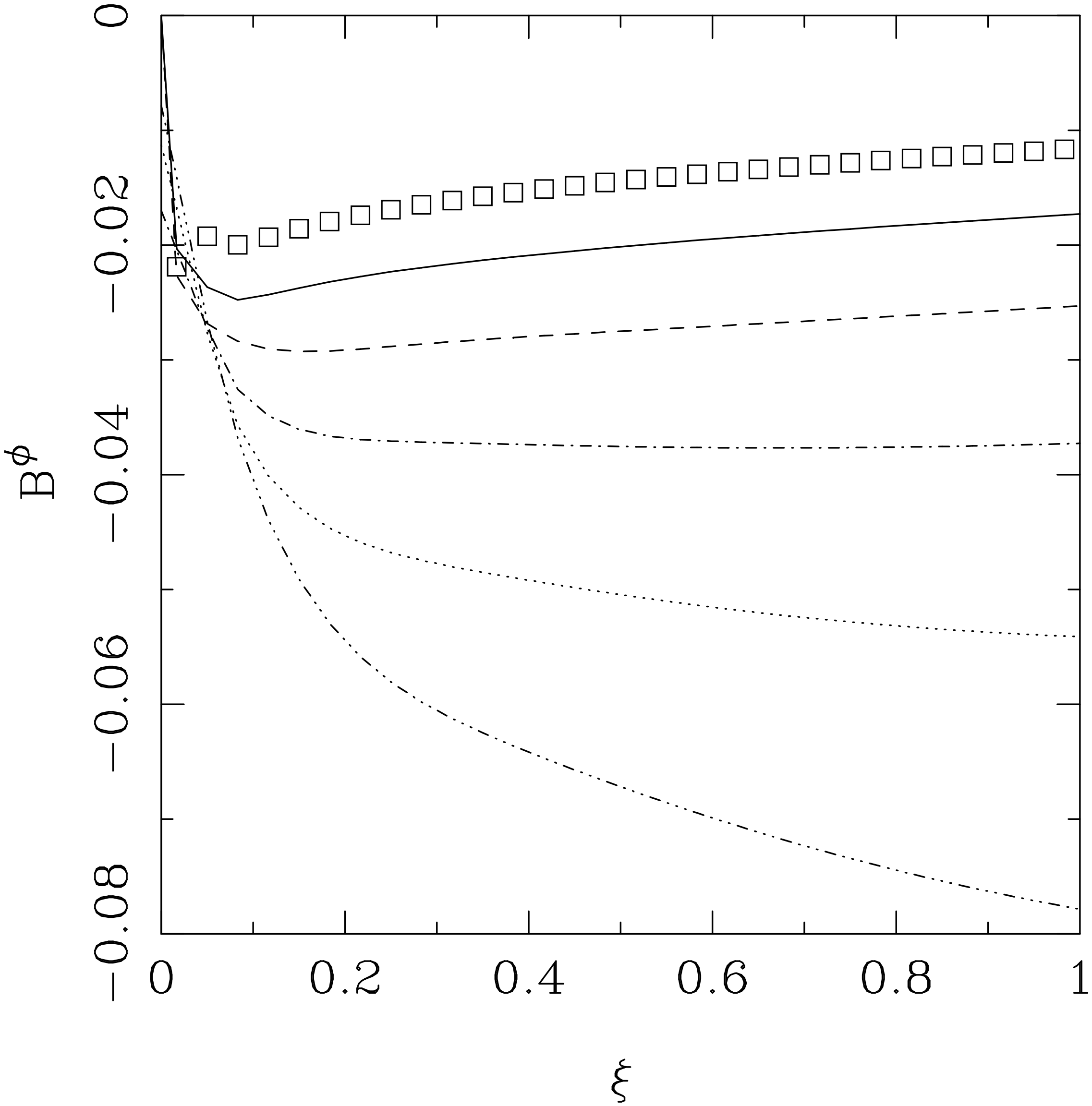
On the axis $\left.\dfrac{\gamma_0 V_0}{\Omega}\right|_{axis} = \dfrac{\varpi_0}{\sqrt{\zeta}}$ (gives $\Omega|_{axis}$ for given γ_{0axis}, ϖ_0).

The choice of ϖ_0, $\Omega(\varpi)$ controls the pitch $B_{0\phi}/(\varpi B_{0z})$, and the values of γ_0 on the axis and the jet surface.

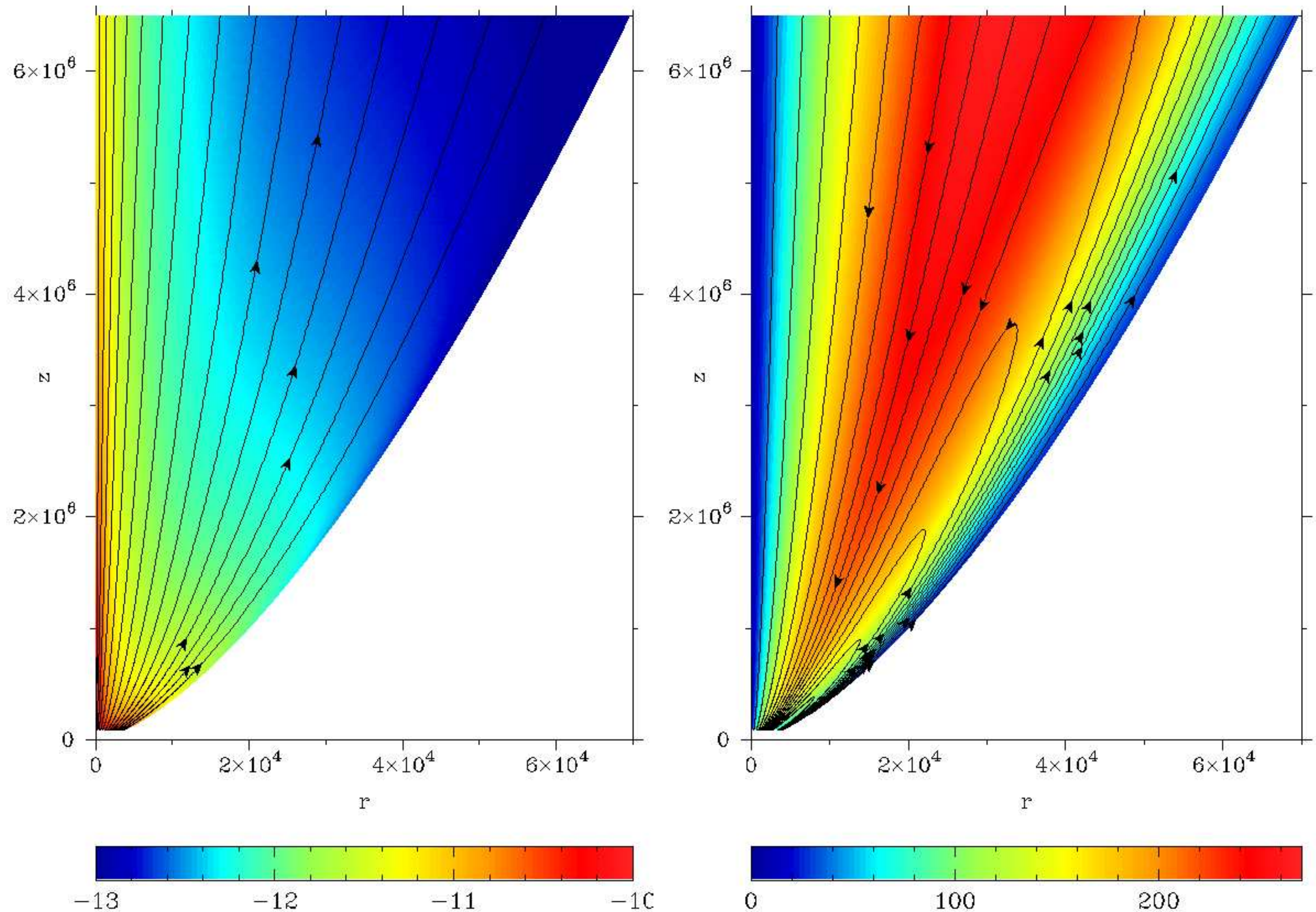




left: density/field lines, right: Lorentz factor/current lines (jet boundary $z \propto r^{1.5}$)
Uniform rotation $\rightarrow \gamma$ increases with r



0
−0.02
−0.04
−0.06
−0.08
B^ϕ
0
0.2
0.4
0.6
0.8
1
ξ



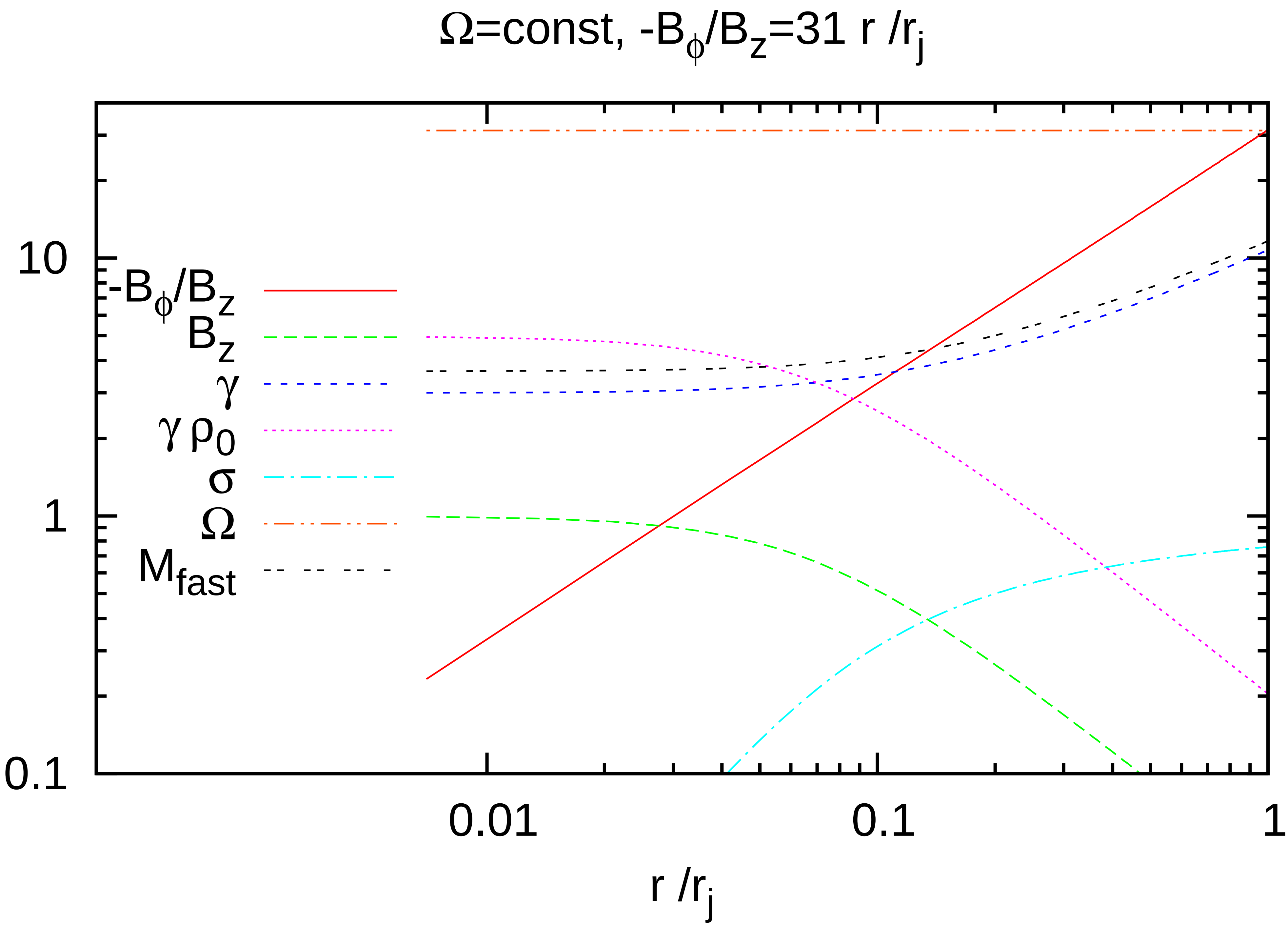
Differential rotation → slow envelope and faster decrease of B_ϕ

• choice of $\rho_{00}(\varpi)$:

This comes from the mass-to-magnetic flux ratio integral $\frac{\gamma_0 \rho_{00} V_0}{B_{0z}}$, which is assumed constant in the simulations. So $\rho_{00} \propto B_{0z}/\gamma_0$.
The constant of proportionality from the value of
$\sigma = \left. \frac{B_{0\phi}^2/\gamma_0^2}{\rho_{00}} \right|_{\varpi=\varpi_j}$.

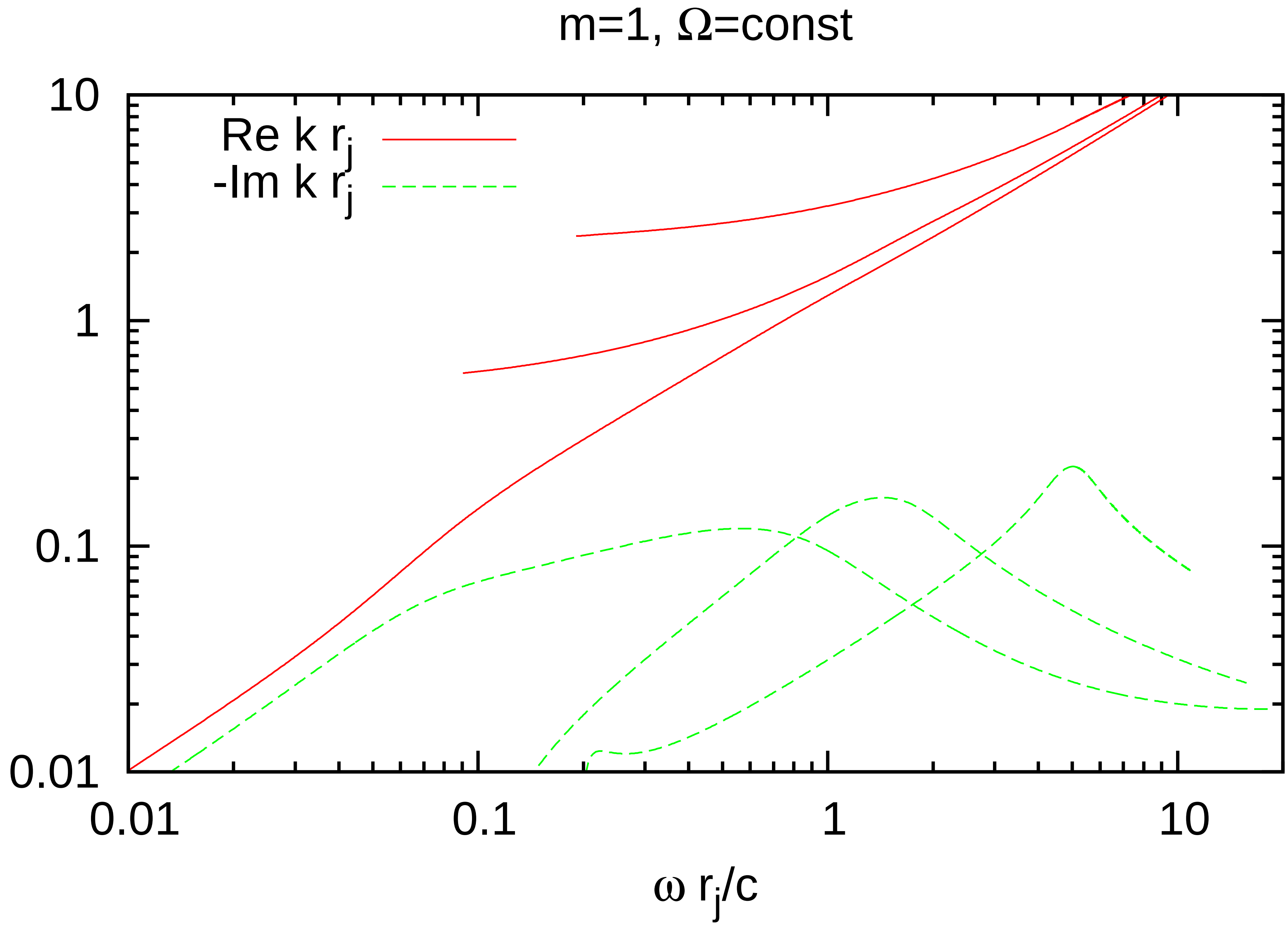
• external medium:

uniform, static, with zero $B_{0\phi}$ and $V_{0\phi} \rightarrow$ Bessel.
In all the following a thermal pressure is assumed, $\xi_e = 1.01$ (the value of ξ_e controls the density ratio).
A cold, magnetized environment gives approximately same results.



Ω=const, -B_φ/B_z=31 r /r_j
10
1
0.1
0.01
0.1
1
r /r_j
-B_φ/B_z
B_z
γ
γ ρ_0
σ
Ω
M_fast

A “surface” and multiple ”body” modes



growth length $= 1/(-\Im k) \sim r_j/0.2 = 5r_j$

nonlinear effects important after a few $10r_j$

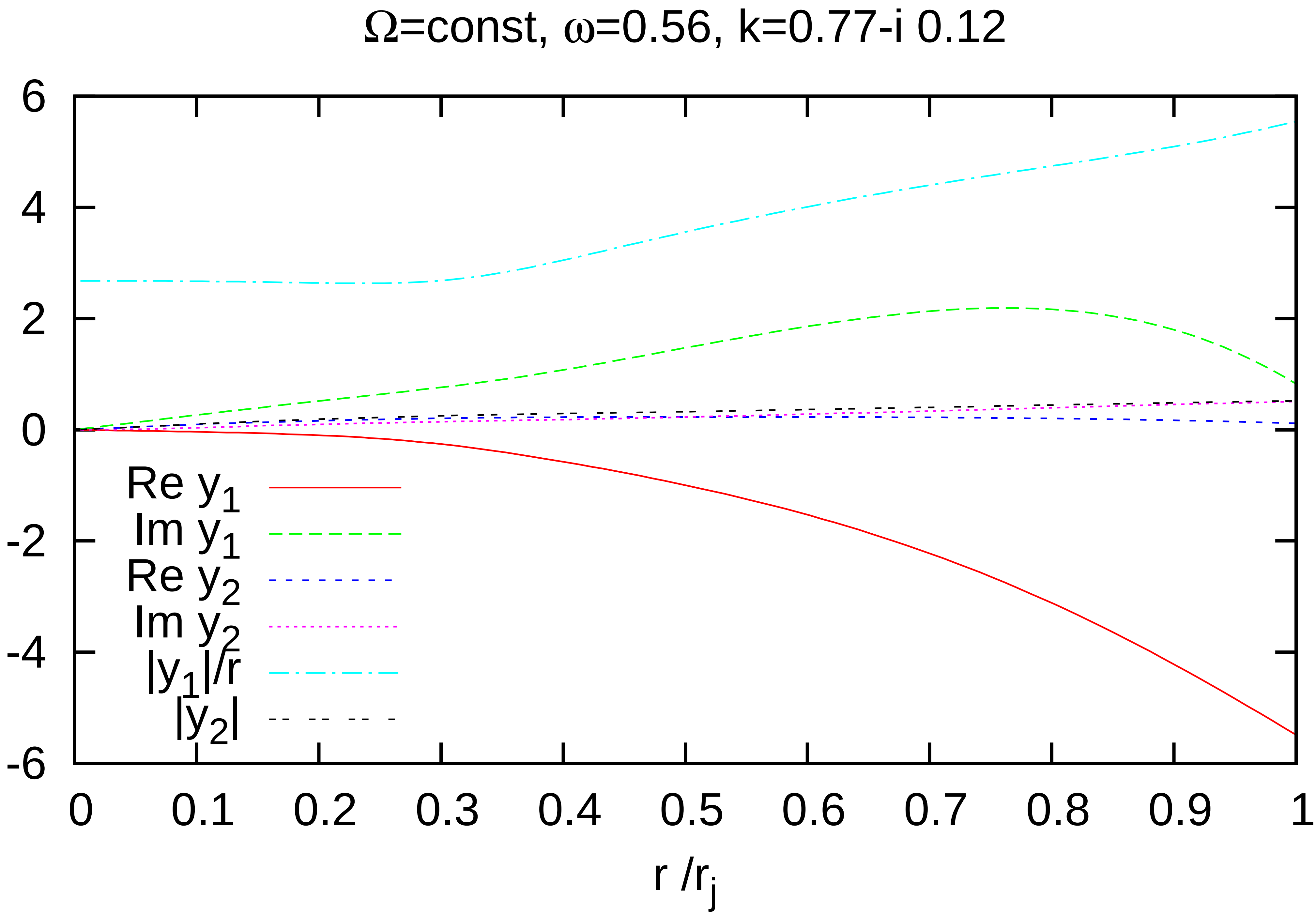
growth time $\approx$ growth length ($c = 1$)

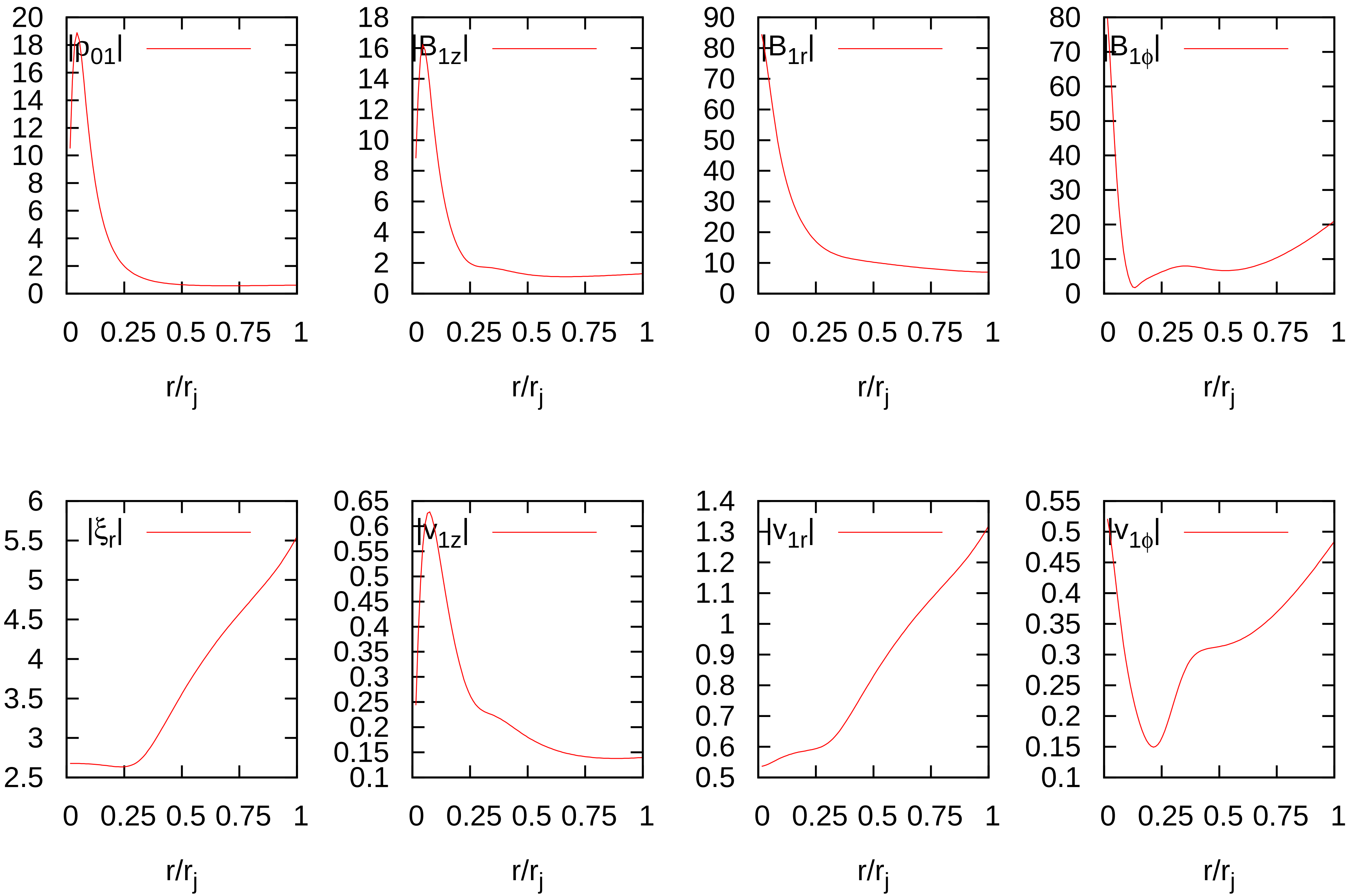
growth rate $\approx -\Im k \sim 0.2/r_j$

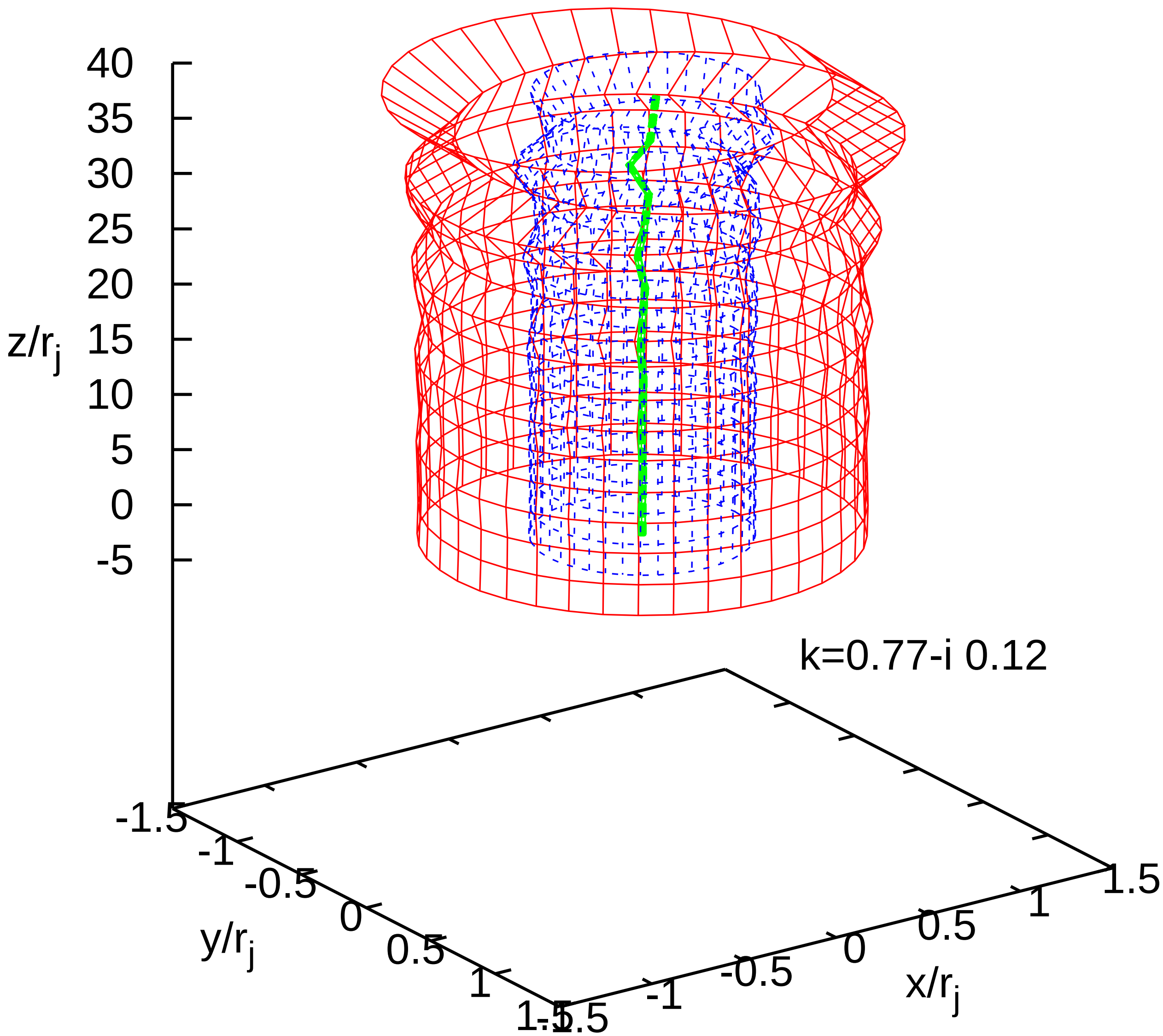
in rough agreement with nonrelativistic linear studies which predict growth rates in comoving frame $\Gamma_{\rm co} \sim \frac{v_A}{10\varpi_0}$ (Appl et al)

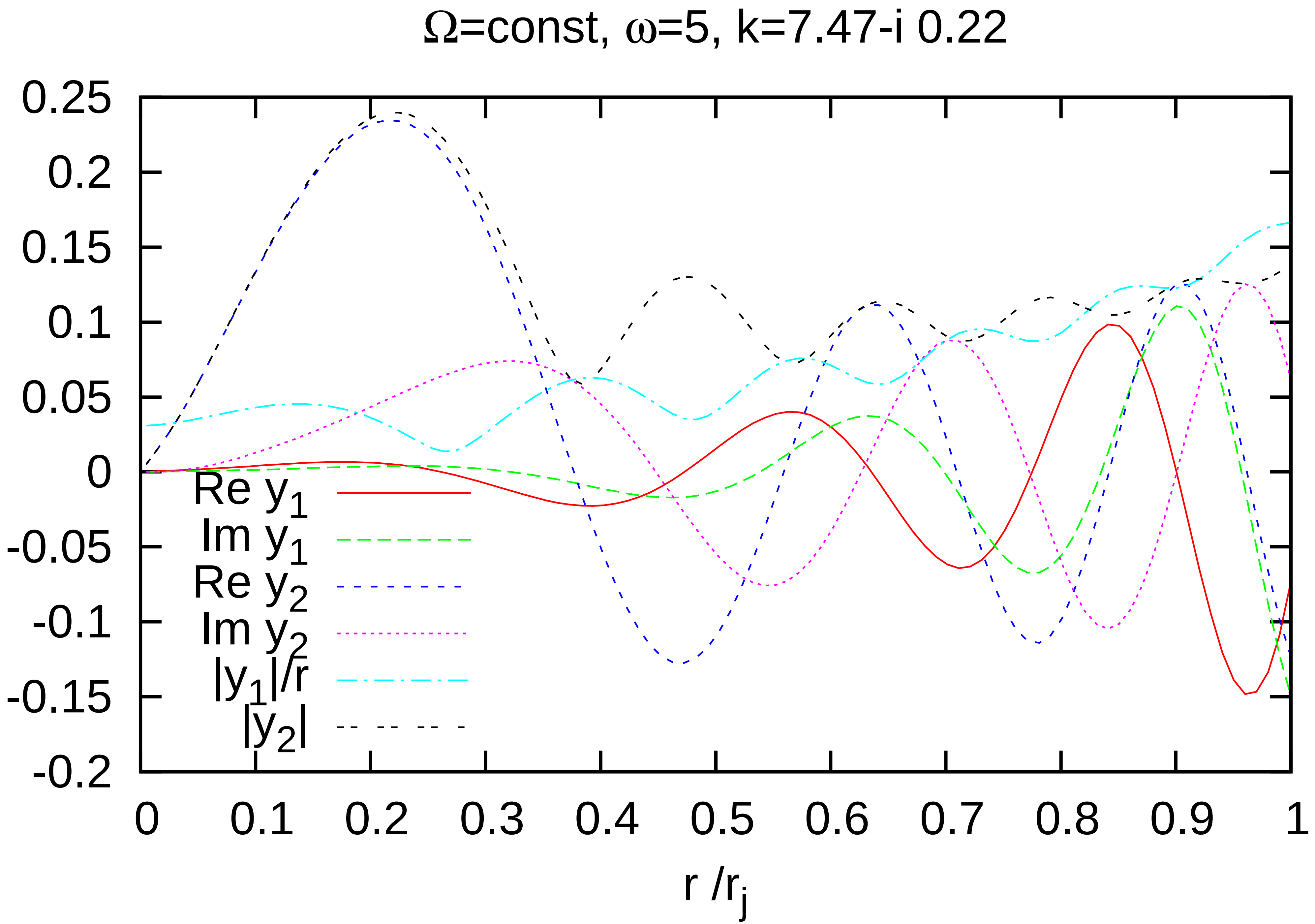
in the lab frame $\Gamma = \frac{\Gamma_{\rm co}}{< \gamma >} \approx 0.2/r_j$

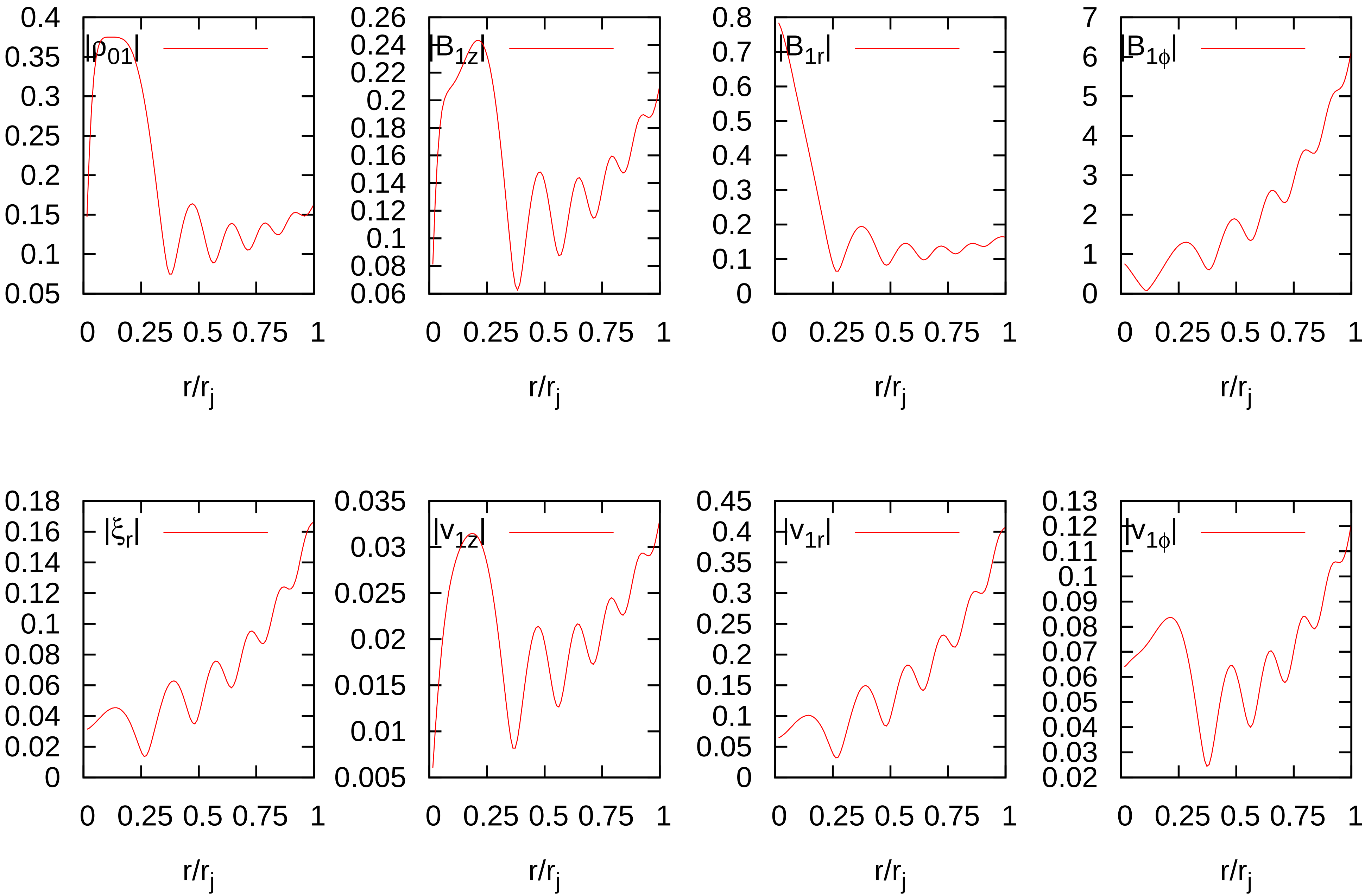
($v_A = \sqrt{\frac{\sigma}{\sigma+1}} \sim \frac{2}{3}, \quad \varpi_0 = 0.1r_j, \quad < \gamma > \sim 5$)

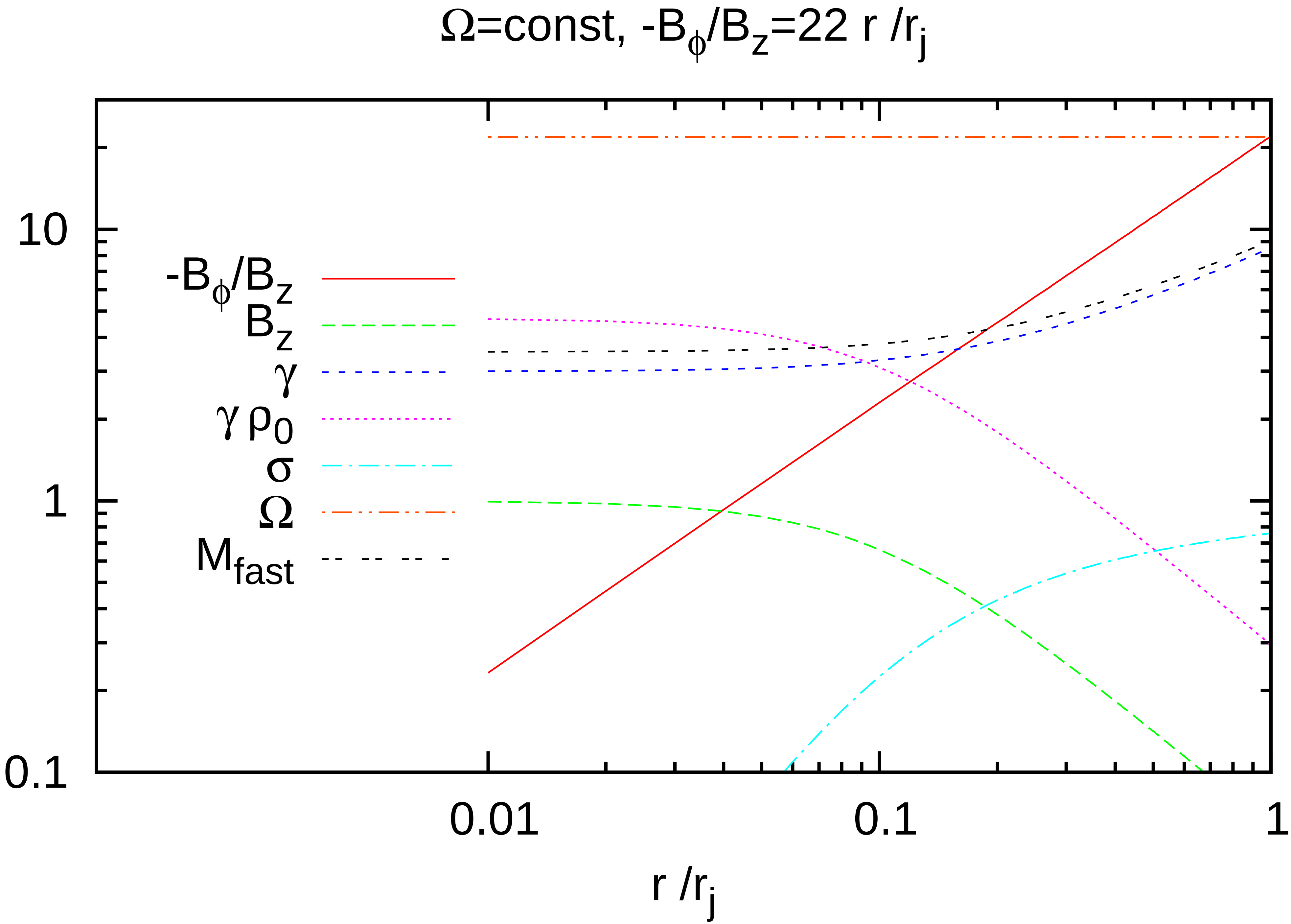

Ω=const, ω=0.56, k=0.77-i 0.12
6
4
2
0
-2
-4
-6
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
r /r_j
Re y_1
Im y_1
Re y_2
Im y_2
|y_1|/r
|y_2|


|ρ01|
|B1z|
|B1r|
|B1φ|
|ξr|
|v1z|
|v1r|
|v1φ|
r/rj

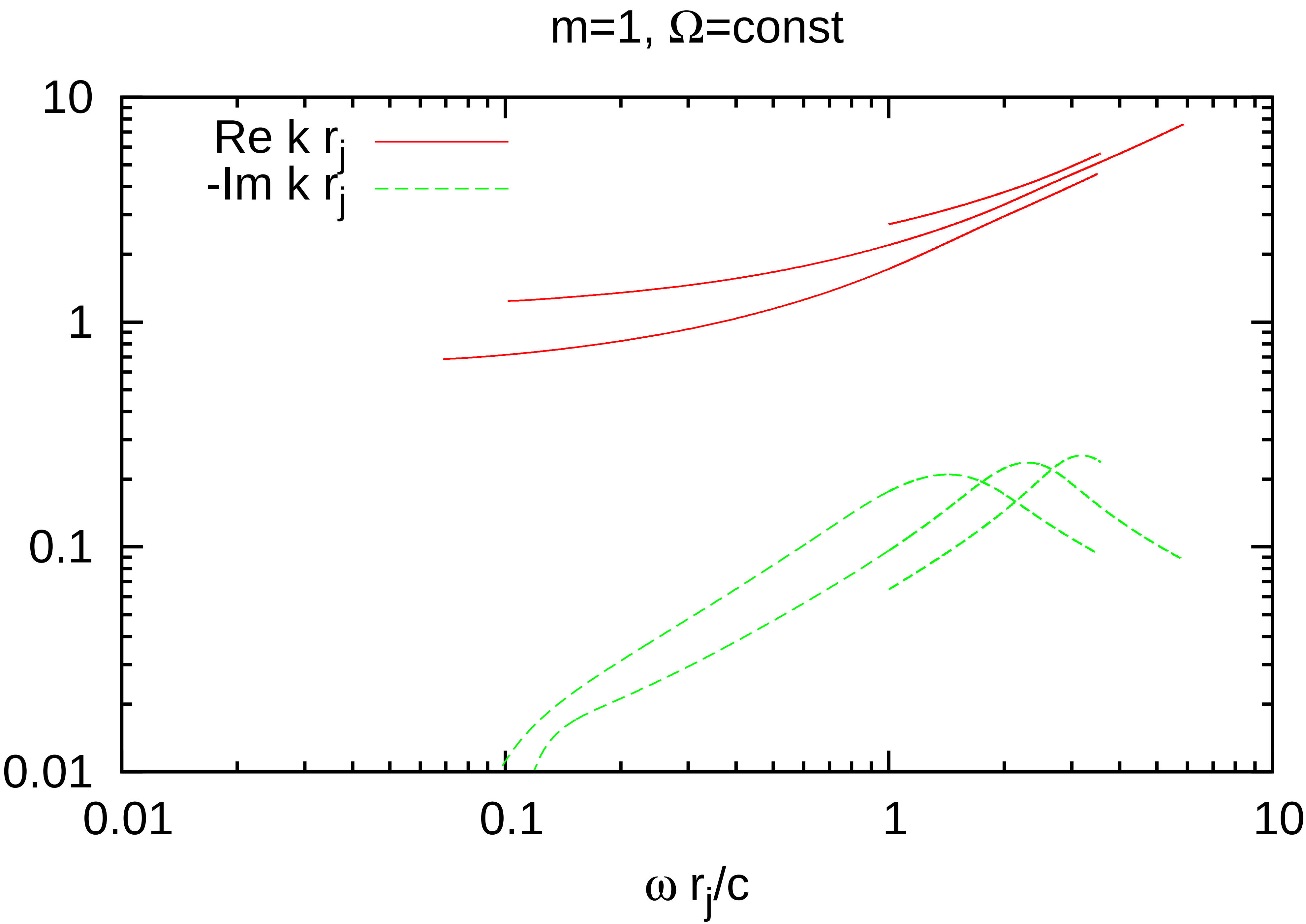

40
35
30
25
20
15
10
5
0
-5
z/r_j
k=0.77-i 0.12
-1.5
-1
-0.5
0
0.5
1
1.5
y/r_j
-1.5
-1
-0.5
0
0.5
1
1.5
x/r_j

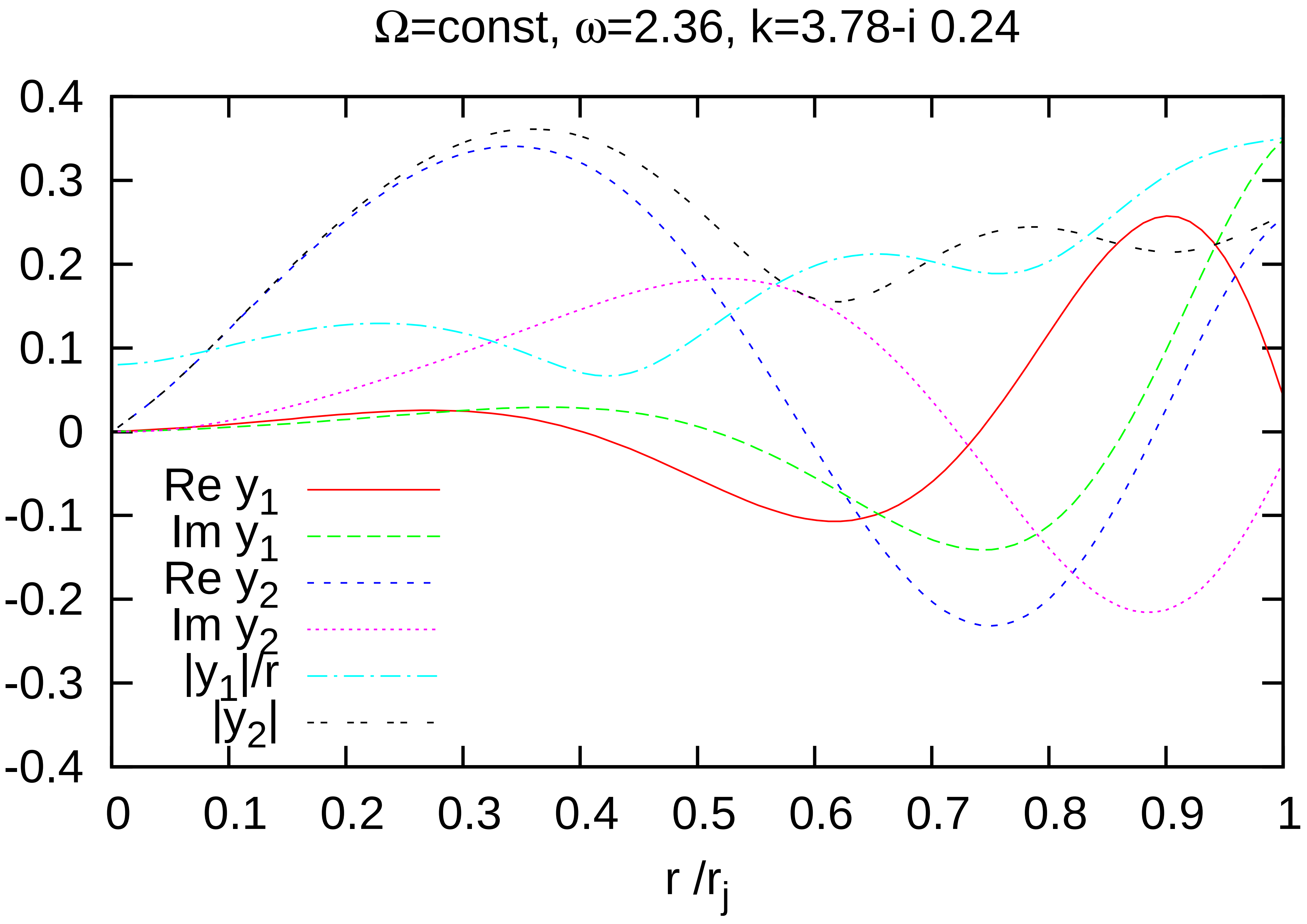

Ω=const, ω=5, k=7.47-i 0.22
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
r /r_j
Re y_1
Im y_1
Re y_2
Im y_2
|y_1|/r
|y_2|


|ρ01|
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
|B1z|
0.26
0.24
0.22
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
|B1r|
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
|B1ϕ|
7
6
5
4
3
2
1
0
0 0.25 0.5 0.75 1
r/rj
|ξr|
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
|v1z|
0.035
0.03
0.025
0.02
0.015
0.01
0.005
|v1r|
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
|v1ϕ|
0.13
0.12
0.11
0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0 0.25 0.5 0.75 1
r/rj



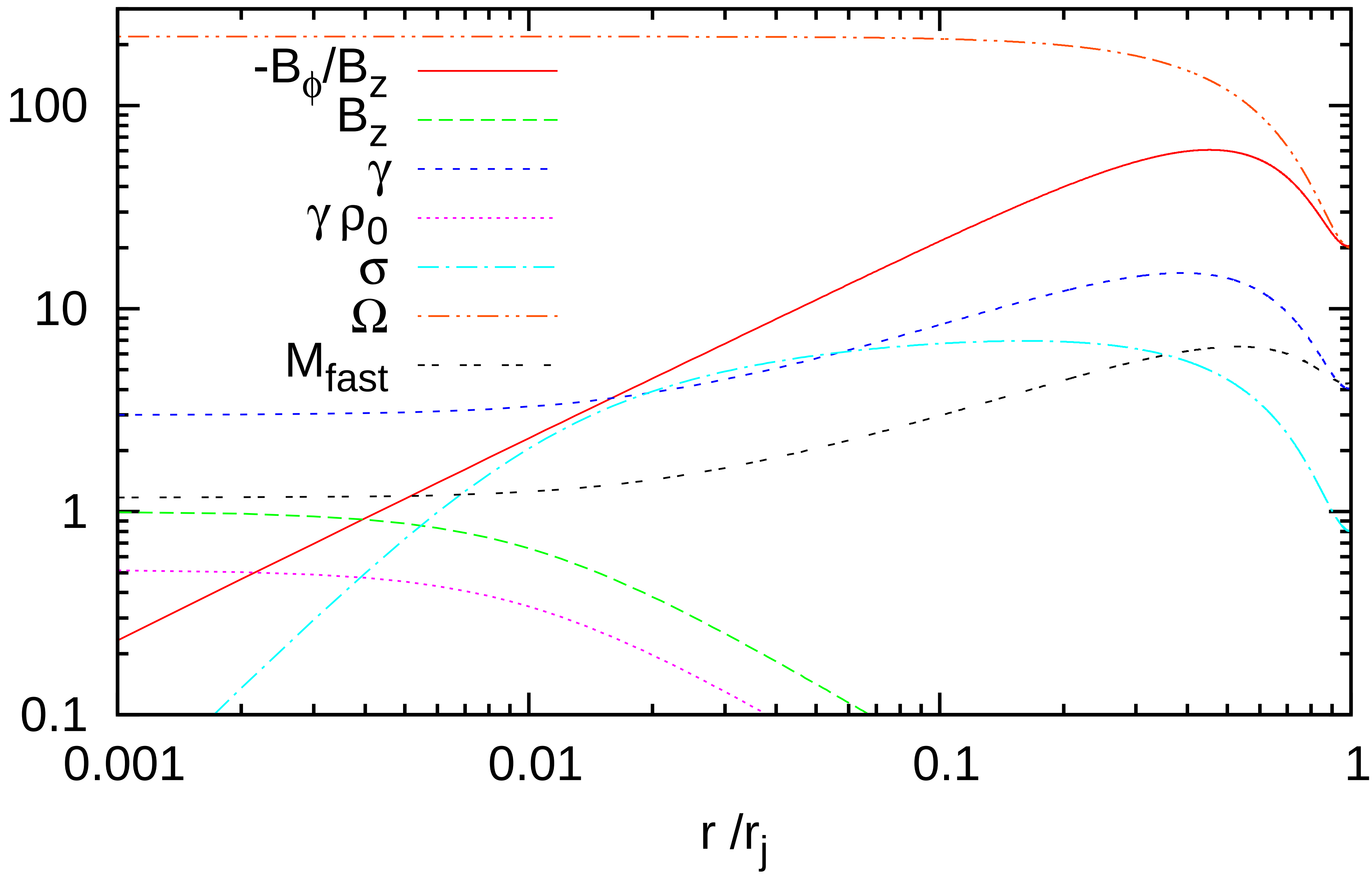
Ω=const, -Bϕ/Bz=22 r /rj
10
1
0.1
0.01
0.1
1
r /rj
-Bϕ/Bz
Bz
γ
γ ρ0
σ
Ω
Mfast


m=1, Ω=const
10
1
0.1
0.01
Re k r_j
-Im k r_j
0.01
0.1
1
10
ω r_j/c



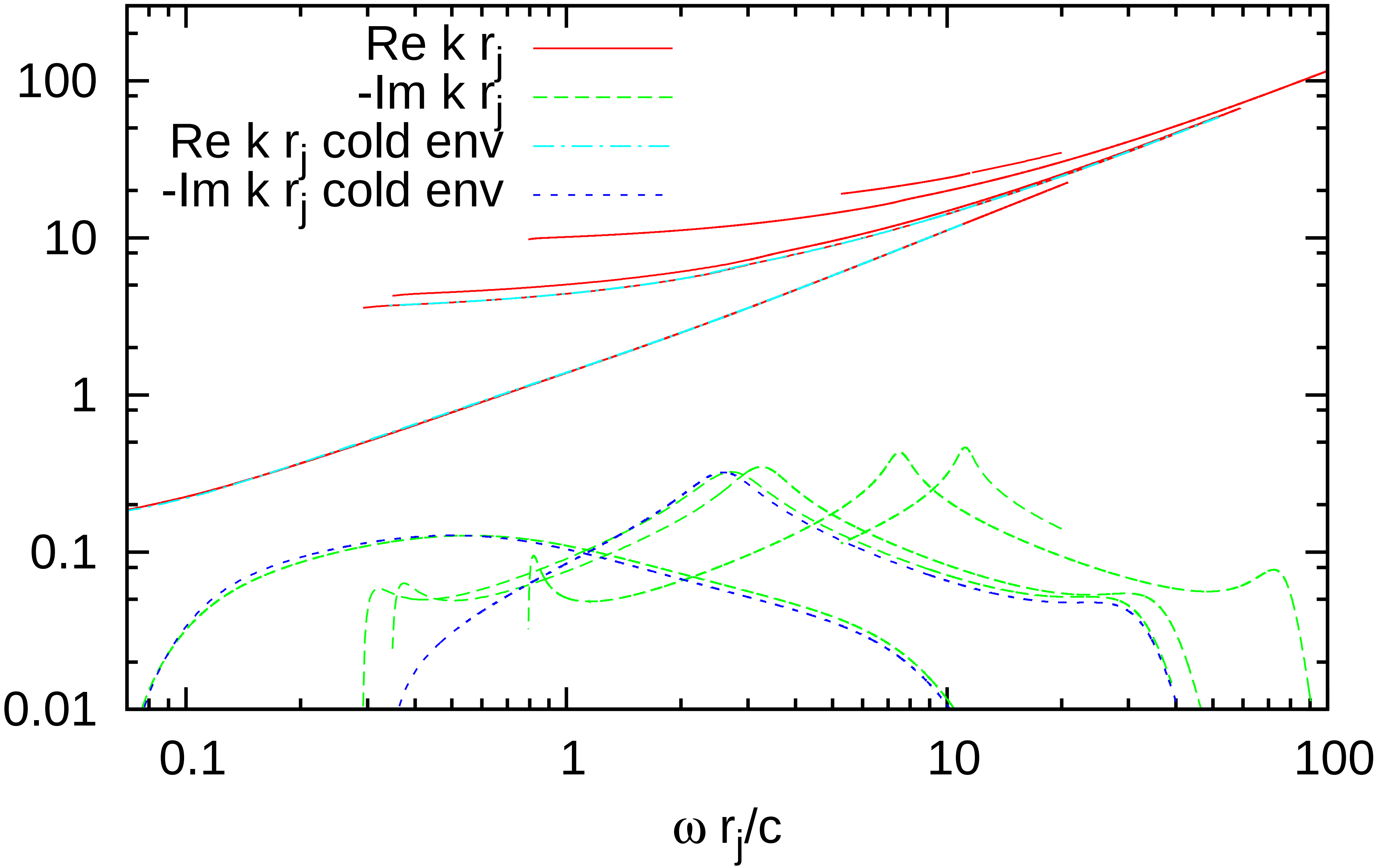
Ω=const, ω=2.36, k=3.78-i 0.24
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
r /r_j
Re y_1
Im y_1
Re y_2
Im y_2
|y_1|/r
|y_2|

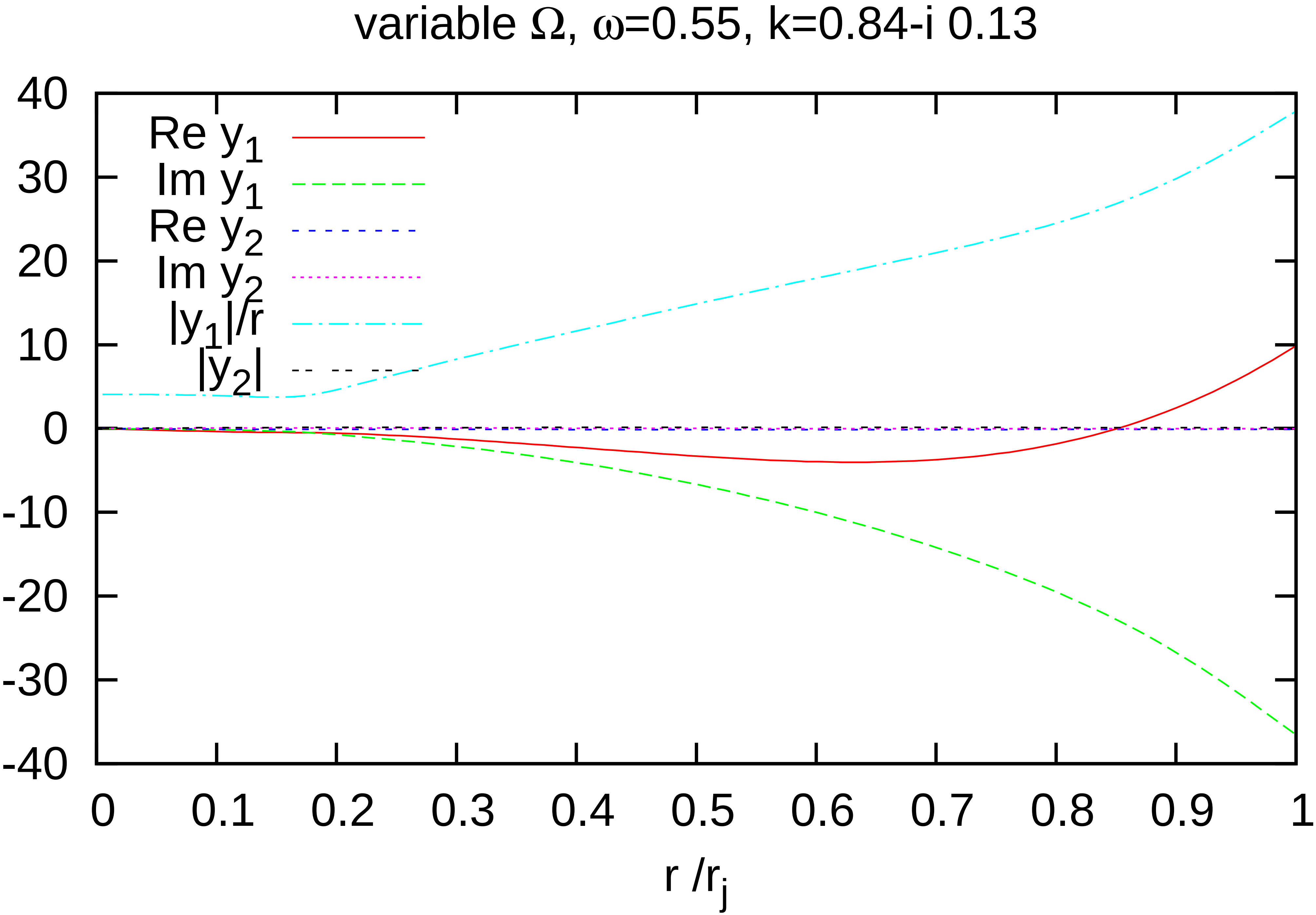
variable Ω



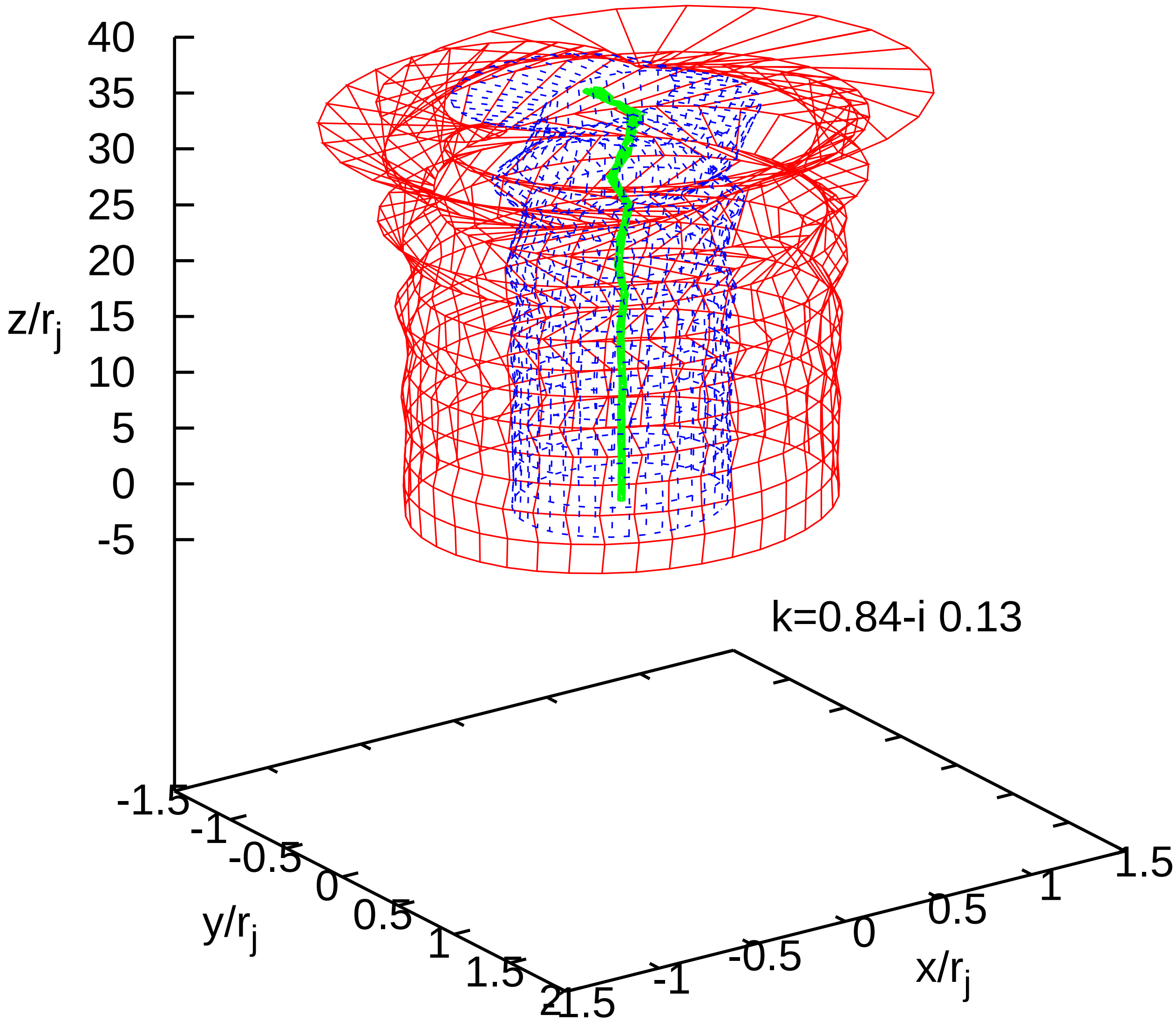
$-B_\phi/B_z$
B_z
γ
$\gamma\rho_0$
σ
Ω
M_{fast}
100
10
1
0.1
0.001
0.01
0.1
1
r/r_j

m=1, variable Ω

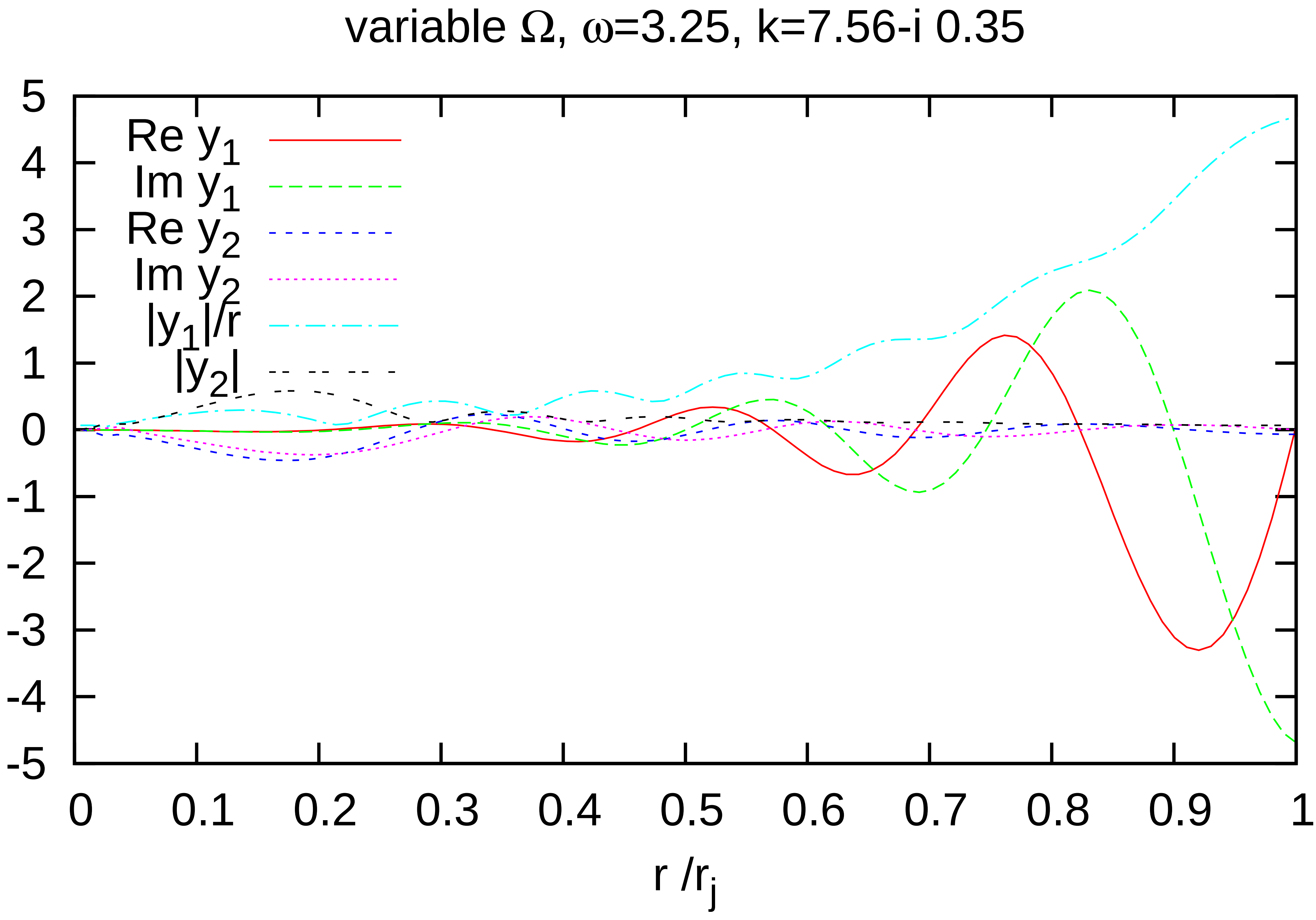




variable Ω, ω=0.55, k=0.84-i 0.13
Re y1
Im y1
Re y2
Im y2
|y1|/r
|y2|
40
30
20
10
0
-10
-20
-30
-40
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
r /rj



40
35
30
25
20
15
10
5
0
-5
z/r_j
k=0.84-i 0.13
-1.5
-1
-0.5
0
0.5
1
1.5
2
y/r_j
-1.5
-1
-0.5
0
0.5
1
1.5
x/r_j


variable Ω, ω=3.25, k=7.56-i 0.35
Re y_1
Im y_1
Re y_2
Im y_2
$|y_1|/r$
$|y_2|$
5
4
3
2
1
0
-1
-2
-3
-4
-5
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
r/r_j

Summary – Discussion – Next steps

⋆ Kink instability in principle is in action

⋆ Low $(|B_\phi|/B_z)_{\rm co}$, low σ, high γ, stabilize

⋆ The flow is significantly disrupted after a few $10r_j$
(nonlinear evolution through simulations only)

- Explore the parameter space for kink and other modes
- colder/moving environment? other jet equilibrium models?
- use the eigenstates as initial conditions in numerical studies
- during acceleration? effect of poloidal curvature ?

Acknowledgments

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European Union
European Social Fund

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EDUCATION AND LIFELONG LEARNING
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MANAGING AUTHORITY

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2007-2013
programme for development
EUROPEAN SOCIAL FUND

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