Magnetically driven relativistic jets

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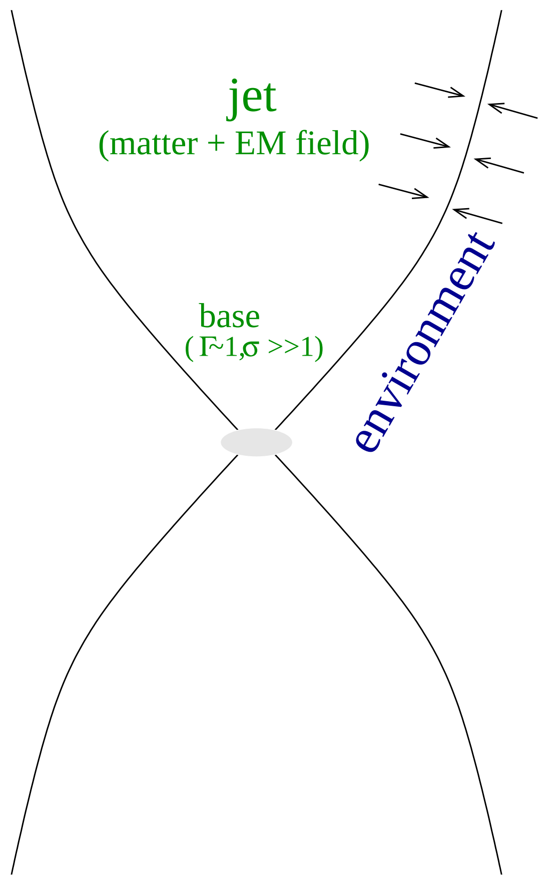

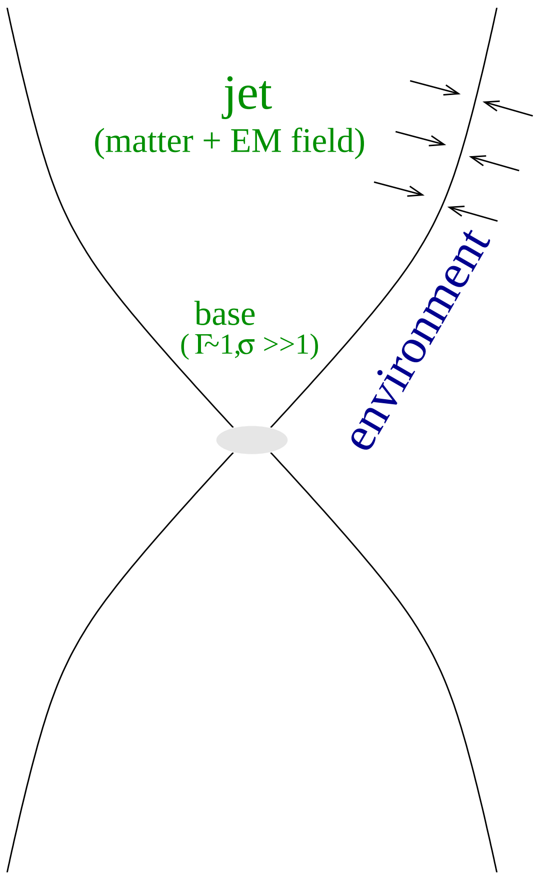

- Extracted energy per time $\dot{\mathcal{E}}$ mainly in the form of Poynting flux (magnetic fields tap the rotational energy of the compact object or disk)

$$\dot{\mathcal{E}} = \frac{c}{4\pi}\,\underbrace{\frac{r}{r_{\rm lc}} B_p}_{E}\, B_\phi \times (\text{ area }) \approx \frac{c}{2} B^2 r^2$$

- Ejected mass per time $\dot{M}$

- The $\mu \equiv \dot{\mathcal{E}}/\dot{M}c^2$ gives the maximum possible bulk Lorentz factor of the flow

- Magnetohydrodynamics: matter (velocity, density, pressure) + large scale electromagnetic field

Numerical simulations

Komissarov, Vlahakis, Königl & Barkov

Assumptions:

- only jet (given boundary conditions at base)
- ideal MHD
- axisymmetry
- cold (not always, but focus on magnetic effects)
- given wall shape (avoid interaction with environment)

Input:

magnetized plasma of a given magnetization (given $\mu = \dot{\mathcal{E}}/\dot{M}c^2$) is ejected into a funnel of a given shape

(use elliptic coordinates — cut the superfast part into sectors)

Output:

- ☞ Γ *vs* distance ? ($\mu = \dot{\mathcal{E}}/\dot{M}c^2 = \underbrace{\dot{\mathcal{E}}_{matter}/\dot{M}c^2}_{\Gamma} + \underbrace{\dot{\mathcal{E}}_{EM}/\dot{M}c^2}_{\Gamma\sigma}$)
- ☞ Γ_∞ and the acceleration efficiency $\dfrac{\Gamma_\infty}{\mu} = \dfrac{\Gamma_\infty \dot{M}c^2}{\dot{\mathcal{E}}} =$?
- ☞ self-collimation (formation of a cylindrical core) ?
- ☞ pressure on the wall ? ($\equiv$ pressure of the jet environment)

Results

Results

...

First what we expect :)

Analytical results

Simplifications using $\Gamma \gg 1$ and $r \gg r_{\rm lc}$ (then $v_\phi/c \ll r/r_{\rm lc}$)
(these are valid in the superfast regime)
(note that at fast $\Gamma \approx \mu^{1/3} \ll \mu$):

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(note that at fast $\Gamma \approx \mu^{1/3} \ll \mu$):

☞ component of the momentum equation along the flow
(wind equation) $\Gamma \approx \mu - \frac{\Psi\Omega^2}{4\pi^2 k c^3}\mathcal{S}$
where the bunching function is $\mathcal{S} = \frac{\pi r^2 B_p}{\int \boldsymbol{B}_p \cdot d\boldsymbol{S}} = \frac{\pi r^2 B_p}{\Psi}$
• acceleration if B_p drops faster than r^{-2}
(monopole flow $\rightarrow$ negligible acceleration)
(prescribed field shape $\rightarrow$ trivial – and incomplete)
• crucial to solve the transfield component of the momentum equation (that controls the shape of the field and thus $\mathcal{S}$)
• role of collimation
• external pressure plays important role

☞ transfield component of the momentum equation

$$\frac{\Gamma^2 r}{\mathcal{R}} \approx \frac{\left(\frac{2I}{\Omega B_p r^2}\right)^2 r \nabla \ln \left|\frac{I}{\Gamma}\right| \cdot \frac{\nabla\Psi}{|\nabla\Psi|}}{1 + \frac{w}{\rho c^2}\frac{4\pi\rho u_p^2}{B_p^2}\frac{r_{\rm lc}^2}{r^2}} - \Gamma^2 \frac{r_{\rm lc}^2}{r^2}\frac{\nabla r \cdot \nabla\Psi}{|\nabla\Psi|},$$

or simply, $\underbrace{\frac{\Gamma^2 r}{\mathcal{R}}}_{inertia} \approx \underbrace{1}_{EM} - \underbrace{\Gamma^2 \frac{r_{\rm lc}^2}{r^2}}_{centrifugal}$

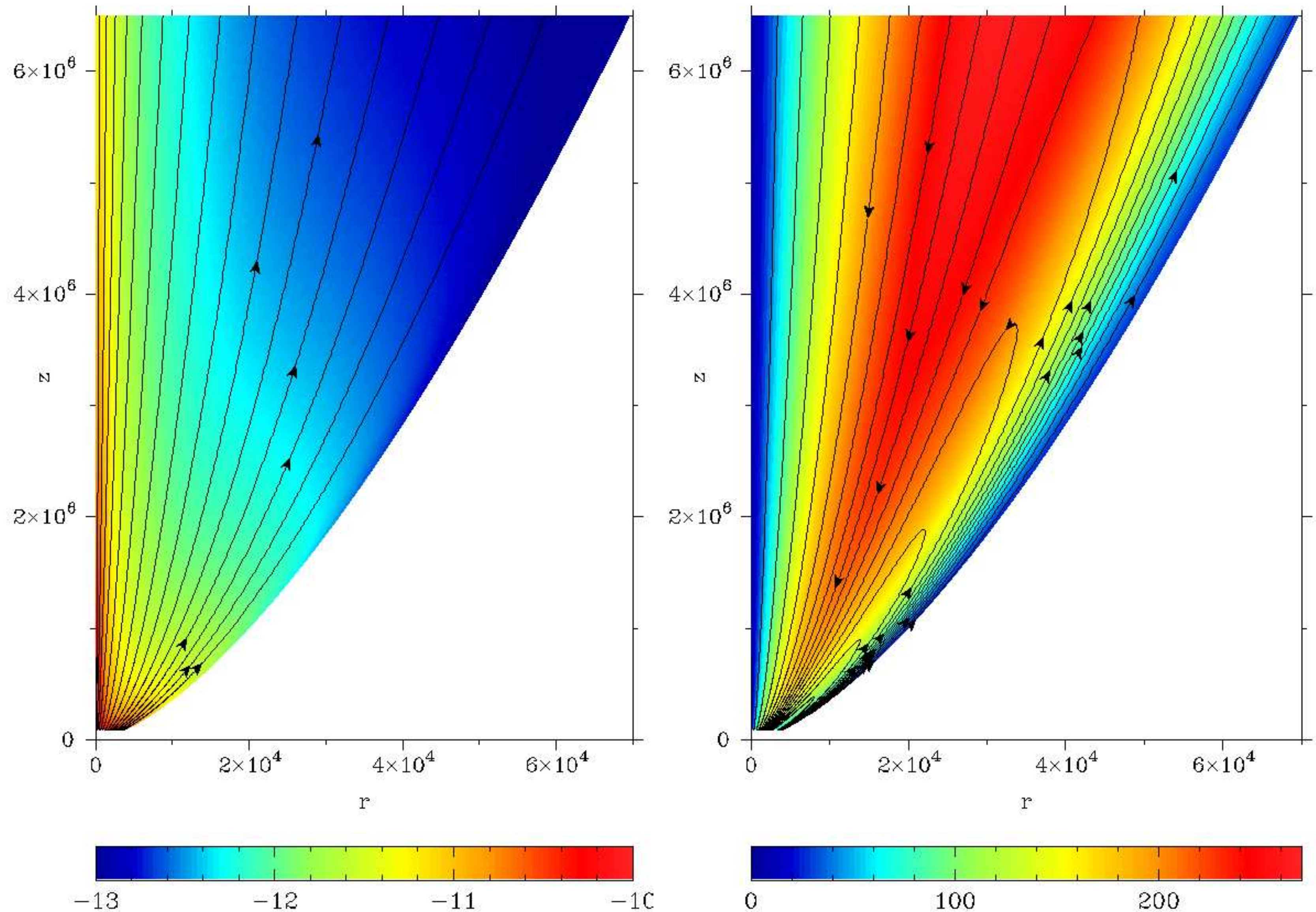
• if centrifugal negligible then $\Gamma \approx z/r$ (since $\mathcal{R}^{-1} \approx -d^2 r/dz^2 \approx r/z^2$) power-law acceleration regime (for parabolic shapes $z \propto r^a$, Γ is a power of r)

• if inetria negligible then $\Gamma \approx r/r_{\rm lc}$ linear acceleration regime

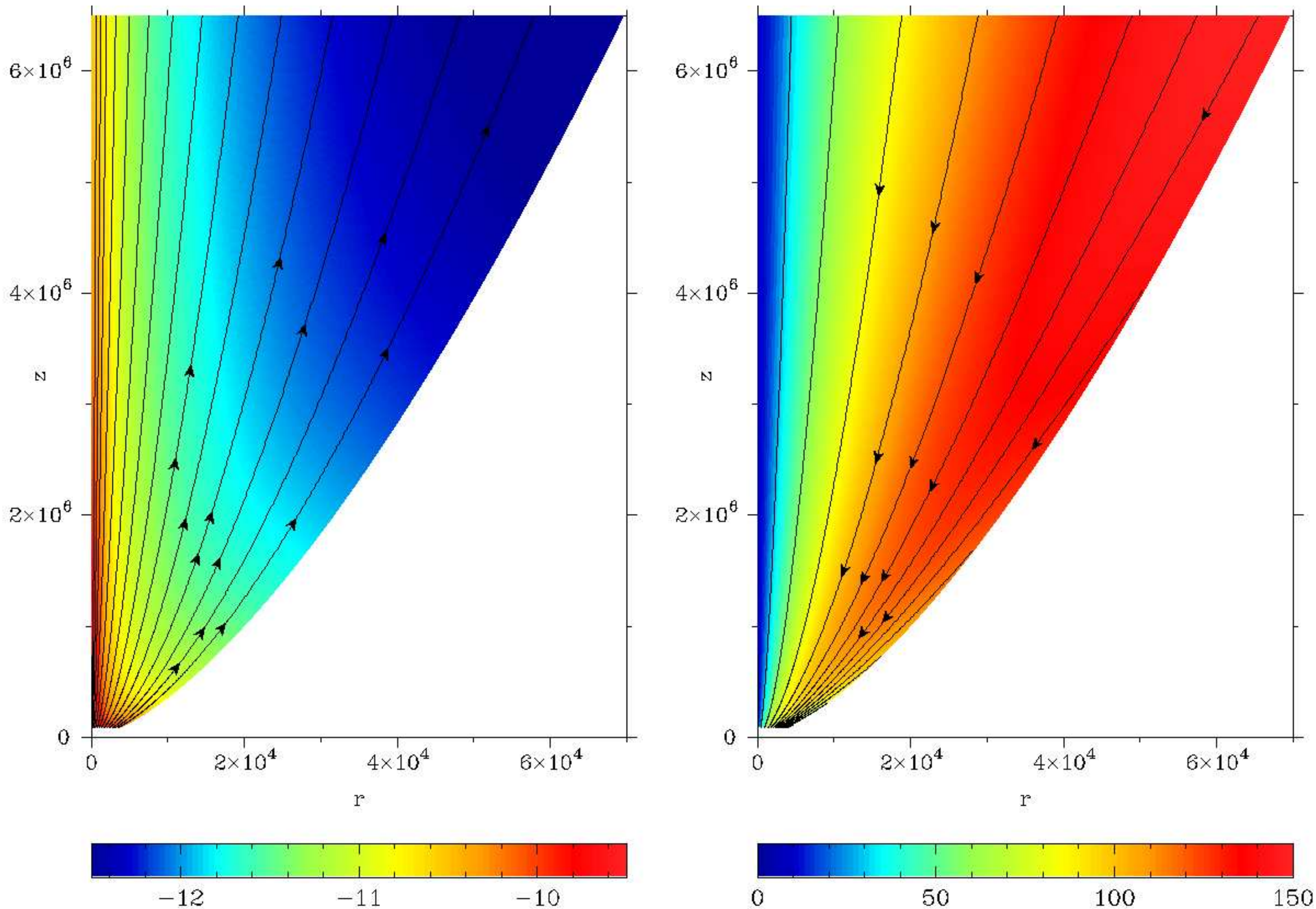
• if electromagnetic negligible then ballistic regime

☞ $p_{\rm ext} = B_{\rm co}^2/8\pi \simeq (B^{\hat\phi})^2/8\pi\Gamma^2 \propto 1/r^2\Gamma^2$
Assuming $p_{\rm ext} \propto z^{-\alpha_p}$ we find $\Gamma^2 \propto z^{\alpha_p}/r^2$.
Combining with the transfield $\frac{\Gamma^2 r}{\mathcal{R}} \approx 1 - \Gamma^2 \frac{r_{\rm lc}^2}{r^2}$ we find the funnel shape (we find the exponent a in $z \propto r^a$).

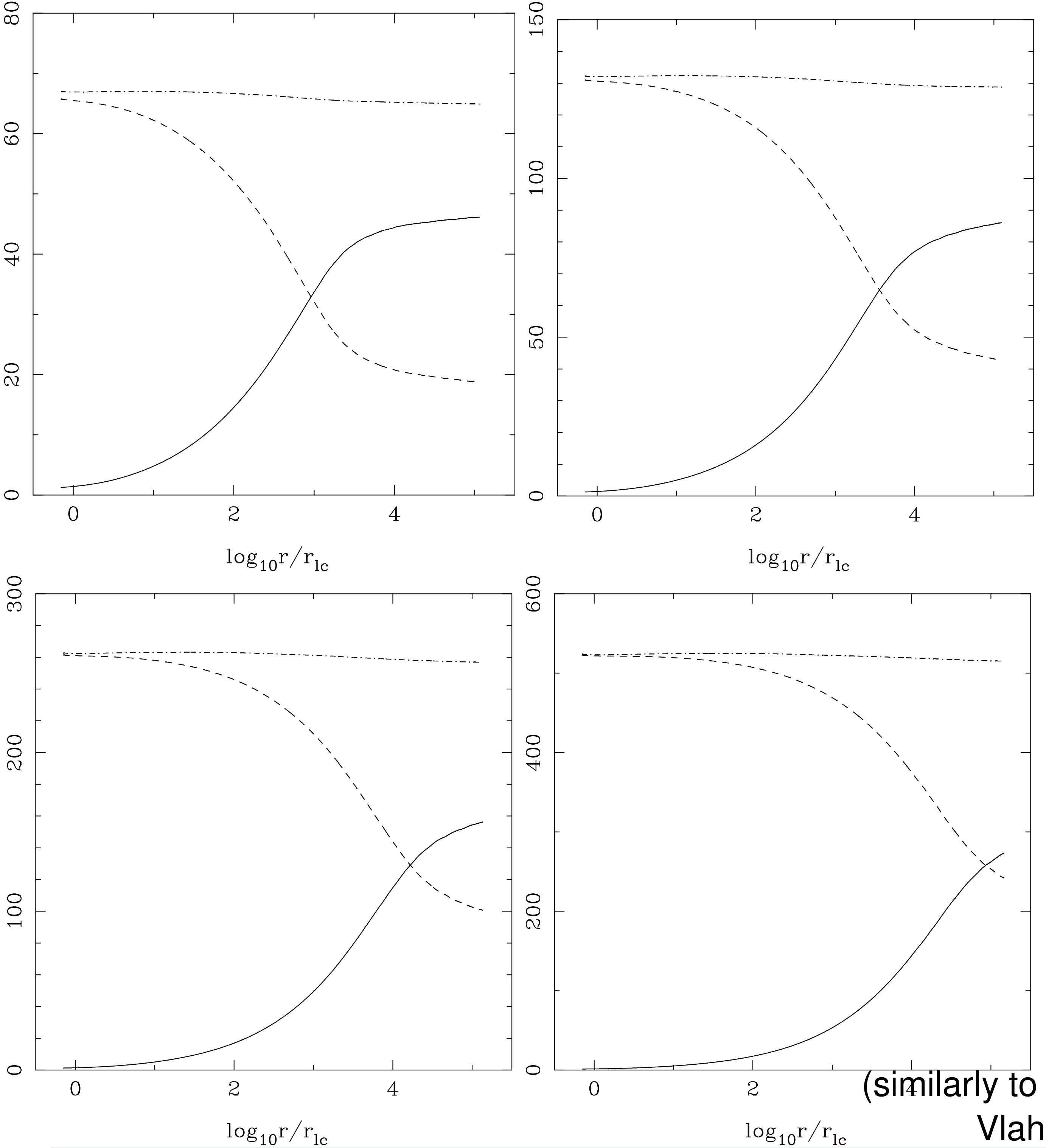
- if $\alpha_p < 2$ (the pressure drops slower than z^{-2}) then
  - ⋆ $a > 2$ (shape more collimated than $z \propto r^2$)
  - ⋆ linear acceleration $\Gamma \propto r$
- if $\alpha_p = 2$ then
  - ⋆ $1 < a \leq 2$ (parabolic shape)
  - ⋆ first $\Gamma \propto r$ and then power-law acceleration $\Gamma \sim z/r \propto r^{a-1}$
- if $\alpha_p > 2$ (pressure drops faster than z^{-2}) then
  - ⋆ $a = 1$ (conical shape)
  - ⋆ linear acceleration $\Gamma \propto r$ (small efficiency)



left: density/field lines, right: Lorentz factor/current lines (wall shape $z \propto r^{1.5}$)
Differential rotation $\rightarrow$ slow envelope



Uniform rotation $\rightarrow \Gamma$ increases with r



Γ (increasing),

$\Gamma\sigma$ (decreasing),

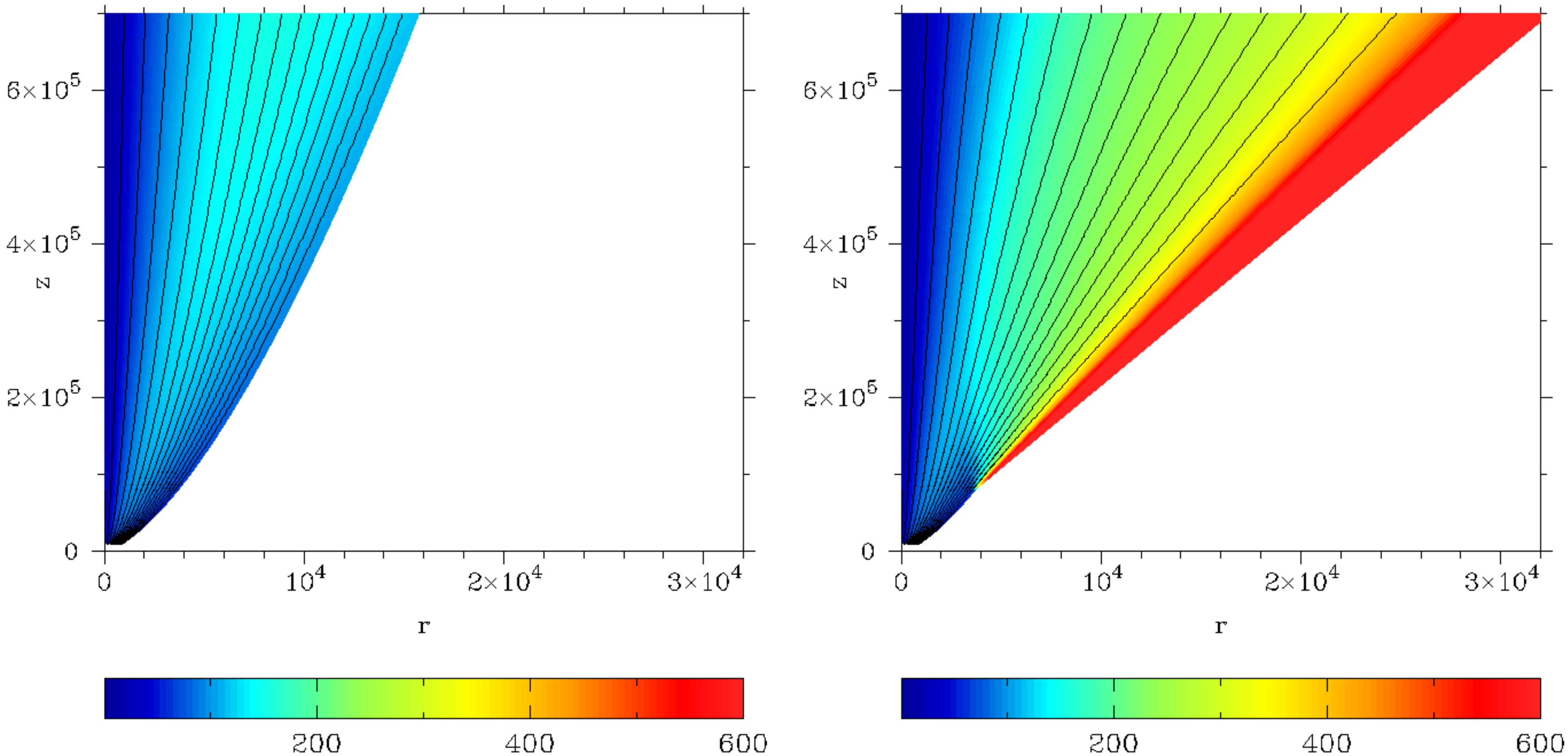
and μ

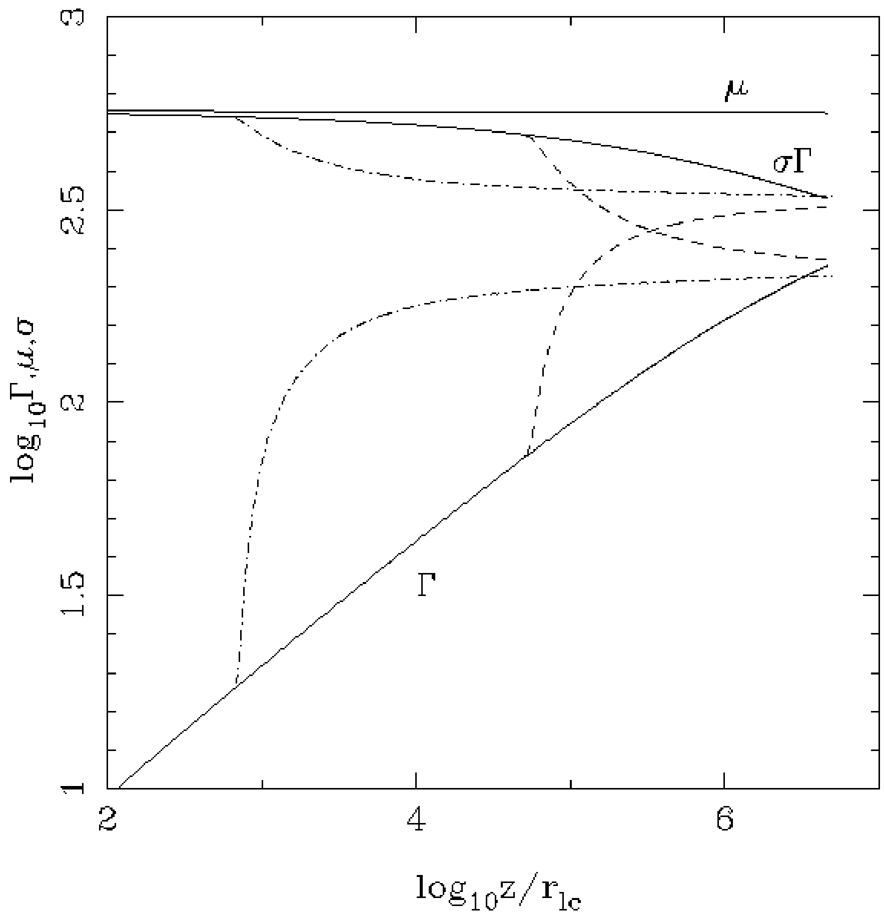
efficiency $> 50\%$

(similarly to the semi-analytical results of Vlahakis & Königl 2003, 2004)

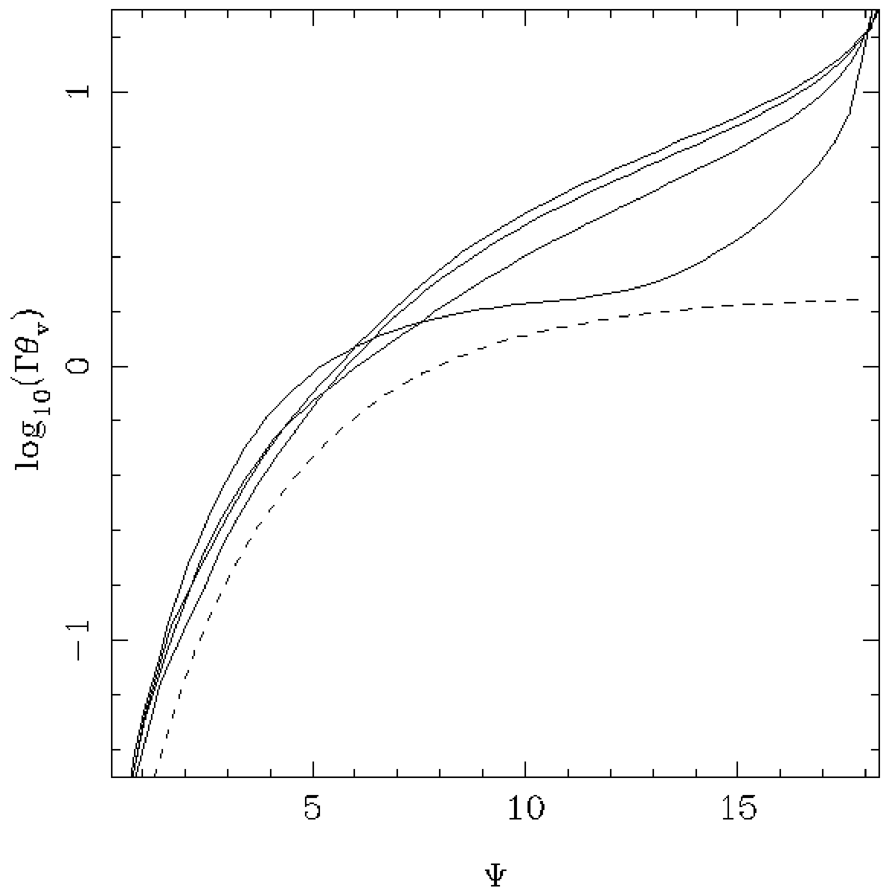
Rarefaction acceleration

Komissarov, Vlahakis & Königl





3
2.5
2
1.5
1
2
4
6
μ
$\sigma\Gamma$
Γ
$\log_{10}\Gamma,\mu,\sigma$
$\log_{10}z/r_{lc}$



1
0
−1
5
10
15
$\log_{10}(\Gamma\theta_v)$
Ψ

Analytics of nonrelativistic flows

In the superfast regime:

$$B_p \approx \frac{2^{1/2}\varpi_{\rm A}^3 \Omega \Psi_A \zeta^{1/2}(1-\zeta)}{\varpi^2}\,, \quad B_\phi \approx -\frac{\varpi_{\rm A}^2 \Omega \Psi_A (1-\zeta)}{\varpi}\,,$$

$$V_p \approx 2^{1/2}\zeta^{1/2}\varpi_{\rm A}\Omega\,, \quad V_\phi \approx \frac{\zeta \varpi_{\rm A}^2 \Omega}{\varpi}\,, \quad \rho \approx \frac{\Psi_A^2 \varpi_{\rm A}^2 (1-\zeta)}{4\pi\varpi^2}\,,$$

$$\zeta \approx \frac{1}{1+2(\varpi/\varpi_{\rm f})^{-2(b-1)}}\,, \quad z \approx z_{\rm f}\left(\frac{\varpi}{\varpi_{\rm f}}\right)^b$$

(ζ is the kinetic-to-total energy flux ratio)

for details see Vlahakis 2009 (in Protostellar Jets in Context, K. Tsinganos, T. Ray, and M. Stute (eds.), ASS Proceedings Series, 205)

On current-driven instabilities

The jet is expected to be unstable if the azimuthal magnetic field dominates the poloidal magnetic field (Kruskal-Shafranov).

In source's frame $\dfrac{|B_\phi|}{B_p} \approx \dfrac{r}{r_{\rm lc}} \gg 1$ — role of inertia?

In the comoving frame $\left(\dfrac{|B_\phi|}{B_p}\right)_{\rm co} \approx \dfrac{|B_\phi|/\Gamma}{B_p} \approx \dfrac{r/r_{\rm lc}}{\Gamma}$

In the power-law regime ($\Gamma \ll r/r_{\rm lc}$) the azimuthal component dominates (unstable)

In the linear acceleration regime ($\Gamma \approx r/r_{\rm lc}$) azimuthal and poloidal components of the magnetic field are comparable

Linear stability analysis

Equilibrium: For $0 < r < r_j$ (jet), $V = 0$ (comoving frame),
$B_z = \dfrac{B_j}{1 + (r/r_0)^2}$, $B_\phi = \dfrac{r}{r_0} B_z$, $\rho = \dfrac{\rho_j}{\left[1 + (r/r_0)^2\right]^2}$, $P = 0$ (cold).

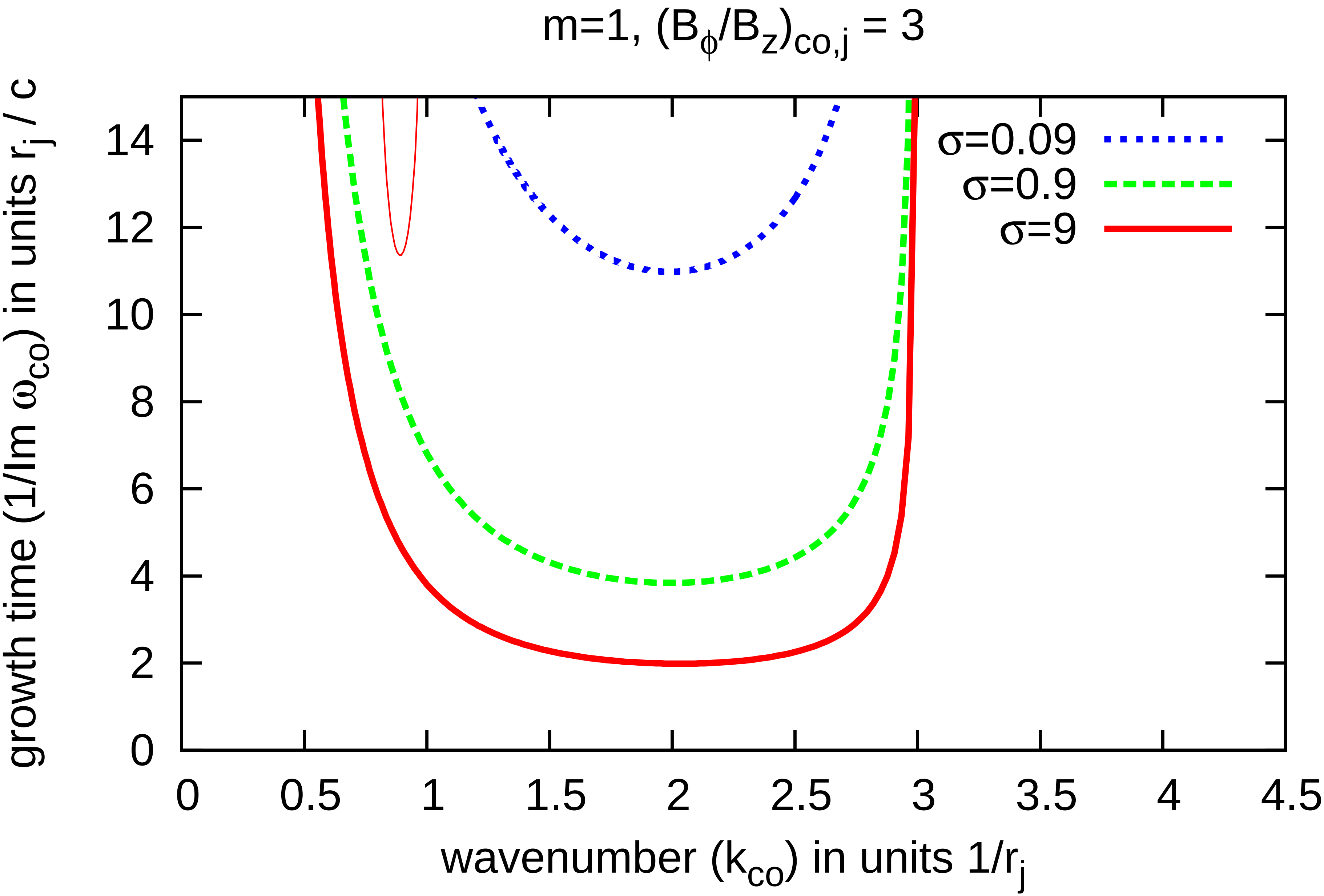
Magnetization $\sigma = \left(\dfrac{B_\phi^2}{4\pi \rho c^2} \right)_{r=r_j}$.

For $r > r_j$ (environment), pressure $p_{\rm ext}$.

Perturbations of the form $f(r) \exp\left[i \left(m\phi + kz - \omega t\right)\right]$.

We linearize the system of RMHD eqs and find $\omega = Re\omega + i Im\omega$ for given k and m.

$1/Im\omega$ is the growth time of the instability.



(thin red line for $(B_\phi/B_z)_{\rm co} = 1$)

In the source's frame growth time is Γ times larger.

Summary

- ⋆ Magnetic driving provides a viable explanation of the dynamics of relativistic jets
  - depending on the external pressure:
    collimation to parabolic shape $z \propto r^a, a > 2$ with $\Gamma \propto r$,
    parabolic shape $z \propto r^a, 1 < a \leq 2$ with $\Gamma \sim z/r \propto r^{a-1}$,
    or conical shape $z \propto r$ with $\Gamma \propto r$
  - bulk acceleration up to Lorentz factors $\Gamma_\infty \gtrsim 0.5 \frac{\mathcal{E}}{Mc^2}$
    (in conical flows only near the axis)

- ⋆ current-driven instabilities depend on the spatial scale of the Lorentz factor (and thus, on the p_{ext})
  - stable jet if acceleration is linear $\Gamma \propto r$ (p_{ext} drops slower than z^{-2}, or initial phase of jets with $p_{\mathrm{ext}} \propto z^{-2}$)
    (becomes unstable when Γ saturates)
  - unstable in the power-law acceleration regime (end-phase of jets with $p_{\mathrm{ext}} \propto z^{-2}$)