Ultrarelativistic magnetohydrodynamic jets in astrophysics

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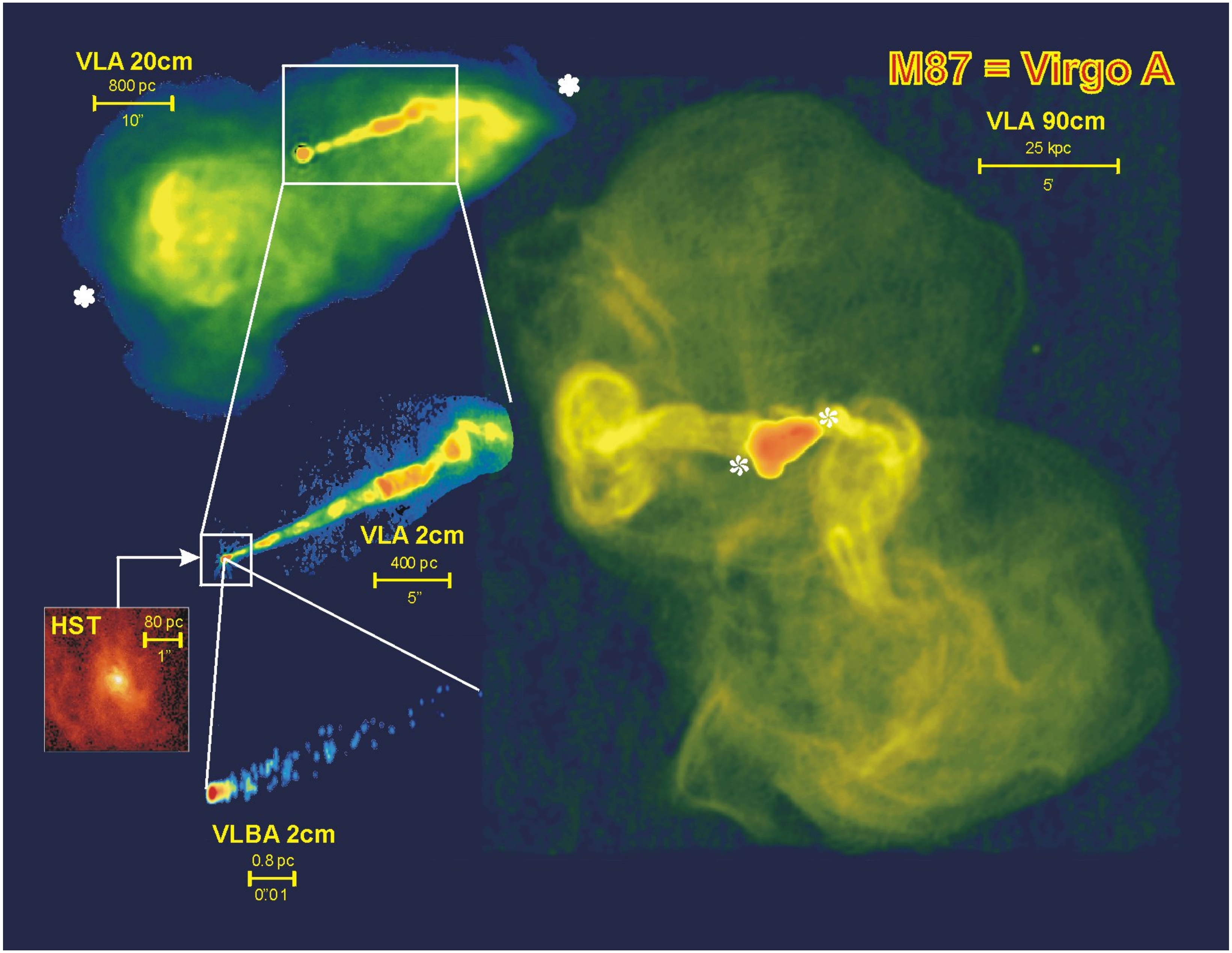
Outline

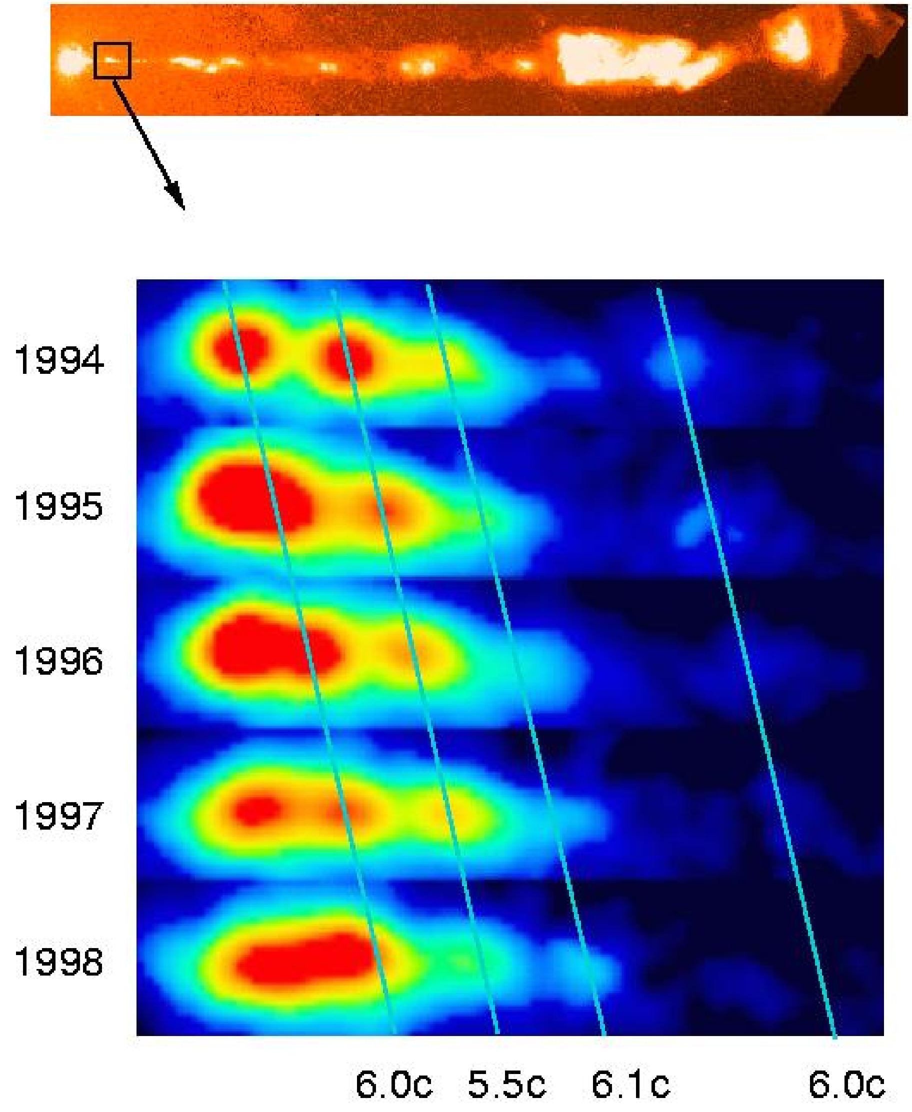
- astrophysical jets
- why magnetic driving
- bulk acceleration – jet shape – external pressure

AGN jets

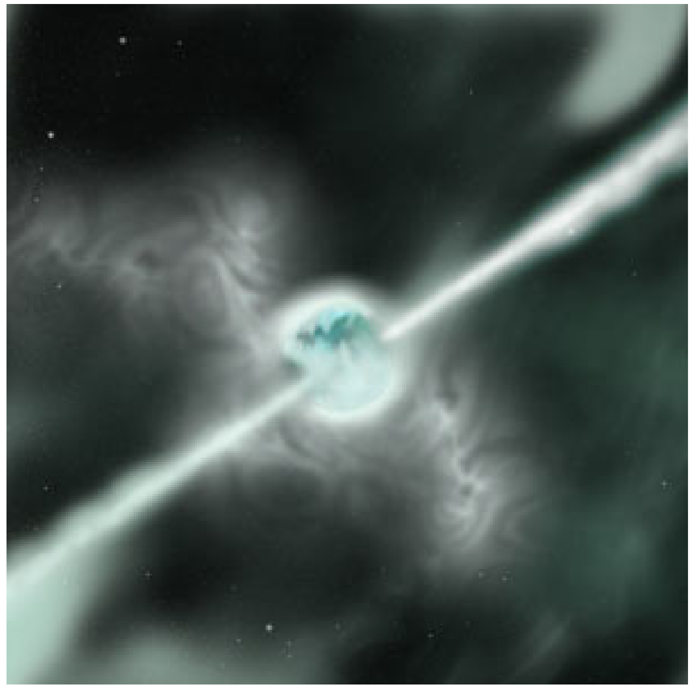


VLA 20cm
800 pc
10"
M87 = Virgo A
VLA 90cm
25 kpc
5'
VLA 2cm
400 pc
5"
HST
80 pc
1"
VLBA 2cm
0.8 pc
0".01

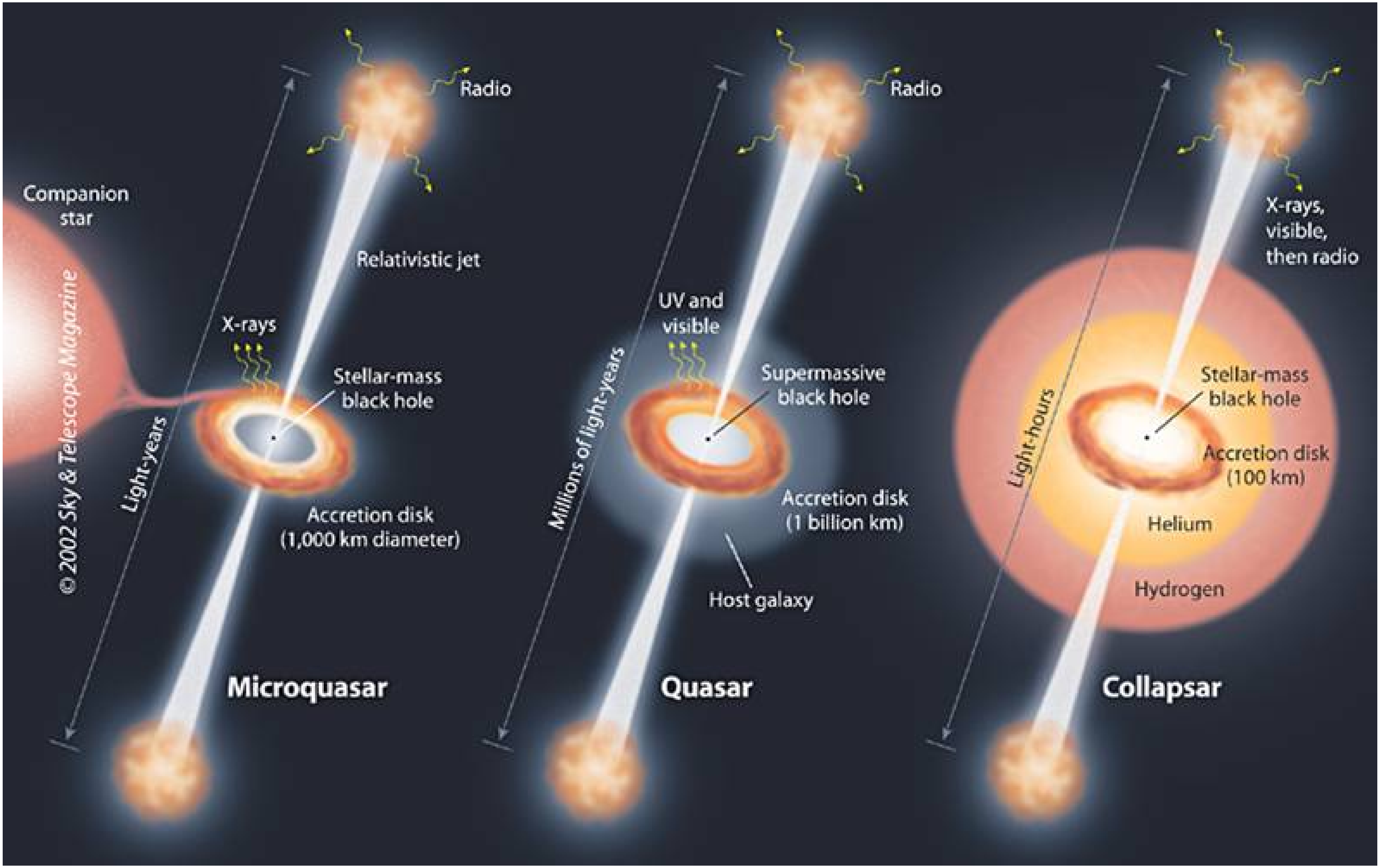
Superluminal Motion in the M87 Jet



Relativistic motion in GRB jets



the only solution to the “compactness problem”


Radio
Companion star
Relativistic jet
X-rays
Stellar-mass black hole
Light-years
© 2002 Sky & Telescope Magazine
Accretion disk (1,000 km diameter)
Microquasar
Radio
UV and visible
Supermassive black hole
Millions of light-years
Accretion disk (1 billion km)
Host galaxy
Quasar
X-rays, visible, then radio
Stellar-mass black hole
Light-hours
Accretion disk (100 km)
Helium
Hydrogen
Collapsar

Thermal driving is problematic

- requires high temperatures – corresponding thermal component of the emission in GRBs? (Zhang & Pe'er 2009)

- fast process – cannot explain pc-scale acceleration in AGN jets (lack of Compton features implies a lower limit on γ at $10^3 r_g$, Sikora et al 2005)

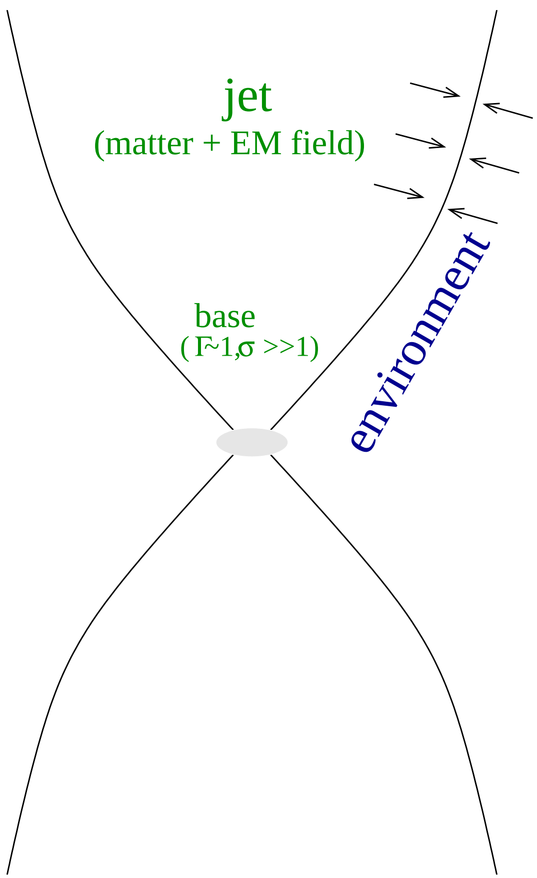
Viable alternative: magnetic driving

Two additional features:

- Extraction of "clean" energy (high energy-to-mass ratio leads to relativistic flows)

- Self-collimation

Magnetized outflows



- Extracted energy per time $\dot{\mathcal{E}}$
mainly in the form of Poynting flux
(magnetic fields tap the rotational energy
of the compact object or disk)

$$\dot{\mathcal{E}} = \frac{c}{4\pi} \underbrace{\frac{r}{r_{\rm lc}} B_p}_{E} B_\phi \times (\text{ area }) \approx \frac{c}{2} B^2 r^2$$

- Ejected mass per time $\dot{M}$

- The $\mu \equiv \dot{\mathcal{E}}/\dot{M}c^2$ gives the maximum
possible bulk Lorentz factor of the flow

- Magnetohydrodynamics:
matter (velocity, density, pressure)
+ large scale electromagnetic field

Numerical simulations

Komissarov, Vlahakis, Königl & Barkov

Assumptions:

- only jet (given boundary conditions at base)
- ideal MHD
- axisymmetry
- cold (not always, but focus on magnetic effects)
- given wall shape (avoid interaction with environment)

Input:

magnetized plasma of a given magnetization (given $\mu = \dot{\mathcal{E}}/\dot{M}c^2$) is ejected into a funnel of a given shape

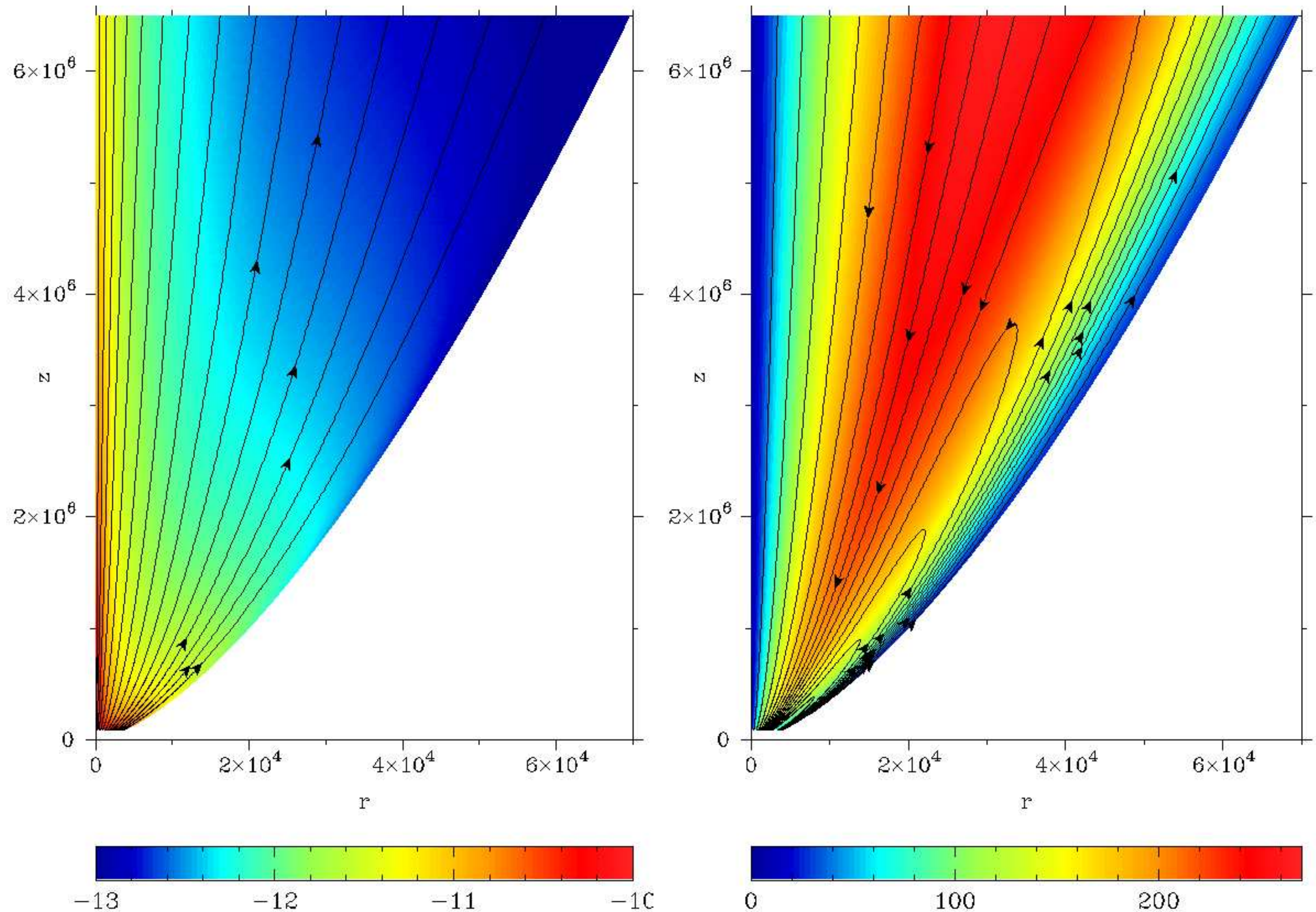
Output:

- ☞ Γ *vs* distance ? $(\mu = \dot{\mathcal{E}}/\dot{M}c^2 = \underbrace{\dot{\mathcal{E}}_{matter}/\dot{M}c^2}_{\Gamma} + \underbrace{\dot{\mathcal{E}}_{EM}/\dot{M}c^2}_{\Gamma\sigma})$

- ☞ Γ_∞ and the acceleration efficiency $\dfrac{\Gamma_\infty}{\mu} = \dfrac{\Gamma_\infty \dot{M}c^2}{\dot{\mathcal{E}}} =$?

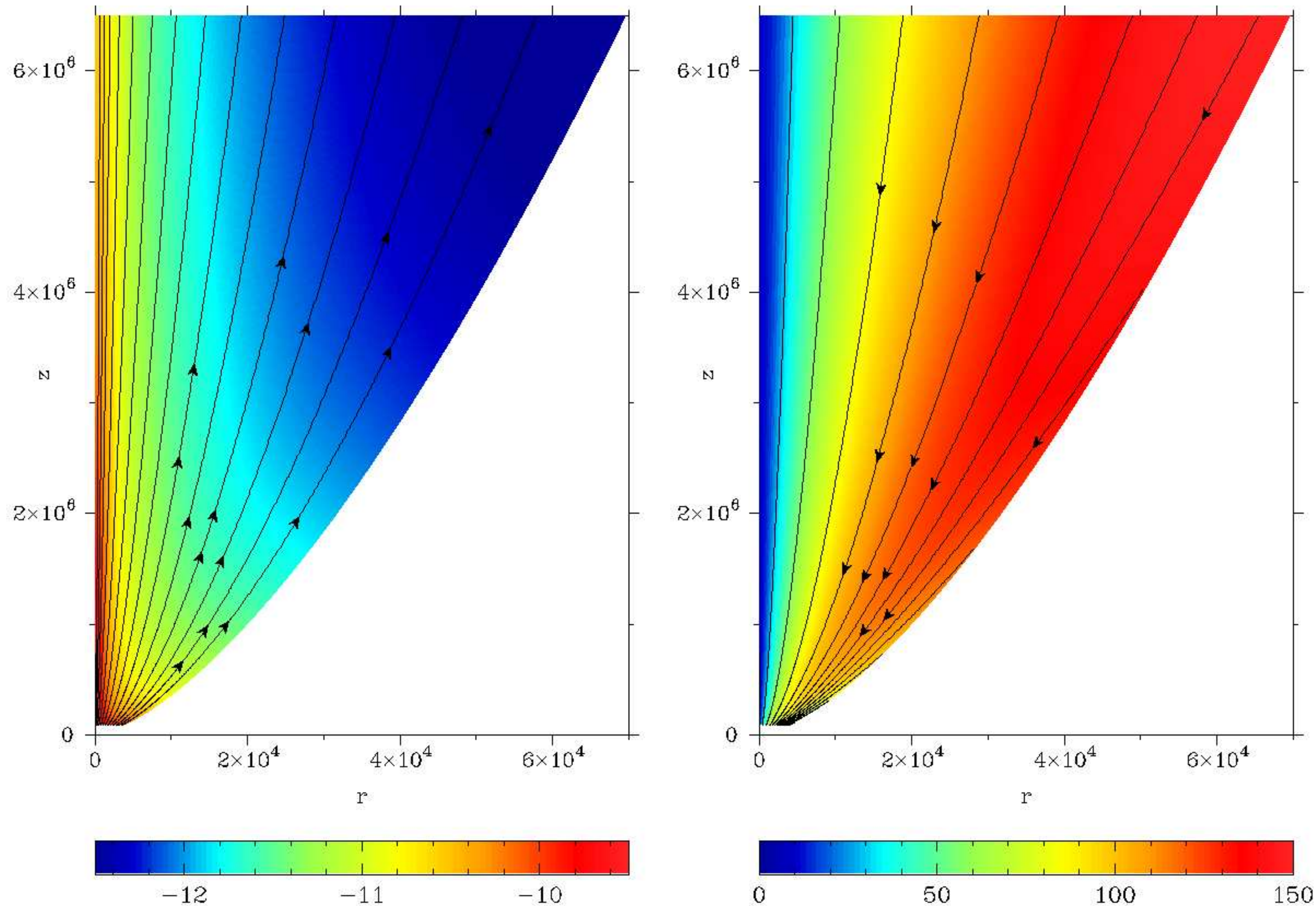
- ☞ self-collimation (formation of a cylindrical core) ?

- ☞ pressure on the wall ? ($\equiv$ pressure of the jet environment)

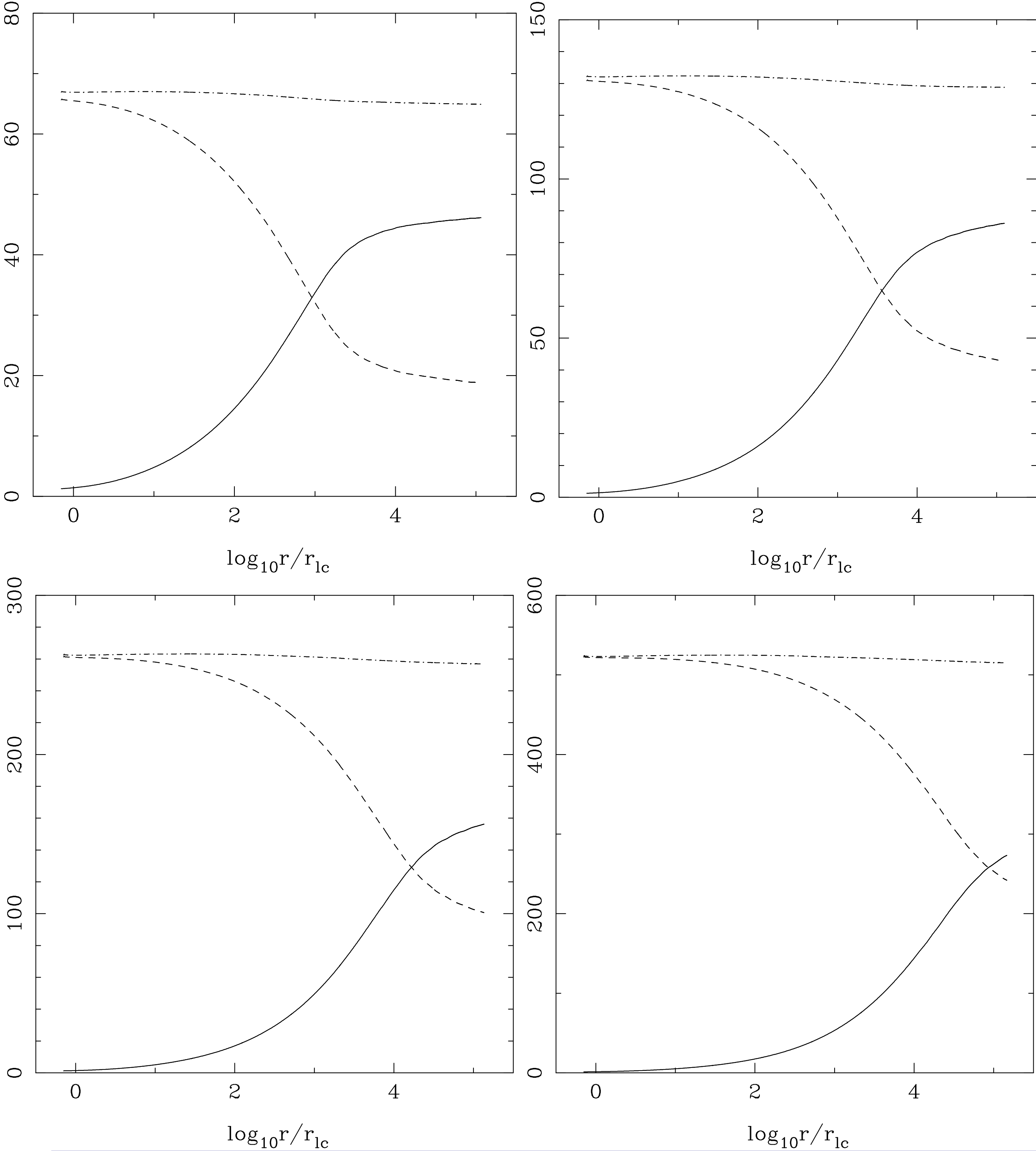


left: density/field lines, right: Lorentz factor/current lines (wall shape $z \propto r^{1.5}$)

Differential rotation $\rightarrow$ slow envelope



Uniform rotation $\rightarrow \Gamma$ increases with r



Γ (increasing),

$\Gamma\sigma$ (decreasing),

and μ

efficiency $> 50\%$

(similar to the results of Vlahakis & Königl 2003,2004; Beskin & Nokhrina 2006)

Analytical scalings

Simplifications using $\Gamma \gg 1$ and $r \gg r_{\rm lc}$ (then $v_\phi/c \ll r/r_{\rm lc}$)
(these are valid in the superfast regime)
(note that at fast $\Gamma \approx \mu^{1/3} \ll \mu$):

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☞ component of the momentum equation

$$\gamma n(\boldsymbol{V}\cdot\nabla)(\gamma w \boldsymbol{V}) = -\gamma^2 n w \nabla \ln h - \nabla p + J^0 \boldsymbol{E} + \boldsymbol{J}\times\boldsymbol{B}$$

along the flow (wind equation) $\Gamma \approx \mu - \frac{\Psi\Omega^2}{4\pi^2 k c^3}\mathcal{S}$

where the bunching function is $\mathcal{S} = \frac{\pi r^2 B_p}{\int \boldsymbol{B}_p \cdot d\boldsymbol{S}} = \frac{\pi r^2 B_p}{\Psi}$

- acceleration if B_p drops (even slightly) faster than r^{-2}
or, if separation between lines increases faster than the cylindrical distance
- crucial to solve the transfield component of the momentum equation (that controls the shape of the field and thus $\mathcal{S}$)
- role of collimation
- external pressure plays important role

☞ transfield component of the momentum equation

$$\frac{\Gamma^2 r}{\mathcal{R}} \approx \frac{\left(\frac{2I}{\Omega B_p r^2}\right)^2 r \nabla_\perp \ln \left|\frac{I}{\Gamma}\right|}{1 + \frac{w}{\rho c^2}\frac{4\pi\rho u_p^2}{B_p^2}\frac{r_{\rm lc}^2}{r^2}} - \Gamma^2 \frac{r_{\rm lc}^2}{r^2}\nabla_\perp r,$$ with $\nabla_\perp \sim 1/r$,

simplifies to $\underbrace{\frac{\Gamma^2 r}{\mathcal{R}}}_{inertia} \approx \underbrace{1}_{EM} - \underbrace{\Gamma^2 \frac{r_{\rm lc}^2}{r^2}}_{centrifugal}$

• if centrifugal negligible then $\Gamma \approx z/r$ (since $\mathcal{R}^{-1} \approx -\frac{d^2 r}{dz^2} \approx \frac{r}{z^2}$)
power-law acceleration regime
(for parabolic shapes $z \propto r^a$, Γ is a power of r)

• if inetria negligible then $\Gamma \approx r/r_{\rm lc}$ linear acceleration regime

• if electromagnetic negligible then ballistic regime

☞ role of external pressure

$p_{\rm ext} = B_{\rm co}^2/8\pi \simeq (B^{\hat{\phi}})^2/8\pi\Gamma^2 \propto 1/r^2\Gamma^2$

Assuming $p_{\rm ext} \propto z^{-\alpha_p}$ we find $\Gamma^2 \propto z^{\alpha_p}/r^2$.

Combining with the transfield $\frac{\Gamma^2 r}{\mathcal{R}} \approx 1 - \Gamma^2 \frac{r_{\rm lc}^2}{r^2}$ we find the funnel shape (we find the exponent a in $z \propto r^a$).

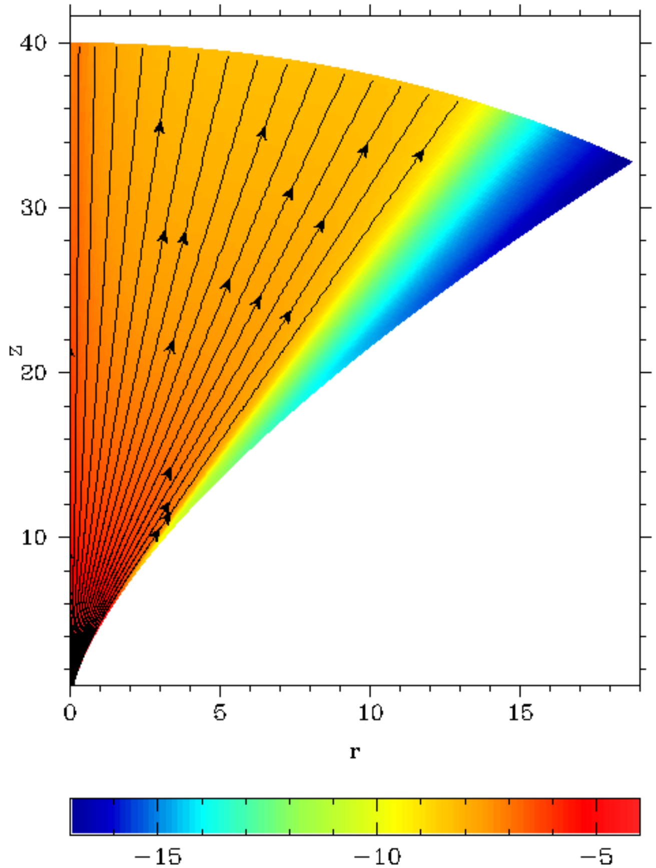
- if the pressure drops slower than z^{-2} then
  - ⋆ shape more collimated than $z \propto r^2$
  - ⋆ linear acceleration $\Gamma \propto r$
- if the pressure drops as z^{-2} then
  - ⋆ parabolic shape $z \propto r^a$ with $1 < a \leq 2$
  - ⋆ first $\Gamma \propto r$ and then power-law acceleration $\Gamma \sim z/r \propto r^{a-1}$
- if pressure drops faster than z^{-2} then
  - ⋆ conical shape
  - ⋆ linear acceleration $\Gamma \propto r$ (small efficiency)

The collimation-acceleration paradigm

- $\mathcal{S} \downarrow$ through stronger collimation of the inner flux surfaces relative to the outer ones
- formation of cylindrical core
- analytical scalings using $\nabla_\perp \sim 1/r$

Other ways to make $\mathcal{S} \downarrow$?

- low $p_{\rm ext}$ in the sub-fast regime doesn't work

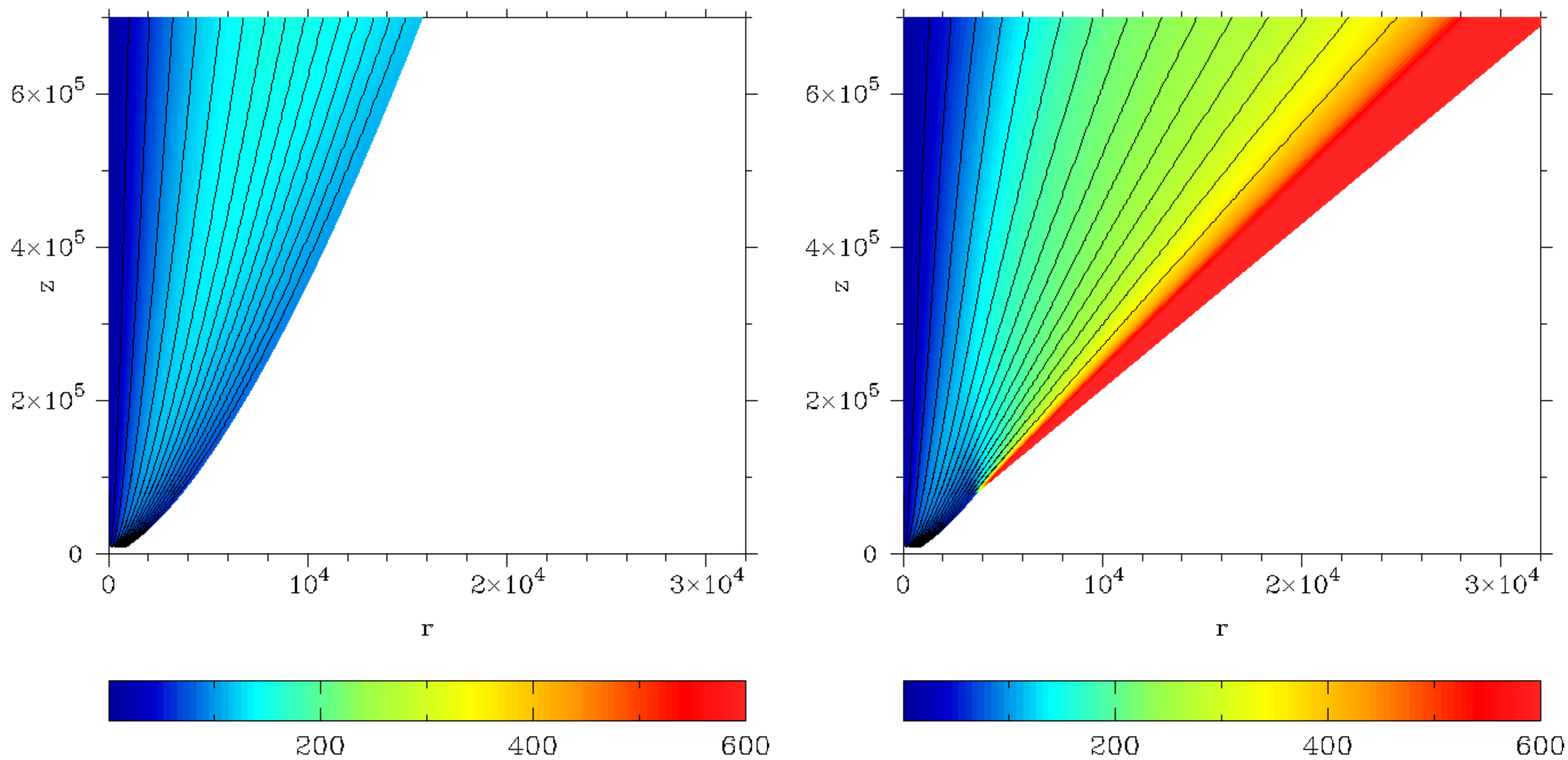


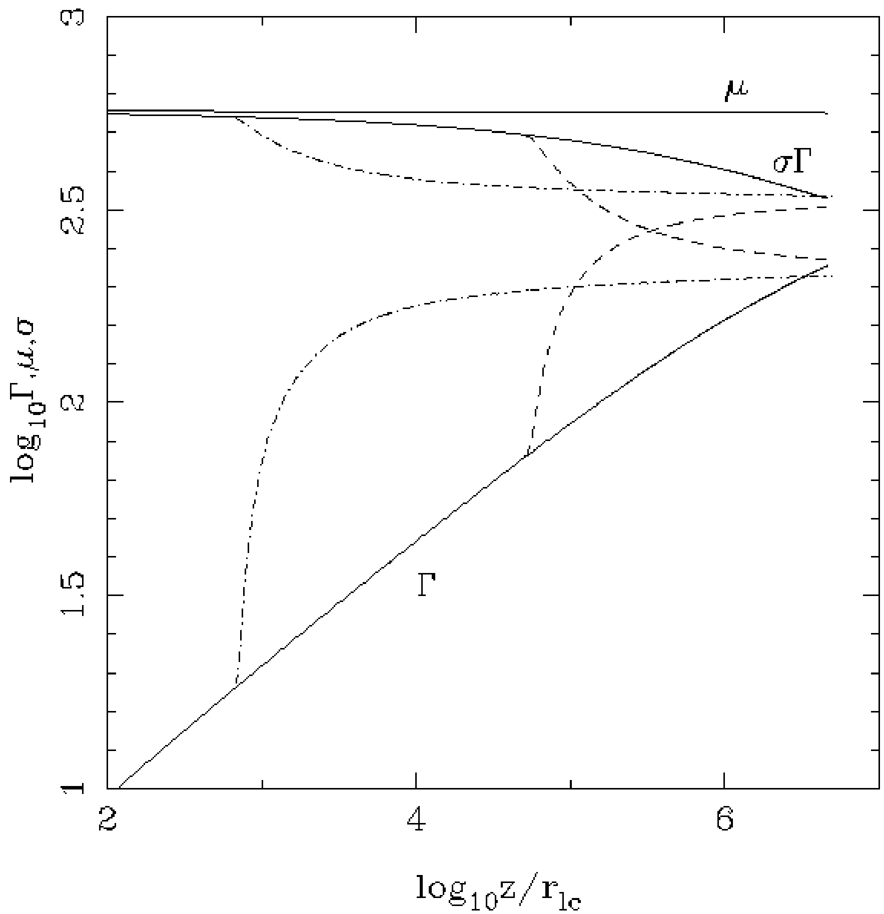
ballistic $\longrightarrow$ loss of causal cannection across the jet
similar to simulations of unconfined winds by Bogovalov 2001

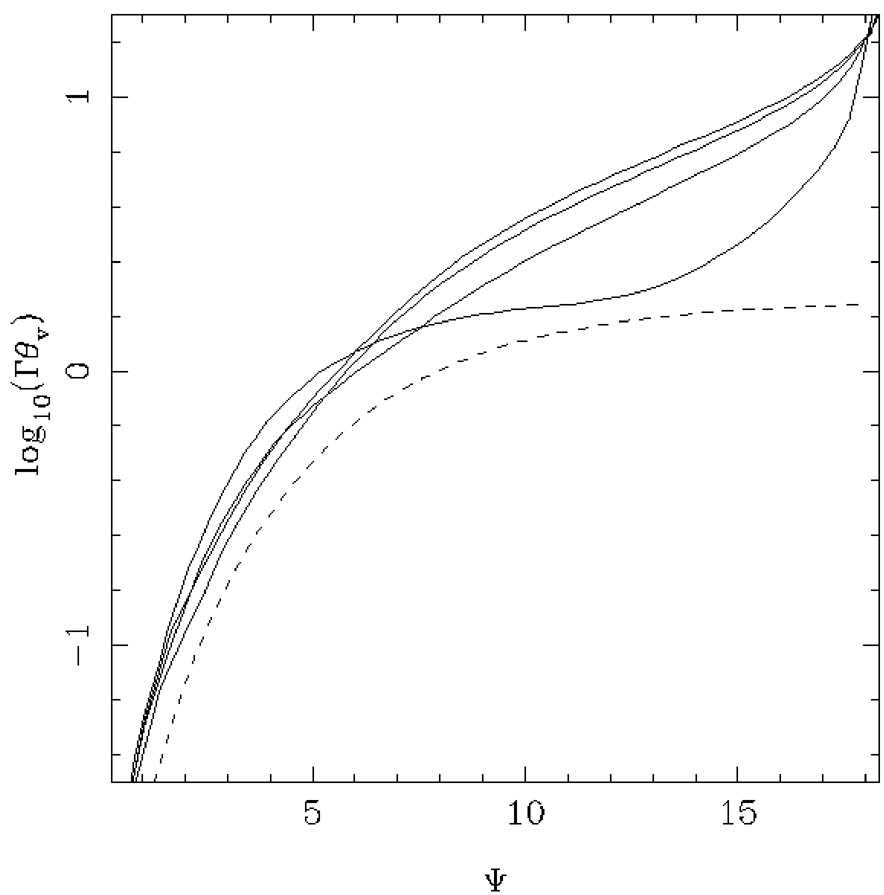
- But it works in the superfast regime
(Tchekhovskoy, Narayan & McKinney 2009)

Rarefaction acceleration

Komissarov, Vlahakis & Königl 2010



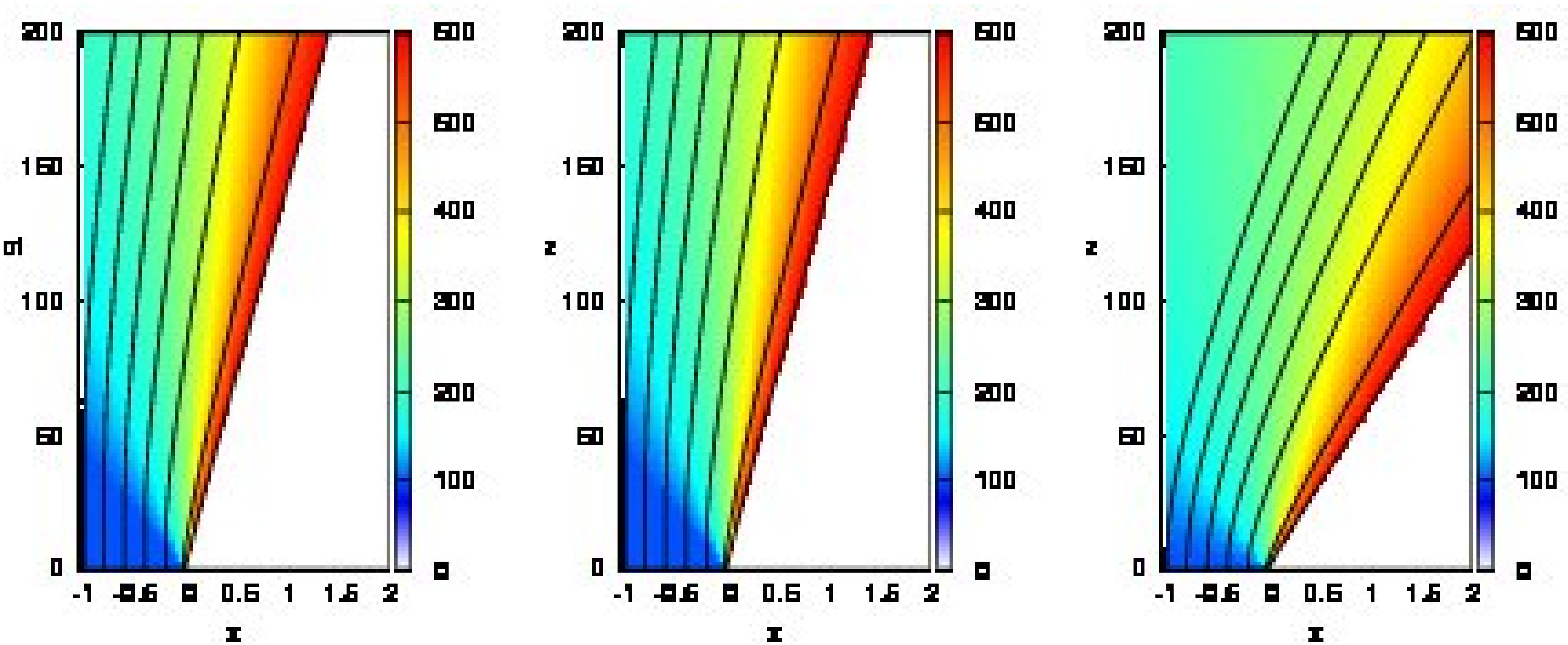




(application to GRB jets)

Steady-state rarefaction wave

Sapountzis & Vlahakis 2010



left: time-dependent rarefaction

middle: steady-state rarefaction

right: combination of rarefaction and nonuniform initial flow

Summary

- ⋆ The collimation-acceleration paradigm provides a viable explanation of the dynamics of relativistic jets
  (similarly to non-relativistic ones)
  - depending on the external pressure:
    collimation to parabolic shape $z \propto r^a, a > 2$ with $\Gamma \propto r$,
    parabolic shape $z \propto r^a, 1 < a \leq 2$ with $\Gamma \sim z/r \propto r^{a-1}$,
    or conical shape $z \propto r$ with $\Gamma \propto r$
  - bulk acceleration up to Lorentz factors $\Gamma_\infty \gtrsim 0.5 \frac{\mathcal{E}}{Mc^2}$
- ⋆ Rarefaction acceleration
  - further increases Γ
  - makes GRB jets with $\Gamma\vartheta \gg 1$

On current-driven instabilities

The jet is expected to be unstable if the azimuthal magnetic field dominates the poloidal magnetic field (Kruskal-Shafranov).

In source's frame $\frac{|B_\phi|}{B_p} \approx \frac{r}{r_{\rm lc}} \gg 1$ — role of inertia?

In the comoving frame $\left(\frac{|B_\phi|}{B_p}\right)_{\rm co} \approx \frac{|B_\phi|/\Gamma}{B_p} \approx \frac{r/r_{\rm lc}}{\Gamma}$

In the power-law regime ($\Gamma \ll r/r_{\rm lc}$) the azimuthal component dominates (unstable)

In the linear acceleration regime ($\Gamma \approx r/r_{\rm lc}$) azimuthal and poloidal components of the magnetic field are comparable

Linear stability analysis

Equilibrium: For $0 < r < r_j$ (jet), $V = 0$ (comoving frame),

$$B_z = \frac{B_j}{1 + (r/r_0)^2},\ B_\phi = \frac{r}{r_0} B_z,\ \rho = \frac{\rho_j}{\left[1 + (r/r_0)^2\right]^2},\ P = 0 \text{ (cold)}.$$

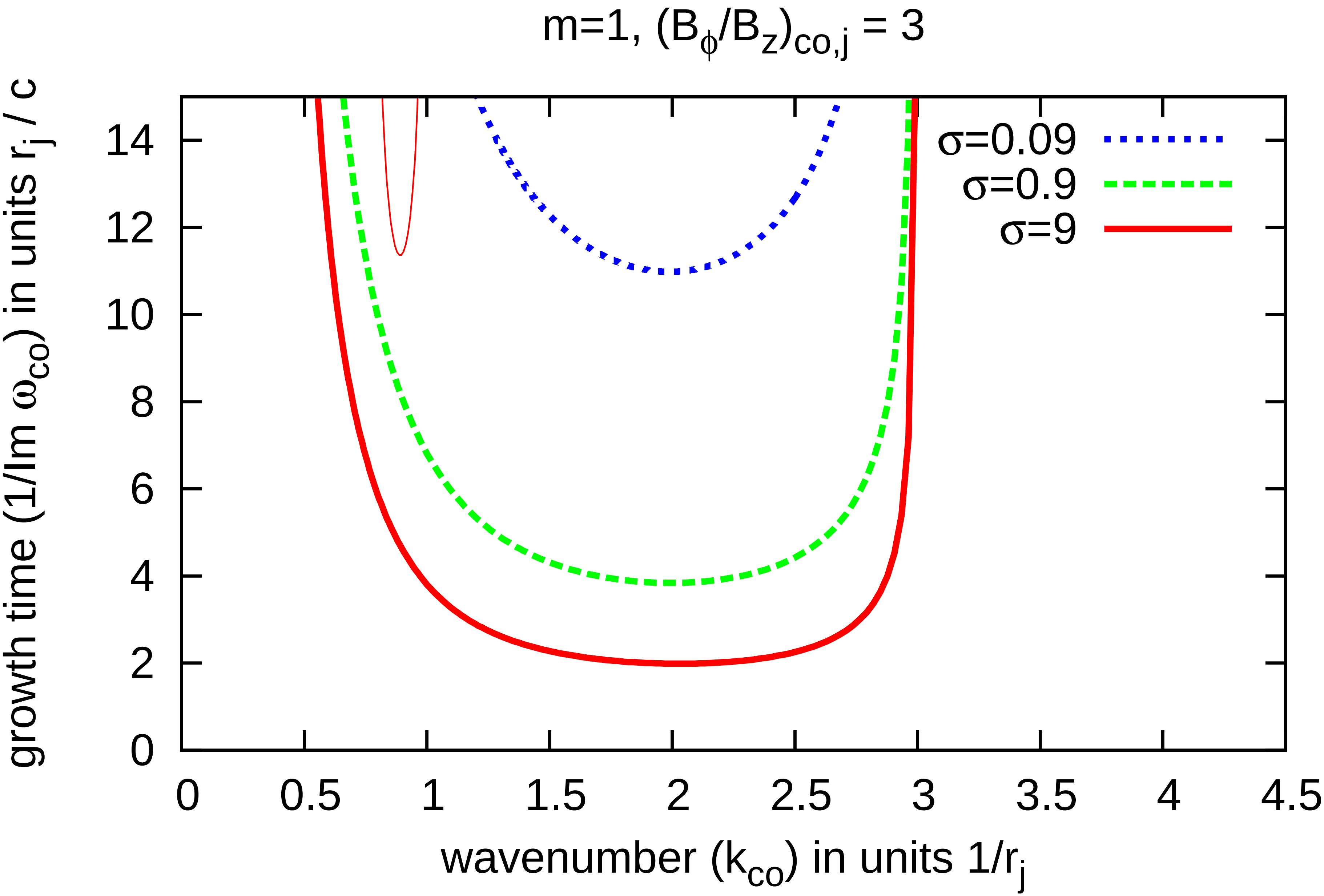
Magnetization $\sigma = \left(\dfrac{B_\phi^2}{4\pi \rho c^2} \right)_{r=r_j}$.

For $r > r_j$ (environment), pressure $p_{\rm ext}$.

Perturbations of the form $f(r) \exp\left[i\left(m\phi + kz - \omega t\right)\right]$.

We linearize the system of RMHD eqs and find $\omega = Re\omega + iIm\omega$ for given k and m.

$1/Im\omega$ is the growth time of the instability.



(thin red line for $(B_\phi/B_z)_{\rm co} = 1$)

In the source's frame growth time is Γ times larger.

Summary

⋆ current-driven instabilities depend on the spatial scale of the Lorentz factor (and thus, on the $p_{\rm ext}$)

- stable jet if acceleration is linear $\Gamma \propto r$ ($p_{\rm ext}$ drops slower than z^{-2}, or initial phase of jets with $p_{\rm ext} \propto z^{-2}$) (becomes unstable when Γ saturates)
- unstable in the power-law acceleration regime (end-phase of jets with $p_{\rm ext} \propto z^{-2}$)